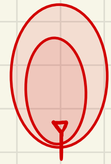
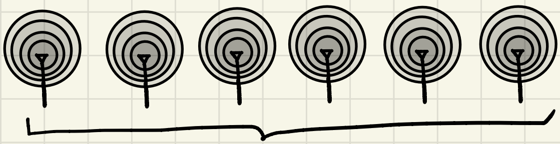
Beamforming and Angle of Arrival (AOA)

- Omnidirectional antennas: radiate signals equally in all directions
  Directional antennas : Direct the radiation more in certain directions and less in others.

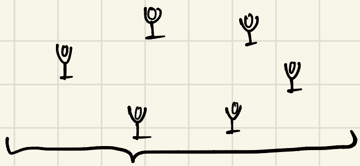


- creating such non-circular radiation patterns ⇒ Beamforming → Spatial Filter
  How ?

- Let's consider an ARRAY of omni-directional antennas (or even microphones)



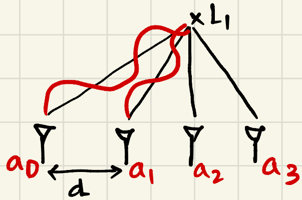
Linear antenna array



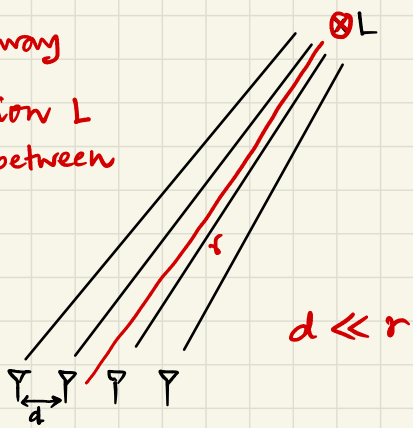
circular mic. array (Alexa)

- say, these antennas transmit all at the same time ?
  ↳ What signals will you receive from different locations ?

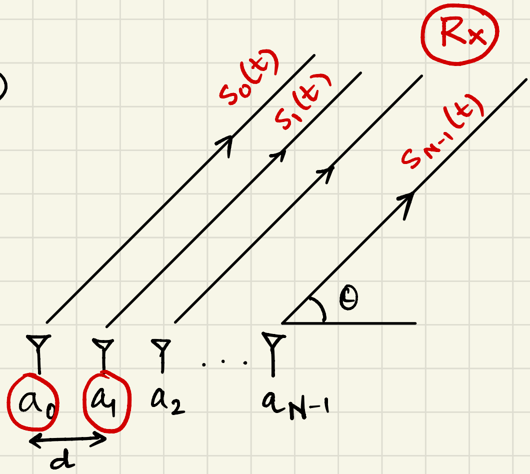
- consider nearby locations first :
  ↳ The aggregate signals at these nearby locations vary based on the location.
  ↳ No pattern is visible as you move.
  ↳ This is called " NEAR FIELD ".



- Now, consider locations that are far away
  ↳ When distance from antennas to location L becomes ≫ than seperation 'd' between the antennas , then the signal paths almost become PARALLEL
  ↳ called "FAR FIELD"
  ↳ Let's analyze far field effects



(→)



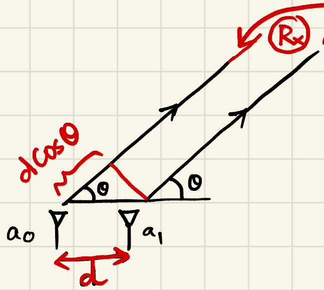
- All antennas transmit same signal.
- Say Rx receives $s_0(t)$ from antenna a_0 ... and $s_i(t)$ from antenna a_i
- Received signal $y(t)$

$$y(t) = \sum_{i=0}^{N-1} s_i(t)$$

(5) Now, assume LoS path (no echo or multipath).

↳ Then what is the difference between $s_0(t)$ and $s_1(t)$?

Ans :



$s_1(t)$ travels $d\cos\theta$ less distance than $s_0(t)$.

⇓

How much phase shift ϕ does this cause?

$c = f\lambda$

λ distance causes 2π phase shift

$\therefore$ $d\cos\theta$ distance causes $\frac{2\pi}{\lambda} d\cos\theta$ phase shift

$s_0(t)$ = Hello how are you ⇒

$s_1(t)$ = $\underbrace{\text{Hello}}_{\frac{2\pi}{\lambda} d\cos\theta}$ how are ⇓ you ⇒

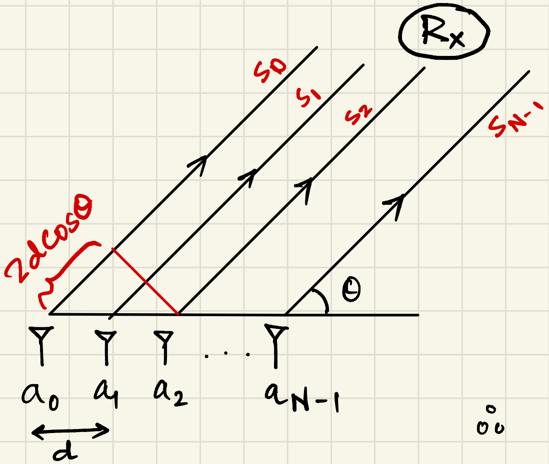
How can we mathematically write that $s_1(t)$ = $s_0(t)$ phase shifted by $\phi = \frac{2\pi}{\lambda} d\cos\theta$

↳ Recall phase shifting ≡ shifting samples of the signal

Thus :

$\underline{s_0(t) = \cos(2\pi f_1 t)}$ $\qquad s_1(t) = \cos(2\pi f_1 t + \phi)$

$$\therefore \boxed{s_1(f) = s_0(f)\, e^{j\phi}}$$



$$s_0 = s_0 e^{j0}$$

$$s_1 = s_0 e^{j\phi} \quad , \quad \phi = \frac{2\pi}{\lambda} d\cos\theta$$

$$s_2 = s_0 e^{j2\phi}$$

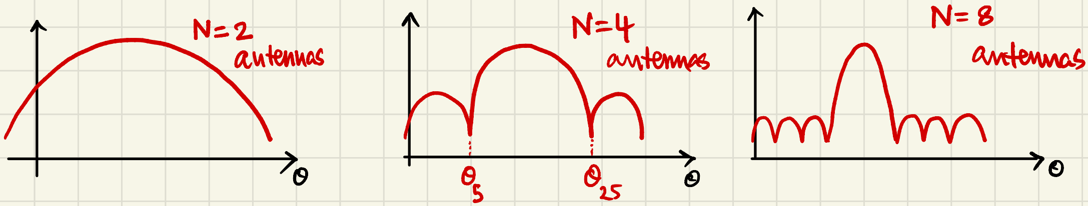
$$\vdots$$

$$s_{N-1} = s_0 e^{j(N-1)\phi}$$

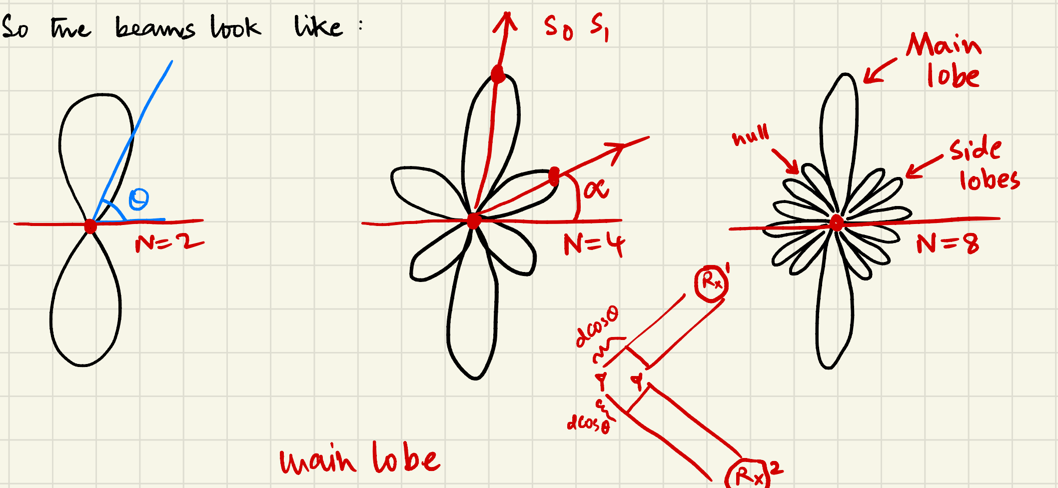
$$\therefore \quad Y = \underbrace{\sum_{k=0}^{N-1} s_0 e^{jk\phi}}$$

$$\cong \underline{s_0\left(\frac{1 - e^{jN\phi}}{1 - e^{j\phi}}\right)} \quad , \quad \phi = \frac{2\pi}{\lambda} d\cos\theta$$

→ Plot Y_f or y_t against θ



→ So the beams look like :



→ Observe, the natural main lobe beam is pointing towards front & back

⊙ Beam Rotation

Now I want the main lobe to point towards my intended direction θ

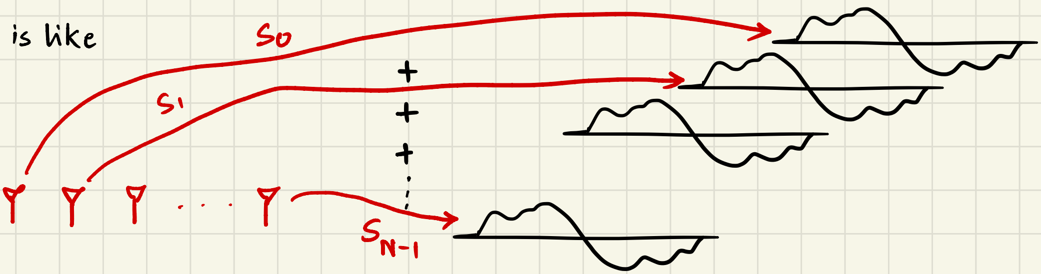
↳ i.e., maximize signal power towards θ.

↳ How? By making signals from all antennas add up "coherently" or "constructively" in the direction θ.

So, first let's see how signals add up along θ

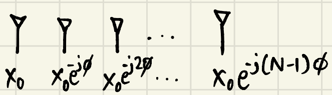
Recall $Y = s_0 + s_0 e^{j\phi} + s_0 e^{j2\phi} + \ldots s_0 e^{j(N-1)\phi}$

This is like



⊙ For max SNR at R_x, phase shift each s_i to compensate for path delay it would experience.

i.e., $[\, x_0 \quad x_0 e^{-j\phi} \quad x_0 e^{-j2\phi} \quad \ldots \quad x_0 e^{-j(N-1)\phi} \,]$



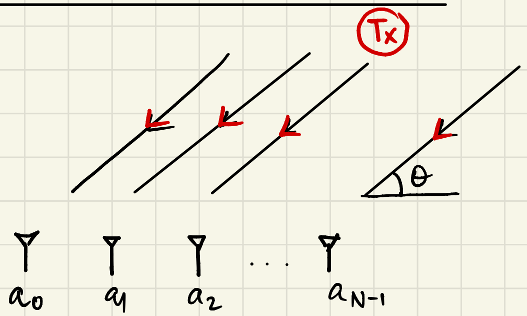
$$\therefore \; Y = \sum_{K=0}^{N-1} \left(s_0 e^{-jK\phi}\right) e^{jK\phi}$$

$$\therefore \; Y = \sum_{K=0}^{N-1} s_0 = N s_0$$

This is called "DELAY-SUM BEAMFORMING".

⊙ Analogy: Stagger runners at the starting line (like phase shifts at tx antennas) to ensure they all run the same distance and finish at the same time (i.e., signals add up coherently).

ANGLE OF ARRIVAL (AOA)



Signal arriving from far field

Antenna array needs to figure out the angle of arrival (Θ) = AoA
Dir. of arrival (DoA).

How can you estimate AoA ? Well, similar concepts as beamforming

- Say received signal is now

$$y_N = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} \cos 2\pi f_1 t \\ \cos(2\pi f_1 t + \phi) \\ \vdots \\ \cos(2\pi f_1 t + (N-1)\phi) \end{bmatrix} \xrightarrow{\text{Freq.}} X(f) \times \begin{bmatrix} e^{j0} \\ e^{j\phi} \\ e^{j2\phi} \\ \vdots \\ e^{j(N-1)\phi} \end{bmatrix}$$

a_0 a_1 . . . a_{N-1}

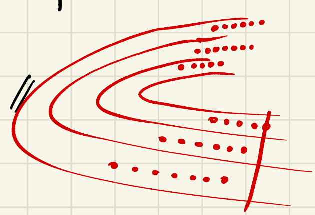
- From this received vector, how do you detect Θ ?
  - Answer: Delay and sum
  - Algorithm :

for $\theta_i = -\pi$ to π // search over all AoA Θ

{

$\alpha_i = \frac{2\pi}{\lambda} d \cos\theta_i$ // calculate phase shift

$$C_{\theta_i} = \left[e^{j0} \; e^{-j\alpha_i} \; e^{-j2\alpha_i} \ldots e^{-j(N-1)\alpha_i} \right] \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-1} \end{bmatrix}$$ //



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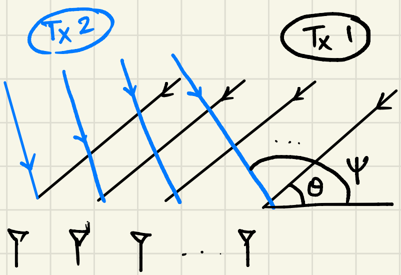
Plot (C_{θ_i} , θ_i) // Plot the AoA spectrum

$$AoA = \theta^*$$

- Now, let's assume multiple transmitters sending in parallel.
  ↳ can we still decode the AoAs ?



say $\phi_1 = \frac{2\pi}{\lambda} d\cos\theta$

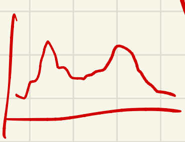
$\phi_2 = \frac{2\pi}{\lambda} d\cos\psi$

$$\underbrace{\begin{bmatrix} Y_0 \\ Y_1 \\ Y_2 \\ \vdots \\ Y_{N-1} \end{bmatrix}}_{\text{Array output}} = \underbrace{\begin{bmatrix} e^{j0} & e^{j0} \\ e^{j\phi_1} & e^{j\phi_2} \\ e^{j2\phi_1} & e^{j2\phi_2} \\ \vdots & \vdots \\ e^{j(N-1)\phi_1} & e^{j(N-1)\phi_2} \end{bmatrix}}_{\text{Steering matrix}} \underbrace{\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}}_{\text{source signal}}$$

$$\bar{Y} = A\bar{s}$$

- Now how can you decode θ and ψ (psi) for $\begin{bmatrix} Y_0 \\ Y_1 \\ \vdots \\ Y_{N-1} \end{bmatrix}$
  ↳ Answer: Looking for a certain phase pattern ⟶ so correlate

$$\begin{bmatrix} e^{j0} & e^{-j\phi_1} & e^{-j2\phi_1} & \cdots & e^{-j(N-1)\phi_1} \end{bmatrix} \begin{bmatrix} Y_0 \\ Y_1 \\ \vdots \\ Y_{N-1} \end{bmatrix} = \begin{bmatrix} e^{j0} & e^{-j\phi_1} & \cdots & e^{-j(N-1)\phi_1} \end{bmatrix} \begin{bmatrix} e^{j0} & e^{j0} \\ e^{j\phi_1} & e^{j\phi_2} \\ e^{j2\phi_1} & e^{j2\phi_2} \\ \vdots & \vdots \\ e^{j(N-1)\phi_1} & e^{j(N-1)\phi_2} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$



Perform this for all values of $\phi_i \in [-\pi, \pi]$

Hope dot product large when AoA phase matches ϕ_i.

⊛ Modelling noise $\bar{Y} = A\bar{s} + \bar{n}$ ← noise vector

$$\begin{bmatrix} Y_0 \\ Y_1 \\ \vdots \\ Y_{N-1} \end{bmatrix} = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_d \\ | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{N-1} \end{bmatrix}$$

$$\begin{bmatrix} - a_i - \end{bmatrix}^{*} \begin{bmatrix} Y_0 \\ Y_1 \\ \vdots \\ Y_{N-1} \end{bmatrix} = \begin{bmatrix} - a_i - \end{bmatrix}^{*} \begin{bmatrix} | & | & & | \\ a_1 & a_i & \cdots & a_d \\ | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix} + \begin{bmatrix} - a_i - \end{bmatrix}^{*} \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{N-1} \end{bmatrix}$$

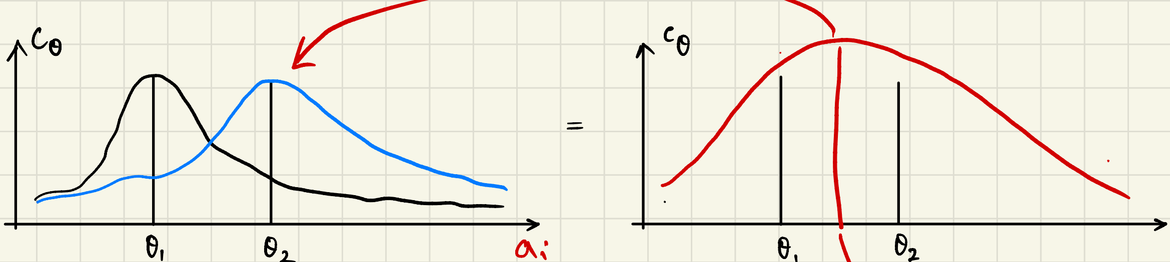
correlating for

$$= \underbrace{\|a_i\|^2 x_1}_{\substack{\text{positive} \\ \text{and large} \\ \text{(coherent)}}} + \underbrace{\sum_{j=0,\, j\neq i}^{N-1} a_i^{*} a_j x_j}_{\substack{\text{hopefully} \\ \text{small} \\ \text{(incoherent)}}} + \underbrace{a_i^{*} n}_{\substack{\text{hopefully} \\ \text{small} \\ \text{(incoherent)}}}$$

⊛ By correlating along all directions a_i, we get an "AoA spectrum"

⊛ Problem is

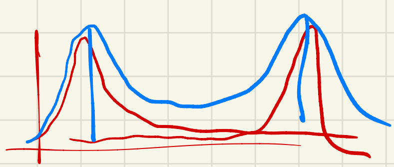
Σ_i black + blue.

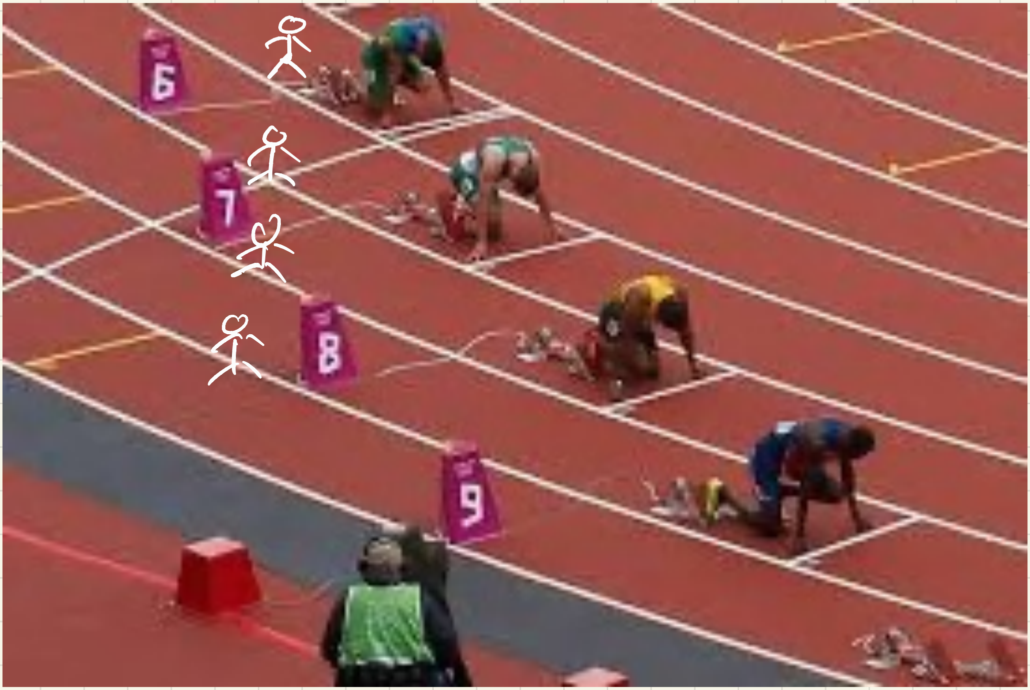


wrong θ

Peak is

Especially when signals are

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