

## CS/ECE 434 Homework 2

Due Apr 10, 11:59 pm

### Problem 1: State true or false with a 1 line justification [24 points]

- a. The DFT spectrum of a signal is symmetric around  $F_s/2$ , where  $F_s$  is the sampling frequency.
- b. GPS location estimation also synchronizes the clock of the mobile device with the satellite clock.
- c. Signal strength is a good indicator of distance in indoor environments.
- d. In RADAR, more the number of access points (APs), better will be the accuracy.
- e. For a static object, estimating gravity well can determine the orientation of the object.
- f. If a transmitter is in the near field, then the location of the object can be determined from a receiver array.
- g. If phase can be measured precisely, then it is adequate to measure the distance between a transmitter and a receiver.
- h. In free fall, an accelerometer will record 0 on all three axes.
- i. Since white noise has a zero mean, we can keep tracking an object's location by integrating IMU data over long period of time.
- j. Order of rotations does not matter. That is:  $R_x(R_y(R_z)) \equiv R_y(R_z(R_x))$ .
- k. A model based localization approach based on RSS (such as that explored by second part of RADAR) will work well in outer space.
- l. Localization approach used in UnLoc will fail in an open area since it will have no remarkable landmarks to reset.

### Problem 2: Explain or argue briefly [21 points]

- a. Explain (5 points): Why is clock synchronization between GPS satellites so crucial to localize mobile devices on the ground.
- b. Argue (5 points): There is some order in which the UnLoc landmarks get estimated (i.e., the ordering is not random).
- c. Argue (5 points): KNN is not always better than NN localization. If so, give an example, if not, argue why not.
- d. Argue both for and against (6 points): In a 2-antenna AoA setup, Alice wants to separate the antennas by  $2\lambda$  distance. What are the benefits and drawbacks over a separation of  $\lambda/2$ ?

### Problem 3: Math nuggets and derivations [20 points]

- Derive the Nyquist sampling theorem from first principles.
- Derive the steering matrix for AoA estimation from first principles.
- For HMMs, derive the expression for  $P(s_k - m_{1:n})$ . Show which one is the forward and which one is the backward algorithm.
- Derive the chain rule for the joint probability,  $P(A, B, C, D)$

### Problem 4: 3D Orientation [15 points]

Assume that the magnitude of acceleration due to gravity is  $g$  and the magnitude of the earth's magnetic field is  $m$ . Also assume that we are at the equator. For a mobile phone assume x-axis is parallel to the phone's width (shorter side), y-axis is parallel to the phone's length (longer side), z-axis penetrates through the phone's glass surface.

- A static mobile phone is supported on a table and shows accelerometer reading:  $[0, 1, 0]g$ . Does this represent the complete orientation of the phone? If yes, what is the orientation matrix. If no, give a reason why not.
- The above phone is showing a magnetometer reading of  $[1, 0, 0]m$ . Is this possible? What is the orientation matrix for the phone?
- A static mobile phone is reporting the following measurements on its accelerometer:  $[-0.3789, 0.2775, 0.8828]g$ . It is reporting the following magnetometer readings:  $[0.5829, 0.8125, -0.0053]m$ . What is its orientation matrix?

### Problem 5: Programming module: Beamforming and AoA [20 points]

- Code up a small simulator which has a device X with N antennas (uniformly separated by distance DIS).
  - Device X is receiving a signal from different angles THETA.
  - The signal is at frequency  $FREQ = 100\text{kHz}$  and is being transmitted by a far field transmitter Y.
  - Now, implement the basic AoA sensing algorithm.
- Assuming you don't know THETA, plot the AoA spectrum for  $DIS = \text{wavelength}$ , half wavelength, and quarter wavelength (note that  $c = f * \text{lambda}$ ).
  - Add increasing random noise to the received signal, and re-plot the AoA spectrum but this time for  $DIS = \text{half wavelength}$  only. What do you observe as you increase noise?
  - Now add a reflection signal coming from a different angle BETA and re-plot the AoA spectrum only for  $DIS = \text{half wavelength}$ . Can you still detect the AoA accurately?
  - Keep adding reflection signals and determine at which point the AoA becomes inaccurate.