

Homework 1: Math Foundations

1 True or False**[6 X 5 = 30 points]**

Write True/False with a brief justification (around 1 sentence). Make assumptions where necessary and state them clearly.

Q1 :

A is a $m \times n$ matrix with $m < n$. The null space $N(A)$ is always 0.

Q2 :

The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, when left multiplied with a second matrix B (i.e. $B * A$), subtracts twice of the first column of B from the second column of B .

Q3 :

For an orthonormal matrix Q (columns of Q are orthogonal to each other and the length of each column is 1), $Q^{-1} = Q^T$.

Q4 :

A signal $S1$ is composed of frequencies f_1, f_2, f_3 and $S2$ consists of frequencies f_2, f_3, f_4 . Then, a signal $S3 = S1 \star S2$ (convolution of $S1$ and $S2$), would consist of union of these frequencies i.e., f_1, f_2, f_3, f_4 .

Q5 :

If b_1, b_2, b_3 form the basis of a space, then $c_1 b_1 + c_2 b_2 + c_3 b_3 = 0$ implies that all c_1, c_2 and c_3 are zero.

Q6 :

The length of the vector $\frac{V}{\|V\|}$ is 0 ($\|V\|$ is the l_2 norm of V).

2 Symmetric Matrices**[5 points]**

Prove that $A^T A$ is a symmetric matrix.

Hint : use the basic properties of transpose, as discussed in class.

3 Column Spaces**[10 points]**

Choose b which gives no solution and another b which gives infinitely many solutions. What are two of those solutions?

$$3x + 2y = 10 \quad (1)$$

$$6x + 4y = b \quad (2)$$

4 Least Squares [10 points]

Consider the following system of equations.

$$x - y = 2 \tag{3}$$

$$x + y = 4 \tag{4}$$

$$2x + y = 8 \tag{5}$$

How many solutions exist for the above system of equations? If a solution exists find one, if not, determine the least squares solution for x and y .

5 Fourier Transform Matrix [10 points]

Compute the 4×4 Fourier Transform matrix W . Use W to transform $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ into the frequency domain.

Also, find the inverse of W and use it to transform $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ into the time domain.

Hint : recall the Fourier Transform equation as taught in class, and write it as a matrix form.

6 Fourier Transform Property [10 points]

Prove that real-input Fourier Transform obeys the symmetry : if $\mathbf{x} = \begin{bmatrix} x_0 \\ x_1 \\ \dots \\ x_{N-1} \end{bmatrix}$ are real numbers, then

for its N -point Fourier Transform \mathbf{X} we have : $X[N - k] = X[k]^*$.

(X^* denotes complex conjugation, e.g. $(e^{j\theta})^* = e^{-j\theta}$.)

7 A Real Fourier Transform [25 points]

7.1 Averaging Filter [5 points]

Filters such as $h = [1/8 \ 1/8 \ 1/8 \ 1/8 \ 1/8 \ 1/8 \ 1/8 \ 1/8]^T$ have an averaging effect, hence called an averaging filter. Use MATLAB to compute h 's 8-point FFT. In your solution, plot the magnitude of the FFT.

Hint : once you compute its FFT as H , you may use this MATLAB command to plot the graph :

`stem(abs(H)); xlabel('H[k]'); ylabel('FFT Magnitude');`

7.2 Human Voice [10 points]

Use your phone to record your own voice, and say “My name is [Your Name].” Use MATLAB to import the saved audio file, and compute its FFT. In your solution, plot the magnitude of the FFT.

Hint : once you compute the voice’s FFT as X , and get the sampling rate F_s from the audio file, you may use this MATLAB command to plot the graph :

```
plot((1 :length(X))/length(X)*Fs, abs(X)); xlabel('Frequency (Hz)'); ylabel('FFT Magnitude');
```

7.3 Convolution

[10 points]

In MATLAB, convolve the average filter in (7.1) with your voice in (7.2). Listen to the sound before and after convolution, to feel the difference. In your solution, plot the magnitude of the FFT of the new signal after convolution.