ECE/CS 434 : Mobile Computing Algorithms and Applications : Homework 2 : Due 11 :59pm, Sun, Feb 28, 2021

Problem 1 [20 points]

Consider the 2^{nd} column of the Fourier matrix, which is $[e^{j0}\ e^{j\theta}\ e^{j2\theta}\ \ldots\ e^{j(N-1)\theta}]^T$.

(a) Prove that the 3^{rd} column is orthogonal to the 2^{nd} column.

(b) Prove that any column is orthogonal to the 2^{nd} column.

(c) Prove that any two columns are orthogonal.

Problem 2 [25 points]

Consider a N dimensional vector $\bar{v}$ expressed in the identity basis.

(a) Express the vector $\bar{v}$ in an orthonormal basis F, where F is a $N \times N$ matrix.
(*Hint : See class notes on how we express a signal in different basis.*)

(b) Let's call the above vector $\bar{w}$. Now create a matrix B such that $B\bar{w}$ scales the i^{th} element of $\bar{w}$ by a scalar b_i. What should be the matrix B ?

(c) Let's denote the vector $B\bar{w}$ as vector $\bar{z}$. Now convert vector $\bar{z}$ back into the original identity basis.

(d) Now write all the above operations on vector $\bar{v}$ in one equation in terms of F and B.

(e) Write the Eigen-decomposition equation of a matrix A, where S contains the eigenvectors of A and Λ is the diagonal matrix containing the eigenvalues.

(f) Given the above exercise you have done, explain in plain English what Eigen-decomposition does to a vector (in other words, what happens when matrix A is multiplied to vector x) ?

Problem 3 [5 points]

In class, we discussed the analogy of expressing a job-interview candidate, Albert, in two different bases ; one was the $\langle$ math, programming, presentation $\rangle$ and the other was $\langle$ machine learning, logic design, project report $\rangle$.

(a) Can you come up with another analogy from the real world where the same "thing" is expressed in 2 different "bases". You must write that one "thing" and the 2 "bases".

(b) In class, we called our analogy the Space-X transform. Please name your own transform for the analogy you came up with above.

Problem 4 [20 points]

(a) You are sampling a signal every 0.25 millisecond. What is the maximum frequency you would be able to see in FFT ?

(b) Suppose you take $N = 1000$ for your FFT. At what frequency resolution would you be able to analyze the signal you are sampling ? A frequency resolution of R Hz means you express the signal at frequencies $[0, R, 2R, ...]$ Hz.

(c) True/False : The FFT of any signal is symmetric around frequency zero. Explain your answer in 1 sentence.

(d) Say X_f is the DFT of a signal x_n. Now, consider $Y_f = X_f.e^{j\phi}$, where ϕ is a constant angle. Is the IDFT(Y_f) a shifted version of the signal x_n ? Briefly argue in favor or against.

Problem 5 [20 points]

(a) Consider a signal $x[n] = \cos(2\pi f_1 n t_s) + 2\sin(2\pi f_1 n t_s) + \sin(4\pi f_1 n t_s)$. Draw the magnitude and phase plots of X_f, which is the DFT of $x[n]$. Assume that f_1 is the fundamental frequency in which you are sampling the signal.

(b) Prove that DFT is linear, i.e., $\text{DFT}(a_1 x[n] + a_2 y[n]) = a_1 X_f + a_2 Y_f$, where X_f and Y_f are the DFTs of $x[n]$ and $y[n]$, respectively.

Problem 6 [10 points]

Use your phone to record your own voice, and say "My name is [Your Name]." Use Python to import the saved audio file, and compute its FFT. Submit the plot of the *magnitude* of the FFT.

Hint : You can use `scipy.io.wavfile.read` [1] to read the audio file and get the sampling rate. If your data has two channels, you can extract 1 with `data = data[:, 0]`. You can then compute the FFT with `scipy.fft` [2].

1. https://docs.scipy.org/doc/scipy/reference/generated/scipy.io.wavfile.read.html
2. https://docs.scipy.org/doc/scipy/reference/tutorial/fft.html#fast-fourier-transforms