

ECE/CS 434 : Mobile Computing Algorithms and Applications :  
Homework 3 : Due 11 :59pm, Thu, Apr 22, 2021

**Problem 1 : State True/False with 1 line justifications [10x2=20 points]**

- a. If the clocks on GPS satellites are all asynchronous, we could model them as additional unknowns and still perform trilateration to acquire a GPS fix.
- b. A signal that is below the noise floor (i.e., amplitude of the signal is less than the noise amplitude) cannot be detected through cross-correlation.
- c. Basic trilateration is a linear operation because distance measurements are linear.
- d. Imagine a transmitter and a receiver pair, separated by a distance  $d$ . Say the received power is  $P_R$  when there is only a line of sight path. Under multipath environments, it is possible for the received power to be greater than  $P_R$  even if the transmit power is unchanged.
- e. The amplitude of the straight-line path is always stronger than the echoes (or multipath).
- f. Echoes of the same signal source, when arriving at the receiver, are correlated.
- g. A car has a magical IMU in it that has no noise in its measurement. The car is driving and the driver activates the IMU. The car should be able to accurately dead-reckon itself as long as it is always in contact with the ground.
- h.  $P(A) = \sum_B P(A|B)P(B)$ 
  - i. Order of rotations does not matter. That is :  $R_x(R_y(R_z)) \equiv R_y(R_z(R_x))$ .
  - j. In active noise cancellation (ANC), the DSP processor must continuously adapt the weight vector  $W = [w_1 \ w_2 \ \dots w_n]^T$  based on the variation of the source signal.

**Problem 2 : Channel, Convolution, and Packets [4x5=20 points]**

- (a) Draw an ideal Channel Impulse Response (CIR) where the channel only has one direct path (the path with the shortest distance) and one echo arriving 10 samples later. Label the diagram clearly.
- (b) Draw an updated CIR with direct path signal + 3 equally spaced echoes, in which the direct path signal travels through a wall. The first two echoes bounce off the wall, and the final echo bounces off the wall and another surface, before arriving at the receiver. Assume amplitudes get attenuated to 0.5 due to penetration and 0.7 due to each reflection. Label the diagram carefully.
- (c) Say that a source signal  $X[N]$  of length  $N$  travels through a  $K$ -tap channel (i.e., there are  $K$  taps including the LoS), and the received signal is  $Y[N]$ . Assuming  $N > K$ , write the channel matrix  $H$ .  
Hint : *Matrix  $H$  should have a good number of zeroes in it, and should satisfy  $Y = HX$ .*
- (d) We know that  $y = h * x + n$ , where “\*” denotes convolution,  $n$  denotes noise,  $x$  is the data packet being sent, and  $y$  being the received signal. Write out the basic steps that the receiver of  $y$  would perform to decode this packet (assuming the packet is carrying a preamble in its header).

### Problem 3 : Beamforming, Triangulation [5x3 + 1x5=20 points]

- (a) Write one similarity and one difference between the steering matrix and the Fourier matrix.
- (b) The received signal at a microphone array is written as  $y = As + n$ , where  $y$  is the received signal vector (of dimension  $M \times 1$  where  $M$  is the number of microphones),  $A$  is the steering matrix,  $s$  are the signal sources, and  $n$  is the noise vector. Write out this equation by showing all the vectors and matrices (assume  $K < M$  different sound sources, and assume only line-of-sight path for each of them). For example, an equation showing all vectors and matrices in  $A \times x = b$  would look like :

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k \\ l \end{bmatrix}$$

- (c) For the same problem above, assume there is only a single source but  $K < M$  echoes. Now write the same equation showing all the vectors and matrices.
- (d) Explain the Delay-Sum algorithm using the equation in part (c) above.
- (e) True/False : The accuracy of the Delay-Sum Algorithm is expected to improve with more microphones as receivers. Answer with a mathematical justification.
- (f) You are performing WiFi triangulation and you have computed the AoA spectrum at 3 WiFi access points. What is the correct way to estimate the location of the user from these AoA spectrums. Hint : *If you are thinking of picking the best AoA angle from each AoA spectrum and computing intersections of these lines, then think again. See what ArrayTrack does.*

### Problem 4 : IMU and Localization [5x4=20 points]

- (a) Argue in favor or against (in 2 sentences or less) :  
If gravity was not pointing vertically downwards, but instead at some angle  $\theta$  with respect to the vertical direction, then we could estimate the IMU orientation without the magnetometer measurement.
- (b) Argue in favor or against (in 2 sentences or less) :  
Assume the noise in an accelerometer is zero-mean. If we double-integrate the accelerometer for a long duration, the dead reckoned trajectory would be correct.
- (c) Assume that the magnitude of acceleration due to gravity is  $g$  and the magnitude of the earth's magnetic field is  $m$ . Also assume that we are at the equator. For a mobile phone assume x-axis is parallel to the phone's width (shorter side), y-axis is parallel to the phone's length (longer side), z-axis penetrates through the phone's glass surface.
- A static mobile phone is supported on a table and shows accelerometer reading :  $[0, 1, 0]g$ . Does this represent the complete orientation of the phone? If yes, what is the orientation matrix. If no, give a reason why not.
  - The above phone is showing a magnetometer reading of  $[1, 0, 0]m$ . Is this possible? What is the orientation matrix for the phone?
  - A static mobile phone is reporting the following measurements on its accelerometer :  $[-0.3789, 0.2775, 0.8828]g$ . It is reporting the following magnetometer readings :  $[0.5829, 0.8125, -0.0053]m$ . What is its orientation matrix?

- (d) Complete the following sentences :
- i. In UnLoc, the order in which the landmark locations are estimated depends on ...
  - ii. In UnLoc, the landmark dead-reckoning error are uncorrelated across different users for the following reasons (mention at least two) :
  - iii. In UnLoc, the landmark dead-reckoning error could be correlated across different users for the following reason :
- (e) List one plausible UnLoc landmark for each sensor below (a WiFi example shown below) :
- *WiFi* : a small spot at which only a specific combination of WiFi APs are audible
  - Accelerometer :
  - Gyroscope :
  - Magnetometer :
  - Microphone :
  - 4G :
  - Bluetooth :

### Problem 5 : Probability, HMM, and Applications [3,4,4,2,4,3 points]

- (a) Suppose random variables A and B are conditionally independent, given C. This means  $P(A, B|C) = P(A|C)P(B|C)$ . One real life example of such conditional independence is the following : people's height and vocabulary are not independent, but they are conditionally independent given that people are from the same class. Can you give one more example.
- (b) Three universities (A, B, C) have 100, 200, 300 students respectively, with 50, 75, and 100 of those students interested in mobile computing. Google randomly picks a university with equal probabilities (there is a 1/3 chance of selecting either A, B or C), and then draws a student's name at random from that university. The student proves to be interested in mobile computing. What is the probability the student comes from university C?
- (c) In a HMM, prove that  $P(s_k|m_{1:n}) = P(s_k|m_{1:k})P(m_{k+1:n}|s_k)$ .
- (d) Complete the sentence :  
When performing HMM for IMU based dead-reckoning,  $P(m_k|s_{1:k}) \neq P(m_k|s_k)$  because ...
- (e) Imagine running an HMM for handwriting recognition. The state variable  $s$  takes on values "A, B, C, ... Z" and we have measurements from people's handwriting, say  $m$ . Describe an example cost function,  $P(m|s)$  in plain language, and then express the same as a mathematical equation.  
Hint : *Treat each known state, say "A", as an image composed of black or white pixels.*
- (f) ArmTrak models the point cloud of the elbow for a given wrist orientation. Describe an arm posture that produces the smallest point cloud (feel free to draw the arm posture).

### Problem 6 : Optimization and Acoustics [5x3 + 1x5=20 points]

- (a) Consider  $f(x) = x_1^2 + x_2^2 - 3x_1x_2 + 2x_1 + 3x_2 + 7$ . Compute  $\nabla f(x)$ ,  $\nabla^2 f(x)$ .
- (b) Is  $f(x)$  convex? Prove using equations.
- (c) Prove that the sum of two convex functions is a convex function.

- (d) Consider  $g(x) = x_1^2 + 2x_2^2 + 5$ . Using a step size of 0.2, show the sequence of  $x_k$  for the steepest gradient descent (SGD) algorithm.
- (e) Is it possible that SGD would not converge with  $g(x)$  even if a finite step size is used? Explain briefly.
- (f) MUTE intends to place IoT relays around the user so that any sound source can be cancelled through the “lookahead” idea. How many relays are sufficient to achieve this? You can assume that all interference sources are in a 2D plane at the same height of the user’s ears.