CS 425 / ECE 428
Distributed Systems
Fall 2022

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Lecture 13-A: Impossibility of Consensus

Announcements

- HW1, HW2 solutions released
- MP1, MP2 Recommended solutions released
- Midterm this Friday (10/7)
    - Written, in class
- Locations:
    - 1002 ECE: if your last name starts with A-L
    - Loomis 141: if your last name starts with M-Z
        - 1110 W Green St, Urbana
- Material: Lecture 1-12
- Practice Midterm Released
- Review session this Thursday evening by TAs

Give it a thought

Have you ever wondered why distributed server vendors always only offer solutions that promise five-9’s reliability, seven-9’s reliability, but never 100% reliable?

The fault does not lie with the companies themselves, or the worthlessness of humanity.

The fault lies in the impossibility of consensus

What is common to all of these?

A group of servers attempting:

- Make sure that all of them receive the same updates in the same order as each other
- To keep their own local lists where they know about each other, and when anyone leaves or fails, everyone is updated simultaneously
- Elect a leader among them, and let everyone in the group know about it
- To ensure mutually exclusive (one process at a time only) access to a critical resource like a file

What is common to all of these?

A group of servers attempting:

- Make sure that all of them receive the same updates in the same order as each other [Reliable Multicast]
- To keep their own local lists where they know about each other, and when anyone leaves or fails, everyone is updated simultaneously [Membership/Failure Detection]
- Elect a leader among them, and let everyone in the group know about it [Leader Election]
- To ensure mutually exclusive (one process at a time only) access to a critical resource like a file [Mutual Exclusion]

So what is common?

- Let’s call each server a “process” (think of the daemon at each server)
- All of these were groups of processes attempting to *coordinate* with each other and reach *agreement* on the value of something
    - The ordering of messages
    - The up/down status of a suspected failed process
    - Who the leader is
    - Who has access to the critical resource
- All of these are related to the *Consensus* problem

What is Consensus?

Formal problem statement

- N processes
- Each process p has

input variable xp : initially either 0 or 1

output variable yp : initially b (can be changed only once)

- Consensus problem: design a protocol so that at the end, either:

1. All processes set their output variables to 0 (all-0’s)
2. Or All processes set their output variables to 1 (all-1’s)

What is Consensus? (2)

- Every process contributes a value
- *Goal is to have all processes decide same (some) value*
  - Decision once made can’t be changed
- There might be other constraints
  - Validity = if everyone proposes same value, then that’s what’s decided
  - Integrity = decided value must have been proposed by some process
  - Non-triviality = there is at least one initial system state that leads to each of the all-0’s or all-1’s outcomes

Why is it Important?

- Many problems in distributed systems are equivalent to *(or harder than)* consensus!
  - Perfect Failure Detection
  - Leader election (select exactly one leader, and every alive process knows about it)
  - Agreement (harder than consensus)

- So consensus is a very important problem, and solving it would be really useful!

- So, is there a solution to Consensus?

Two Different Models of Distributed Systems

- Synchronous System Model and Asynchronous System Model
- Synchronous Distributed System
  - Each message is received within bounded time
  - Drift of each process' local clock has a known bound
  - Each step in a process takes lb < time < ub

  E.g., A collection of processors connected by a communication bus, e.g., a Cray supercomputer or a multicore machine

Asynchronous System Model

- Asynchronous Distributed System
  - No bounds on process execution
  - The drift rate of a clock is arbitrary
  - No bounds on message transmission delays

  E.g., The Internet is an asynchronous distributed system, so are ad-hoc and sensor networks

- *This is a more general (and thus challenging) model than the synchronous system model. A protocol for an asynchronous system will also work for a synchronous system (but not vice-versa)*

Possible or Not

- In the synchronous system model
  - Consensus is solvable

- In the asynchronous system model
  - Consensus is impossible to solve
  - Whatever protocol/algorithm you suggest, there is always a worst-case possible execution (with failures and message delays) that prevents the system from reaching consensus
  - Powerful result (*see the FLP proof*)
  - Subsequently, <u>safe</u> or <u>probabilistic</u> solutions have become quite popular to consensus or related problems.

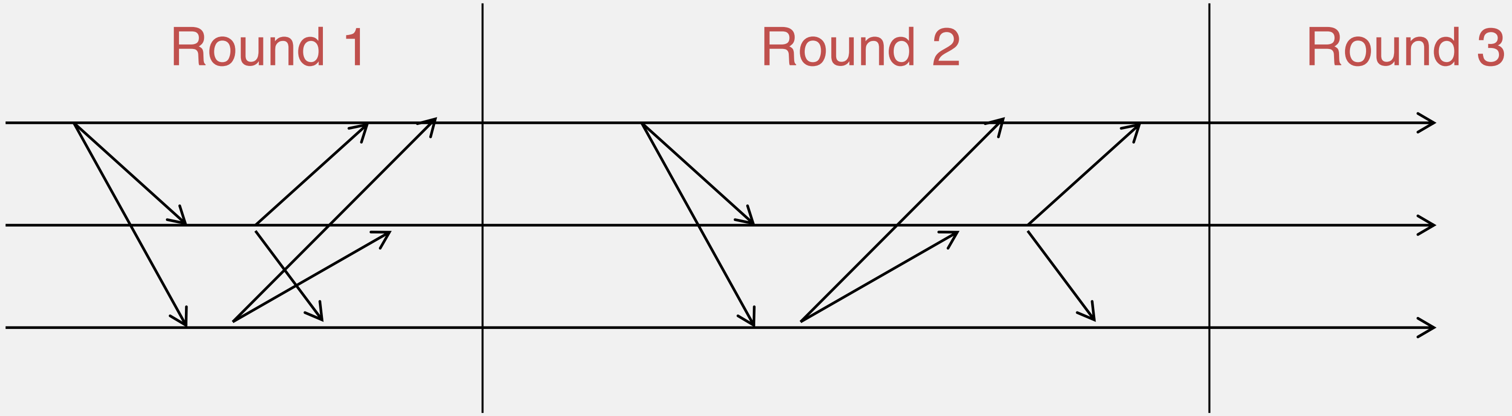
Let’s Try to Solve Consensus!

- Uh, what’ s the **system model**? (assumptions!)
- Synchronous system: bounds on
  - Message delays
  - Upper bound on clock drift rates
  - Max time for each process step

  e.g., multiprocessor (common clock across processors)
- Processes can fail by stopping (fail-stop or crash failures)

Consensus in Synchronous Systems

- For a system with at most f processes crashing
  - All processes are synchronized and operate in "rounds" of time. Round length >> max transmission delay.
  - the algorithm proceeds in $f+1$ rounds (with timeout), using reliable communication to all members
  - $Values^r_i$: the set of proposed values known to p_i at the beginning of round r.



Consensus in Synchronous System

Possible to achieve!

- For a system with at most f processes crashing
  - All processes are synchronized and operate in "rounds" of time
  - the algorithm proceeds in $f+1$ rounds (with timeout), using reliable communication to all members. Round length >> max transmission delay.
  - $Values^r_i$: the set of proposed values known to p_i at the beginning of round r.

- Initially $Values^0_i = \{\}$; $Values^1_i = \{v_i\}$

for round = 1 to $f+1$ do

multicast ($Values^r_i - Values^{r-1}_i$) // iterate through processes, send each a message

$Values^{r+1}_i \leftarrow Values^r_i$

for each V_j received

$Values^{r+1}_i = Values^{r+1}_i \cup V_j$

end

end

d_i = minimum($Values^{f+2}_i$) // consistent minimum based on say, id (not minimum value)

Why does the Algorithm work?

- After $f+1$ rounds, all non-faulty processes would have received the same set of Values. Proof by contradiction.
- Assume that two non-faulty processes, say p_i and p_j , differ in their final set of values (i.e., after $f+1$ rounds)
- Assume that p_i possesses a value v that p_j does not possess.
  - → p_i must have received v in the very last round
    - → Else, p_i would have sent v to p_j in that last round
  - → So, in the last round: a third process, p_k, must have sent v to p_i, but then crashed before sending v to p_j.
  - → Similarly, a fourth process sending v in the last-but-one round must have crashed; otherwise, both p_k and p_j should have received v.
  - → Proceeding in this way, we infer at least one (unique) crash in each of the preceding rounds.
  - → This means a total of $f+1$ crashes, while we have assumed at most f crashes can occur => contradiction.

Consensus in an Asynchronous System

- Impossible to achieve!
- Proved in a now-famous result by Fischer, Lynch and Patterson, 1983 (FLP)
  - Stopped many distributed system designers dead in their tracks
  - A lot of claims of "reliability" vanished overnight

Recall

Asynchronous system: All message delays and processing delays can be arbitrarily long or short.

Consensus:

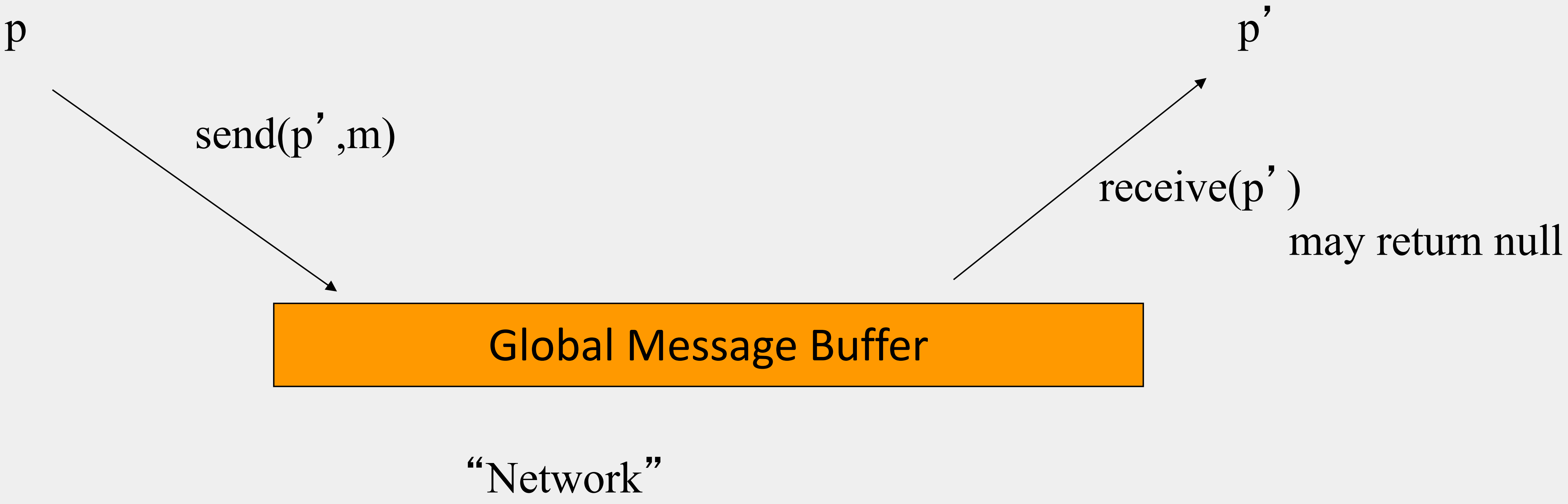
- Each process p has a state
  - program counter, registers, stack, local variables
  - input register xp : initially either 0 or 1
  - output register yp : initially b (undecided)
- Consensus Problem: design a protocol so that either
  - all processes set their output variables to 0 (all-0's)
  - Or all processes set their output variables to 1 (all-1's)
  - Non-triviality: at least one initial system state leads to each of the above two outcomes

Proof Setup

- For impossibility proof, OK to consider

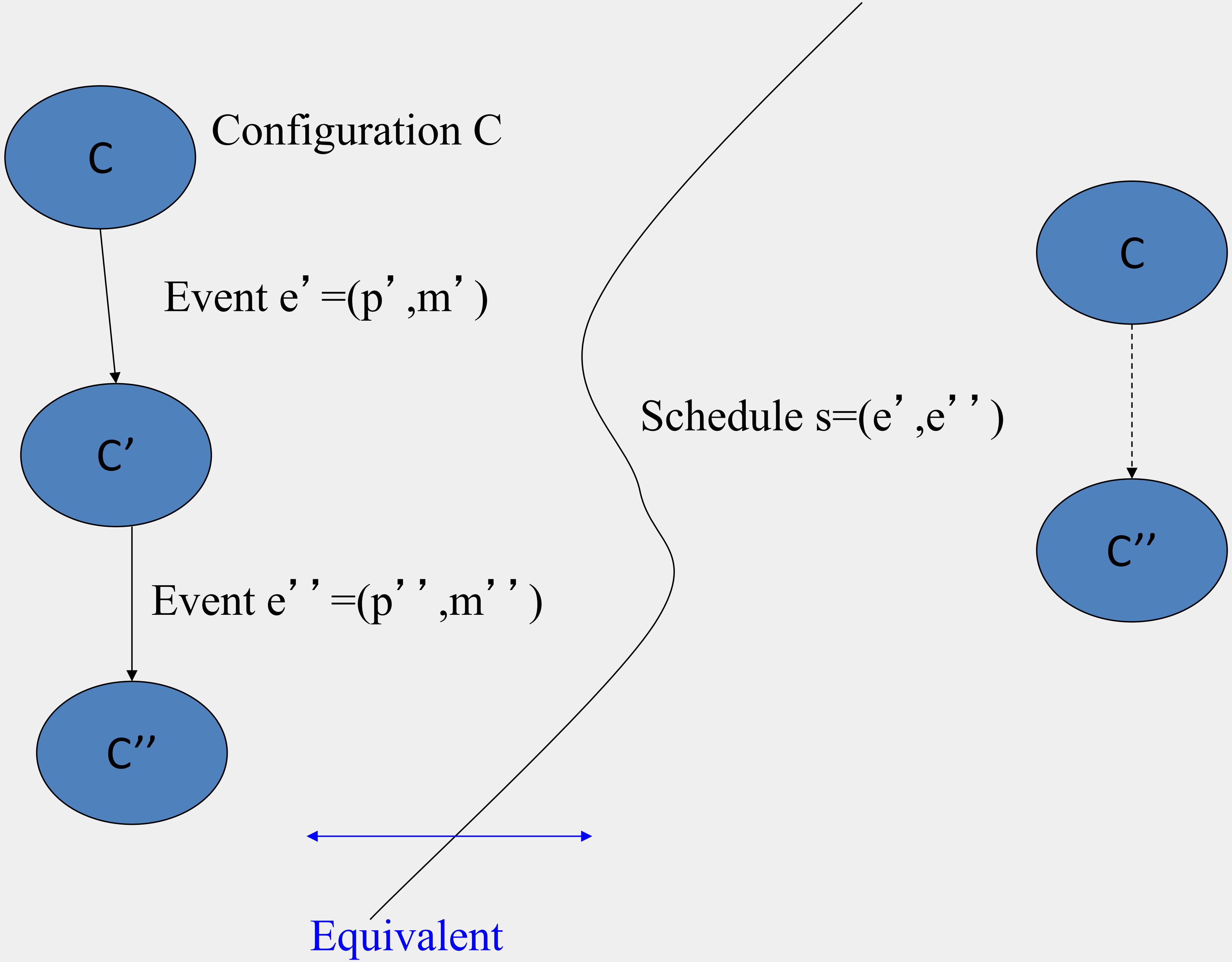
1. more restrictive system model, and
2. easier problem
   - Why is this is ok?

Network


p
p’
send(p’,m)
receive(p’)
may return null
Global Message Buffer
“Network”

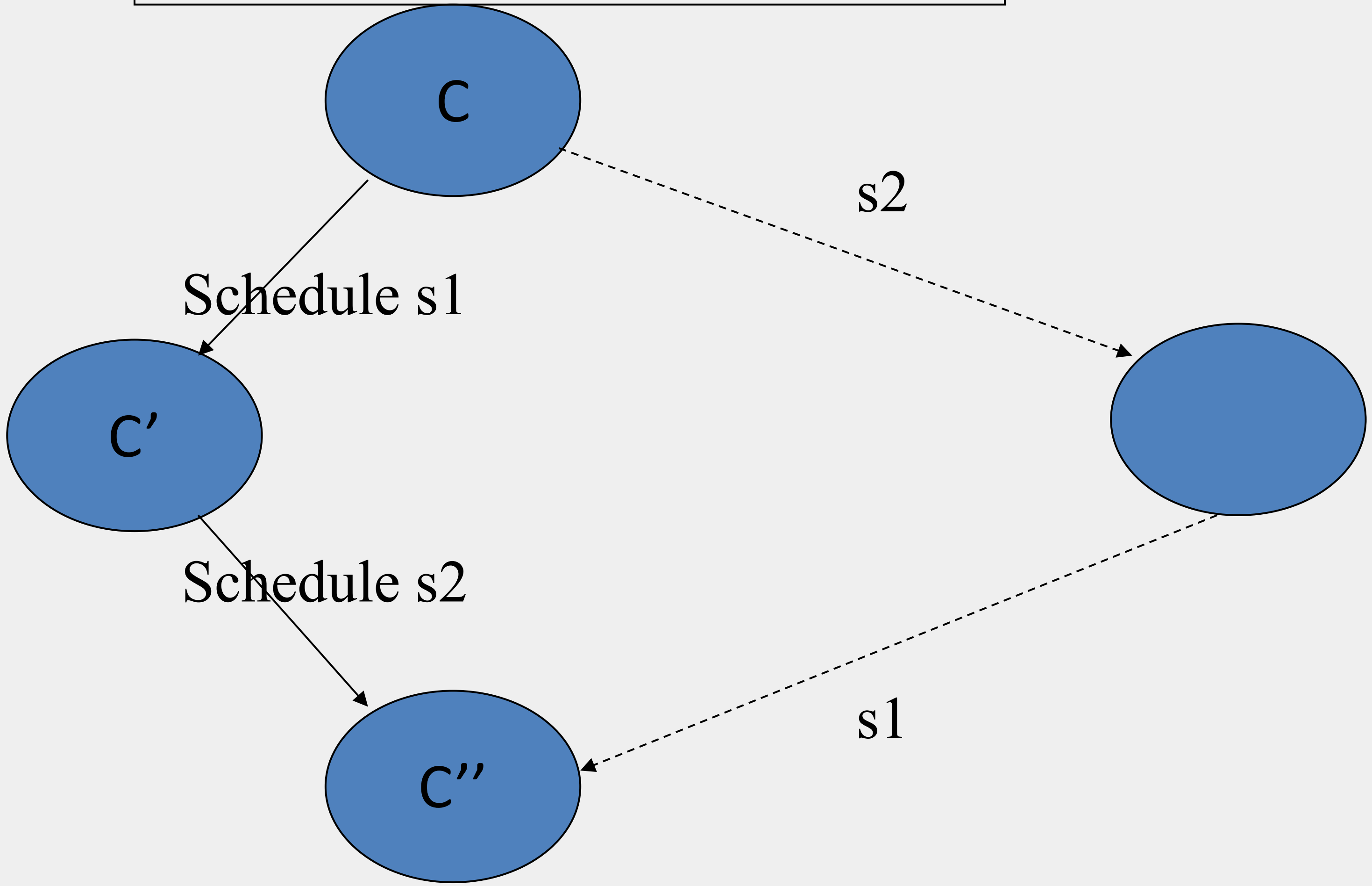
States

- State of a process
- **Configuration**=global state. Collection of states, one for each process; alongside state of the global buffer.
- Each Event (<u>different</u> from Lamport events) is atomic and consists of three steps
  - receipt of a message by a process (say p)
  - processing of message (may change recipient's state)
  - sending out of all necessary messages by p
- Schedule: sequence of events


C
Configuration C
Event e’ =(p’ ,m’)
C’
Event e’ ’ =(p’ ’ ,m’ ’)
C’’
Equivalent
Schedule s=(e’ ,e’ ’)
C
C’’

Lemma 1

Disjoint schedules are commutative



s1 and s2 involve disjoint sets of receiving processes, and are each applicable on C

Easier Consensus Problem

Easier Consensus Problem:

some process eventually sets yp to be 0 or 1

Only one process crashes – we're free to choose which one

Easier Consensus Problem

- Let config. C have a set of decision values V reachable from it
  - If $|V| = 2$, config. C is bivalent
  - If $|V| = 1$, config. C is 0-valent or 1-valent, as is the case

- Bivalent means outcome is unpredictable

What the FLP proof shows

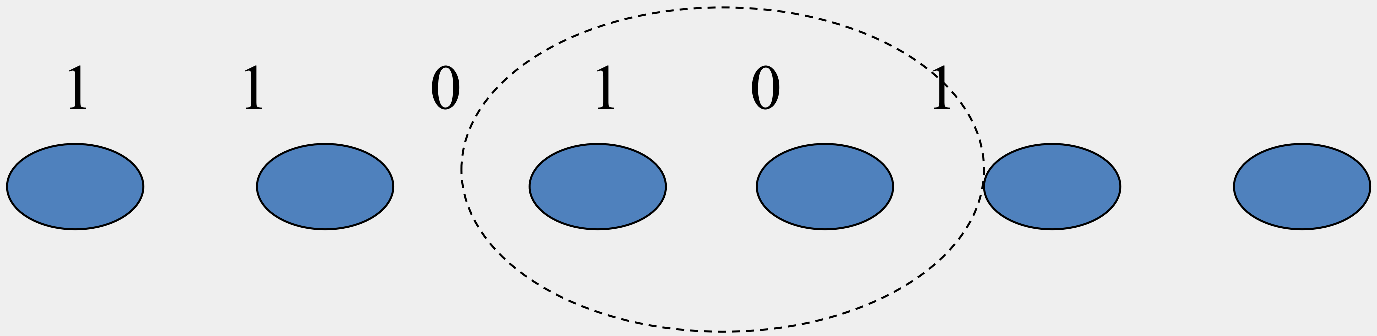
1. There exists an initial configuration that is bivalent

2. Starting from a bivalent config., there is always another bivalent config. that is reachable

Lemma 2

Some initial configuration is bivalent

- Suppose all initial configurations were either 0-valent or 1-valent.
- If there are N processes, there are 2^N possible initial configurations
- Place all configurations side-by-side (in a lattice), where adjacent configurations differ in initial xp value for <u>exactly one</u> process.

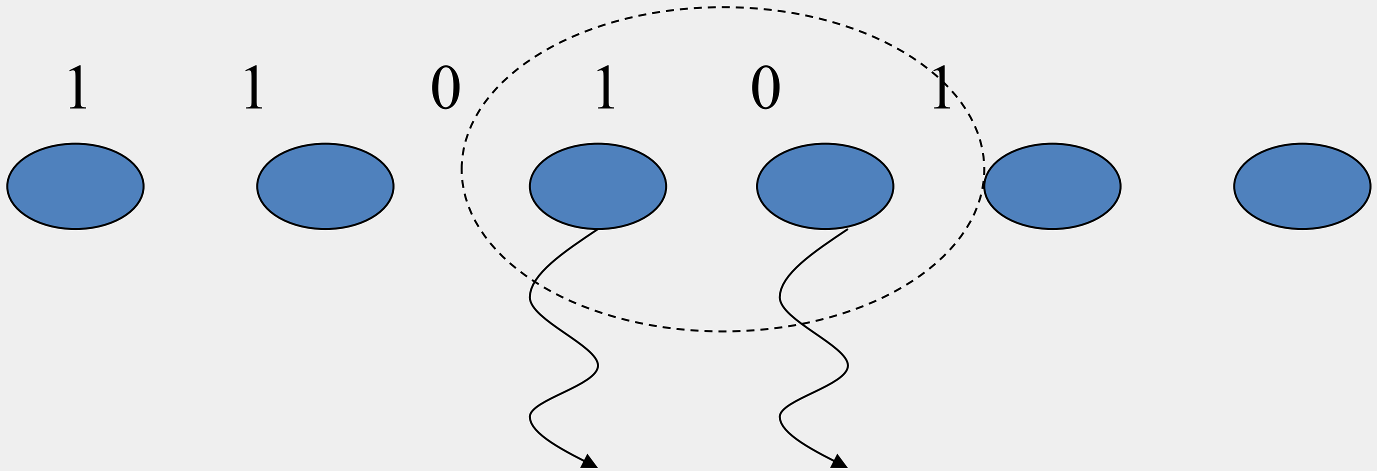


- There has to be some adjacent pair of 1-valent and 0-valent configs.

Lemma 2

Some initial configuration is bivalent

- There has to be some adjacent pair of 1-valent and 0-valent configs.
- Let the process p, that has a different state across these two configs., be the process that has crashed (i.e., is silent throughout)



Both initial configs. will lead to the same config. for the same sequence of events

Therefore, both these initial configs. are <u>bivalent</u> when there is such a failure

What we’ll show

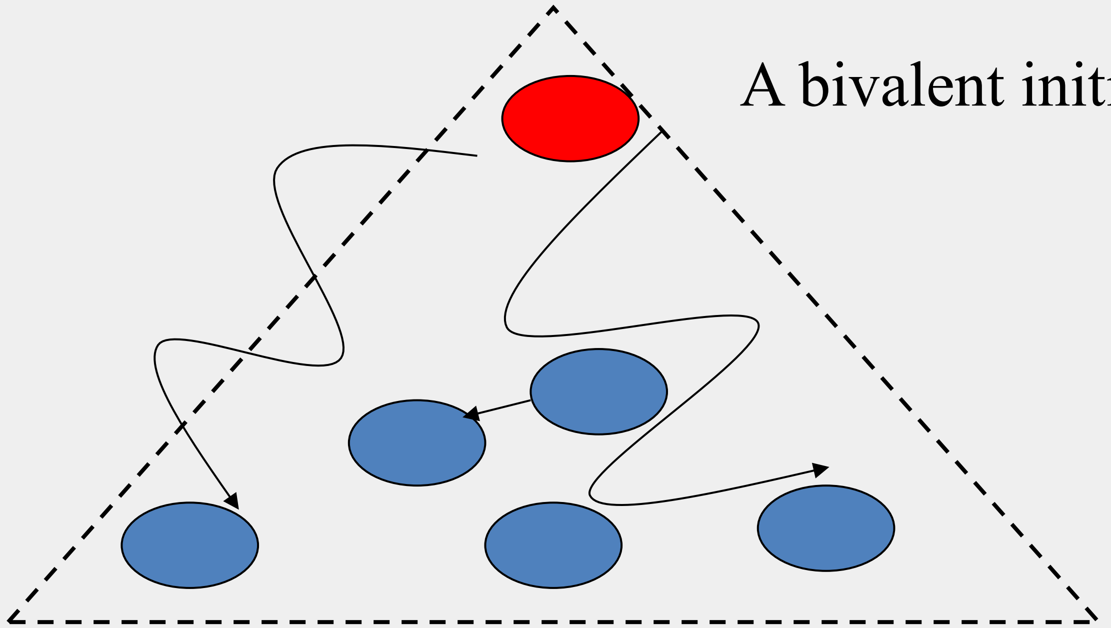
1. There exists an initial configuration that is bivalent

2. Starting from a bivalent config., there is always another bivalent config. that is reachable

Lemma 3

Starting from a bivalent config., there is always another bivalent config. that is reachable

Lemma 3

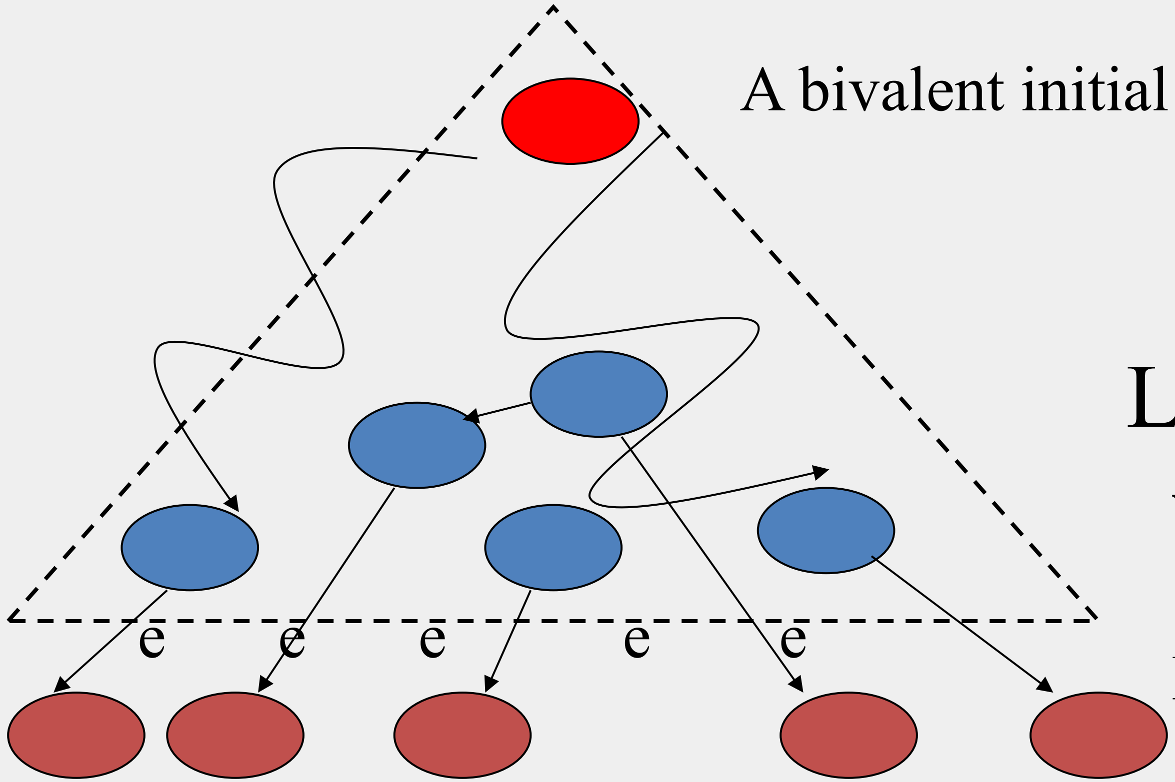


A bivalent initial config.

let e=(p,m) be some event applicable to the initial config.

Let *C* be the set of configs. reachable **without** applying e

Lemma 3



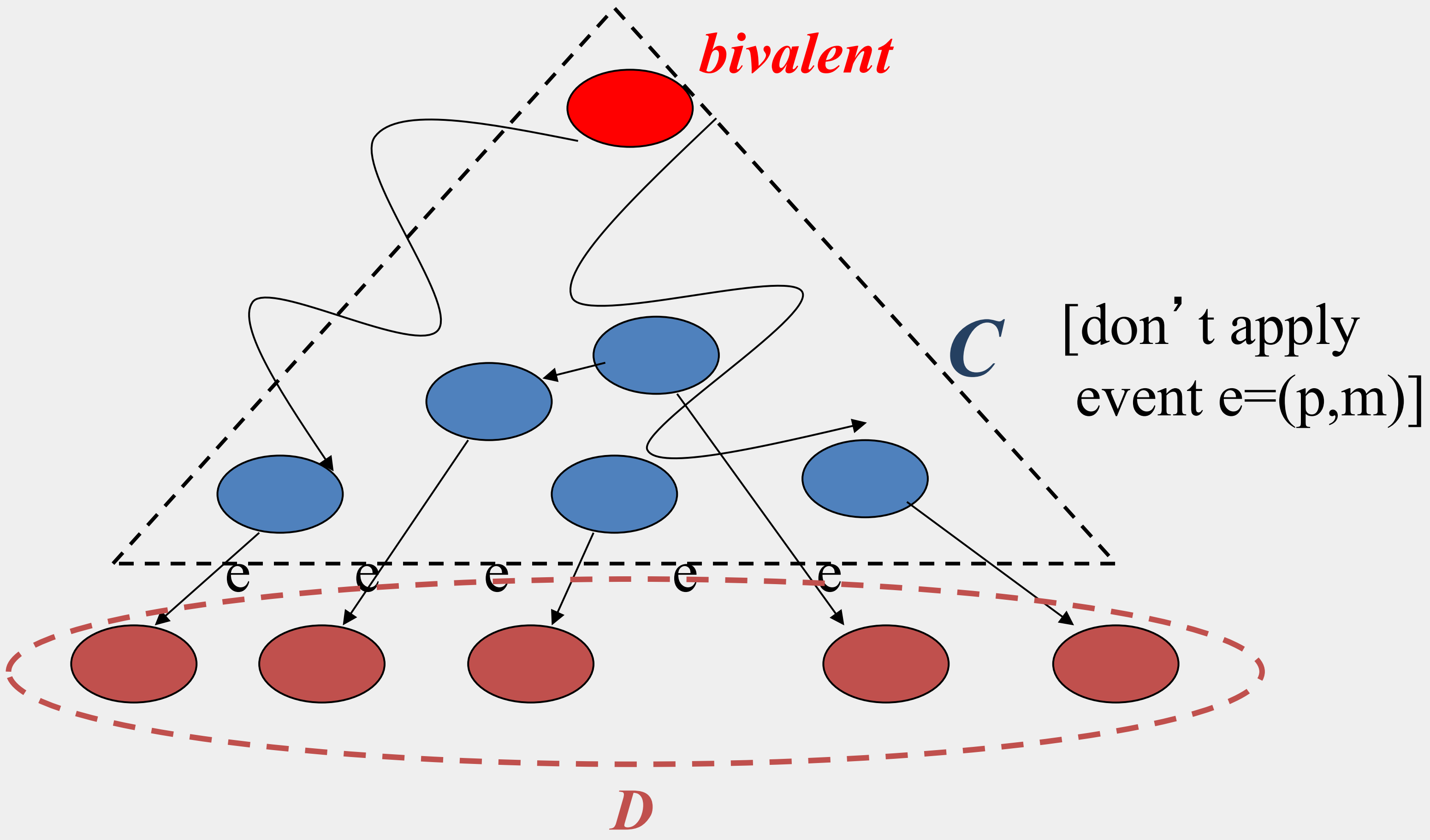
A bivalent initial config.

let e=(p,m) be some event applicable to the initial config.

Let ***C*** be the set of configs. reachable **without** applying e

Let ***D*** be the set of configs. obtained by **applying e** to some config. in ***C***

Lemma 3

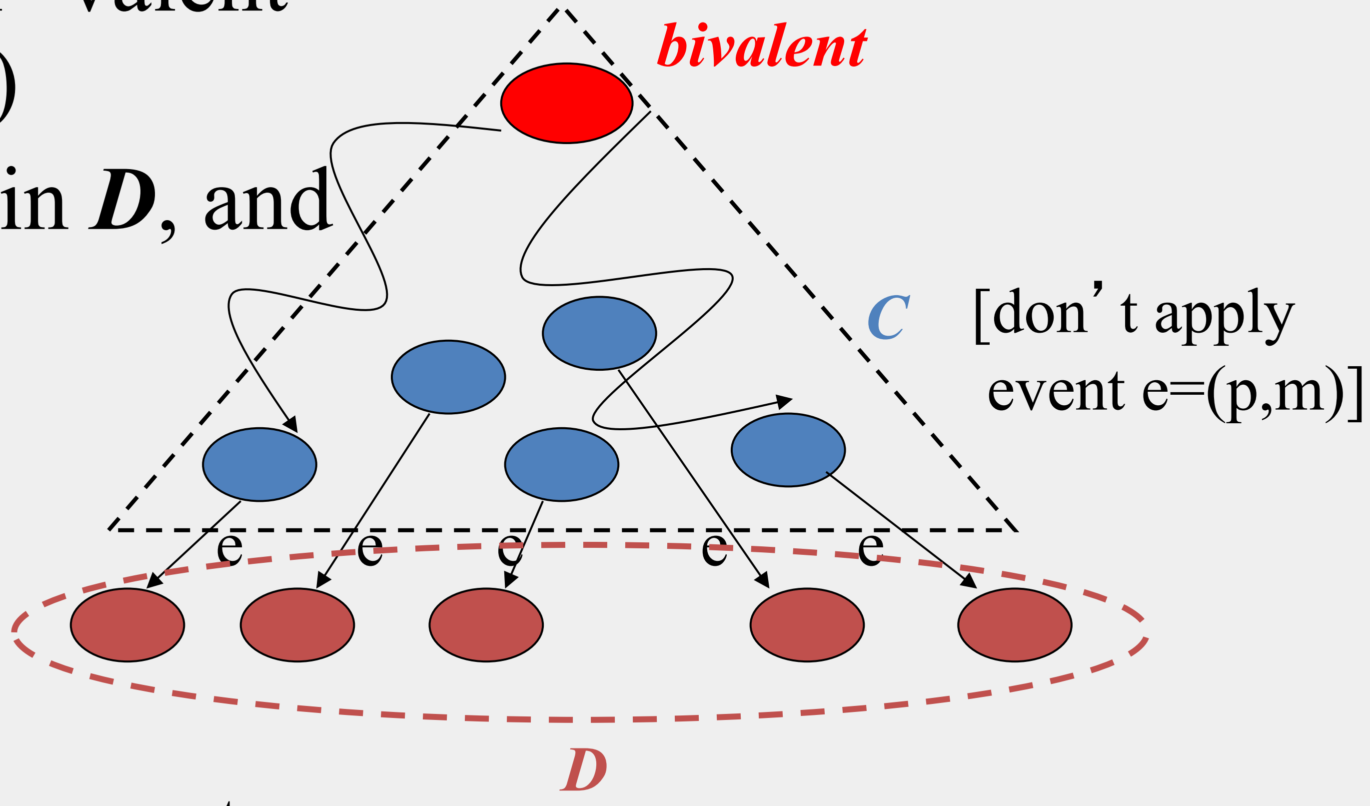


Claim. Set D contains a bivalent config.

Proof. By contradiction. That is,
suppose ***D*** has only 0- and 1- valent states (and no bivalent ones)

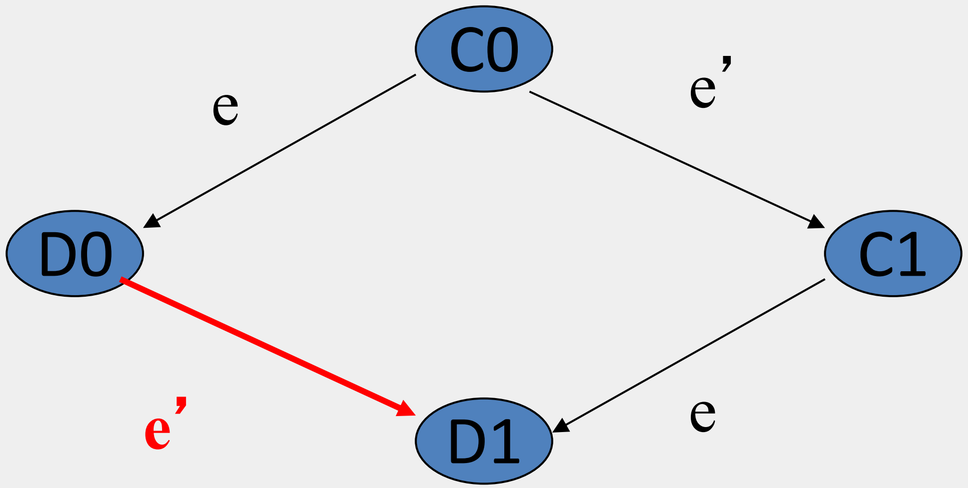
- There are states D0 and D1 in ***D***, and C0 and C1 in ***C*** such that
  - D0 is 0-valent, D1 is 1-valent
  - D0=C0 foll. by e=(p,m)
  - D1=C1 foll. by e=(p,m)
  - And C1 = C0 followed by some event e’=(p’,m’)

(why?)

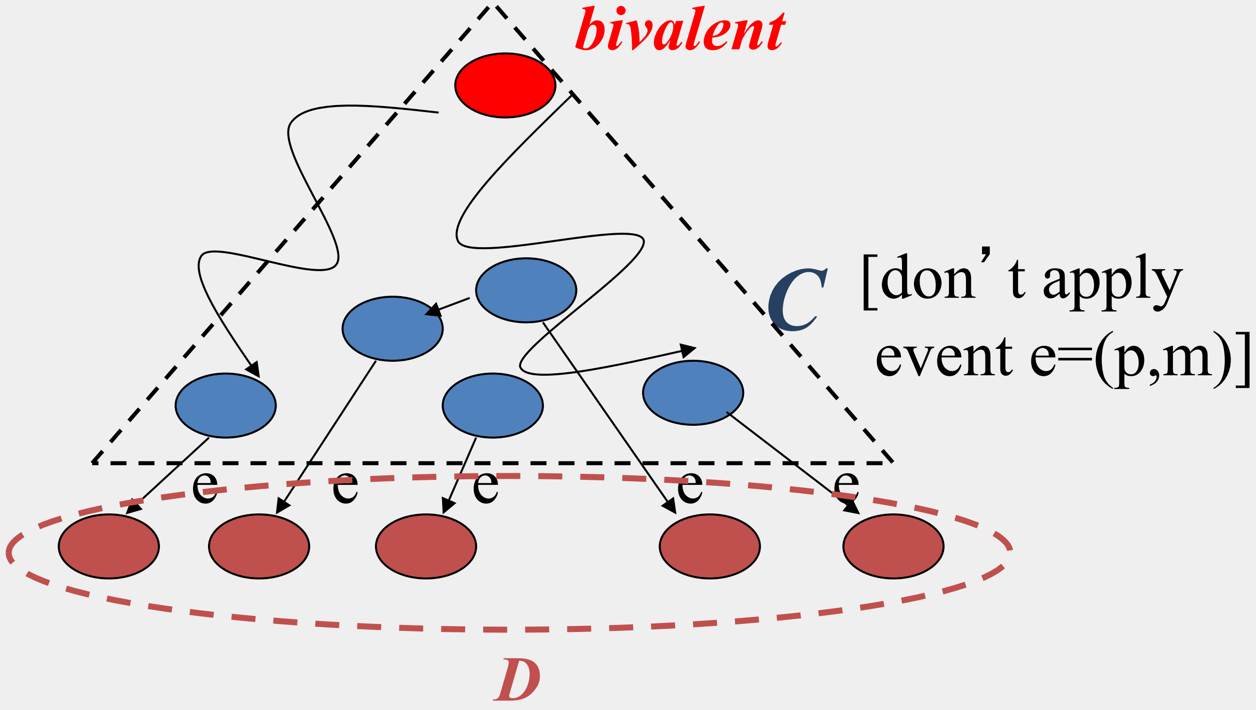


Proof. (contd.)

- Case I: p’ is not p
- Case II: p’ same as p



Why? (Lemma 1)
But D0 is then bivalent!



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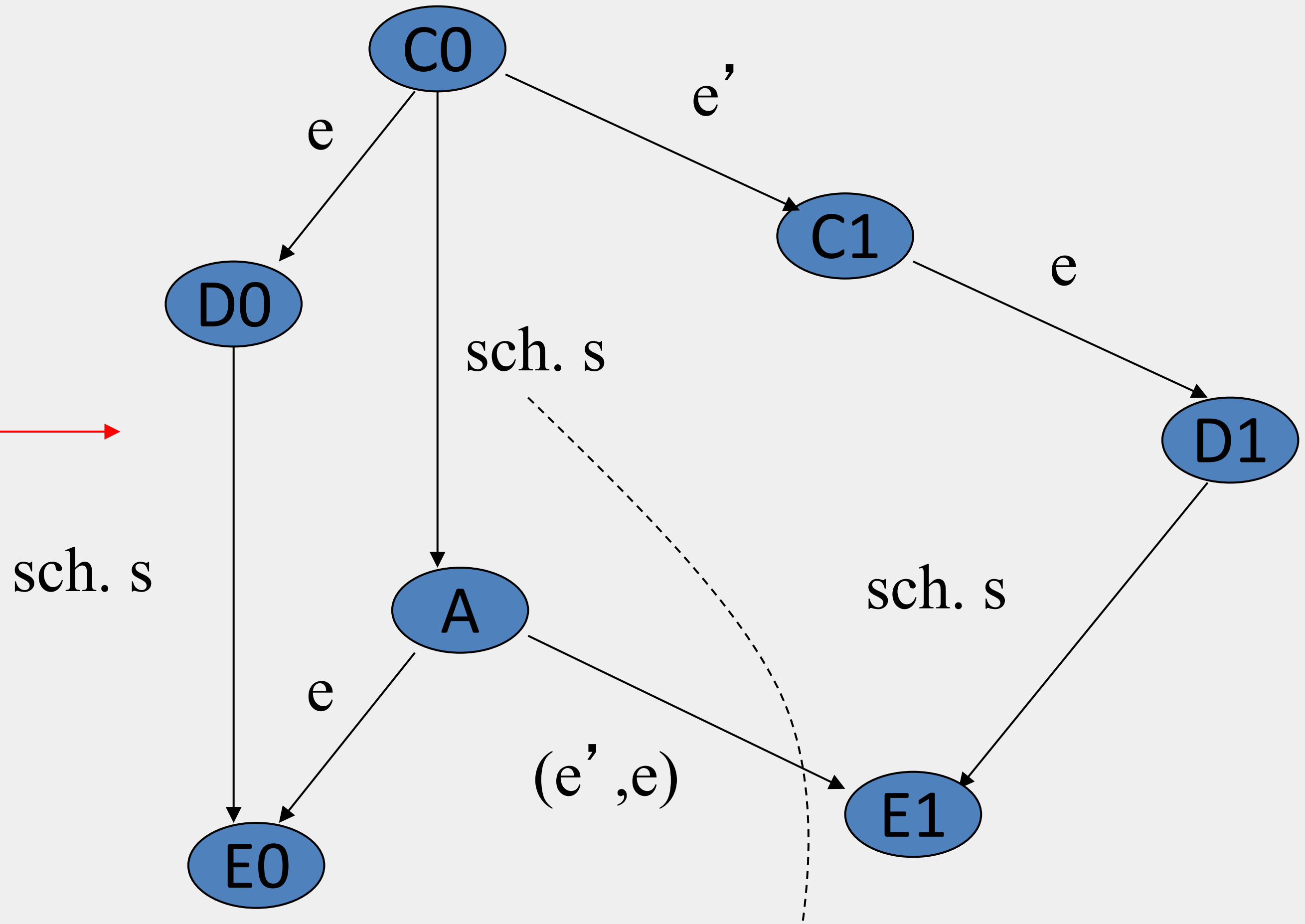

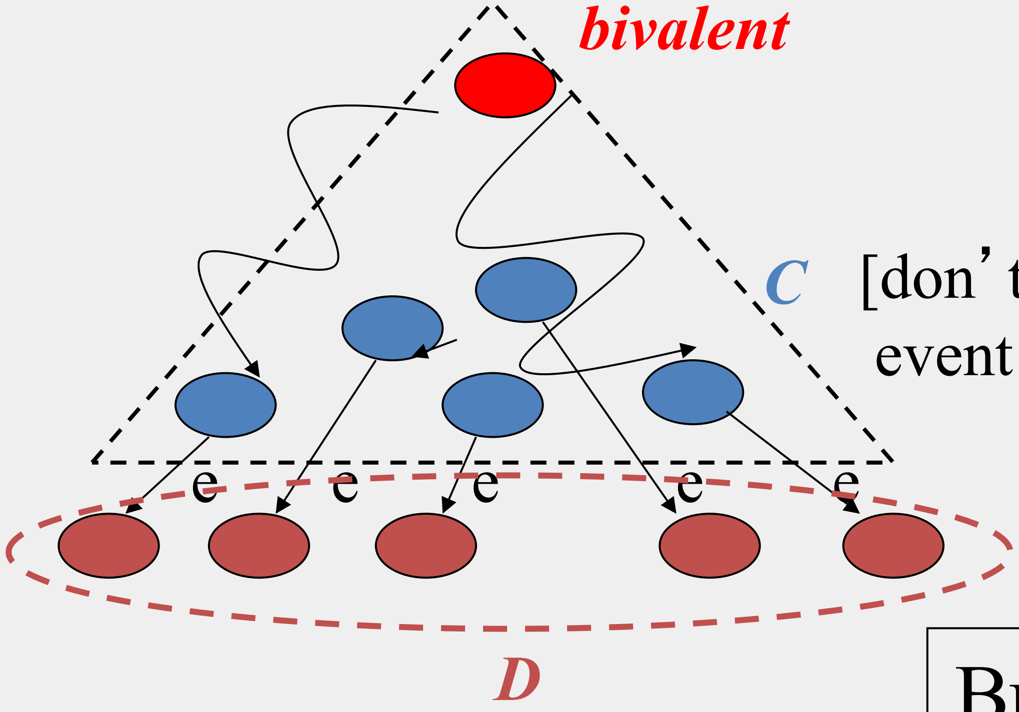

[don’t apply event e=(p,m)]

sch. s
- finite
- **deciding run** from C0
- *p takes no steps*

But A is then bivalent!

Lemma 3

Starting from a bivalent config., there is always another bivalent config. that is reachable

Putting it all Together

- Lemma 2: There exists an initial configuration that is bivalent
- Lemma 3: Starting from a bivalent config., there is always another bivalent config. that is reachable

- Theorem (Impossibility of Consensus): There is always a run of events in an asynchronous distributed system such that the group of processes never reach consensus (i.e., stays bivalent all the time)

Optional Exercises/Questions to Test your Own Knowledge

1. Is the consensus problem the same as majority voting? If not, what are the differences?
2. What is a trivial solution to consensus?
3. Why is consensus solvable for synchronous systems?.
4. . A synchronous consensus algorithm with N=5 processes has only 2 rounds, but can have up to 2 failures. Show how this algorithm fails to solve consensus.
5. Why does the FLP proof treat the network as a giant "buffer"?
6. What is a commutative schedule?
7. What is the lattice of states and why is it important in the FLP proof?
8. How does FLP show that given a bivalent state, one can reach another bivalent state?
9. In FLP's last lemma, why is it ok to prevent process p from taking any steps for a while, or event e from occurring for a while?

Summary

- Consensus Problem
  - Agreement in distributed systems
  - Solution exists in synchronous system model (e.g., supercomputer)
  - Impossible to solve in an asynchronous system (e.g., Internet, Web)
    - Key idea: with even one (adversarial) crash-stop process failure, there are always sequences of events for the system to decide any which way
    - Holds true regardless of whatever algorithm you choose!
  - FLP impossibility proof
- One of the most fundamental results in distributed systems

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Lecture 13-B: Paxos

What is Consensus?

Formal problem statement

- N processes
- Each process p has

    input variable xp : initially either 0 or 1

    output variable yp : initially b (can be changed only once)

- Consensus problem: design a protocol so that at the end, either:

    1. All processes set their output variables to 0 (all-0's)
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What is Consensus? (2)

- Every process contributes a value
- *Goal is to have all processes decide same (some) value*
  - Decision once made can't be changed
- There might be other constraints
  - Validity = if everyone proposes same value, then that's what's decided
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Why is it Important?

- Many problems in distributed systems are equivalent to *(or harder than)* consensus!
  - Perfect Failure Detection
  - Leader election (select exactly one leader, and every alive process knows about it)
  - Agreement (harder than consensus)

- So consensus is a very important problem, and solving it would be really useful!

- Consensus is
  - Possible to solve in synchronous systems
  - Impossible to solve in asynchronous systems

Can't we just solve Consensus?

- Yes, we can!
- (Whut?)

Yes we Can!

- Paxos algorithm
  - Most popular “consensus-solving” algorithm
  - Does not solve consensus problem (which would be impossible, because we already proved that)
  - But provides <u>safety</u> and <u>eventual liveness</u>
  - A lot of systems use it
    - Zookeeper (Yahoo!), Google Chubby, and many other companies

- Paxos invented by? (take a guess)

Yes we Can!

- Paxos invented by Leslie Lamport

- Paxos provides safety and eventual liveness
  - Safety: Consensus is not violated
  - Eventual Liveness: If things go well sometime in the future (messages, failures, etc.), there is a good chance consensus will be reached. But there is no guarantee.

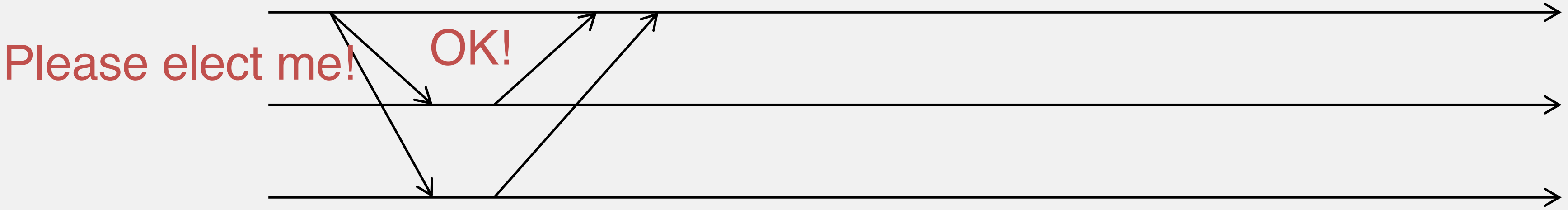
- FLP result still applies: Paxos is not *guaranteed* to reach Consensus (ever, or within any bounded time)

Political Science 101, i.e., Paxos Groked

- Paxos has rounds; each round has a unique ballot id
- Rounds are asynchronous
  - Time synchronization not required
  - If you're in round *j* and hear a message from round *j+1*, abort everything and move over to round *j+1*
  - Use timeouts; may be pessimistic
- Each round itself broken into phases (which are also asynchronous)
  - Phase 1: A leader is elected (Election)
  - Phase 2: Leader proposes a value, processes ack (Bill)
  - Phase 3: Leader multicasts final value (Law)

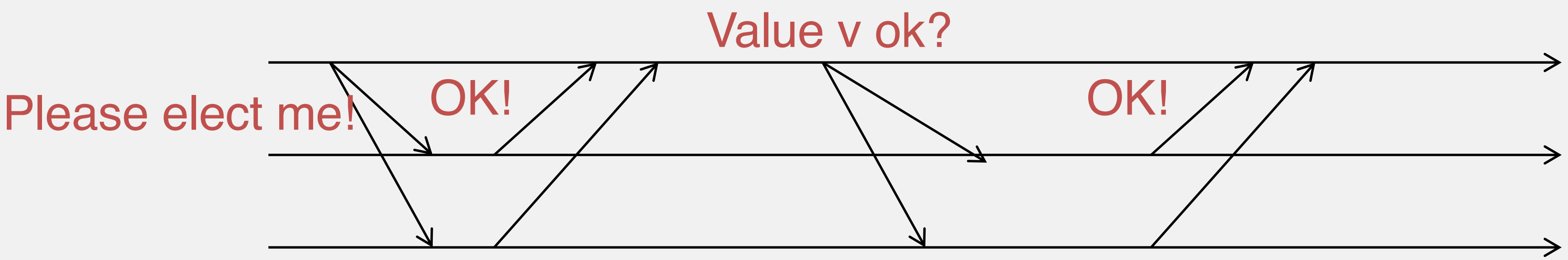
Phase 1 – election

- Potential leader chooses a unique ballot id, higher than seen anything so far
- Sends to all processes
- Processes wait, respond once to highest ballot id
  - If potential leader sees a higher ballot id, it can't be a leader
  - Paxos tolerant to multiple leaders, but we'll only discuss 1 leader case
  - Processes also log received ballot ID on disk
- If a process has in a previous round decided on a value v', it includes value v' in its response
- If majority (i.e., quorum) respond OK then you are the leader
  - If no one has majority, start new round
- (If things go right) A round cannot have two leaders (why?)



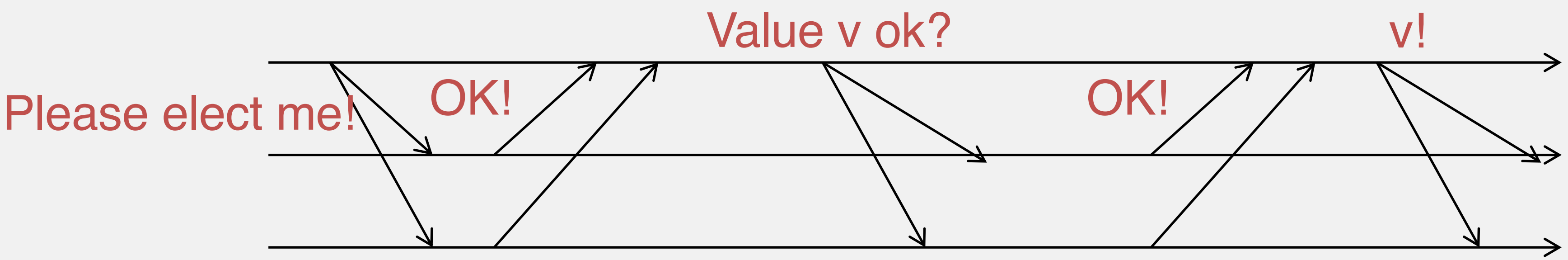
Phase 2 – Proposal (Bill)

- Leader sends proposed value v to all
  - use v=v' if some process already decided in a previous round and sent you its decided value v'
  - If multiple such v' received, use latest one
- Recipient logs on disk; responds OK



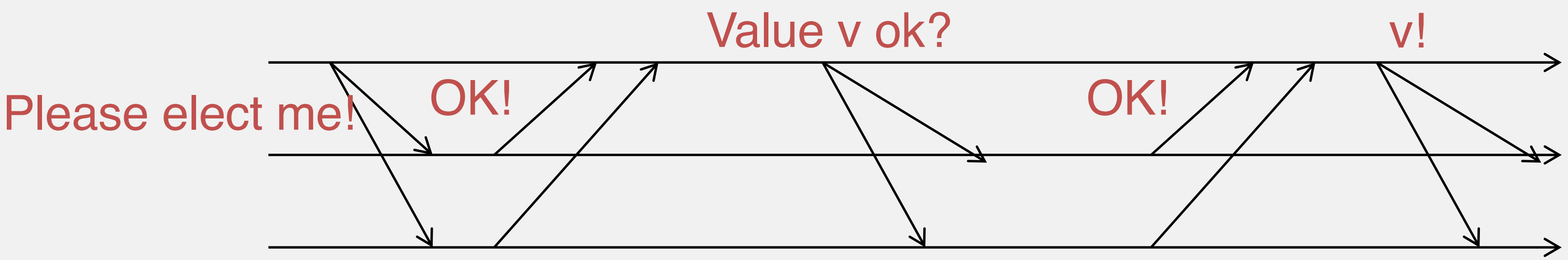
Phase 3 – Decision (Law)

- If leader hears a majority of OKs, it lets everyone know of the decision
- Recipients receive decision, log it on disk



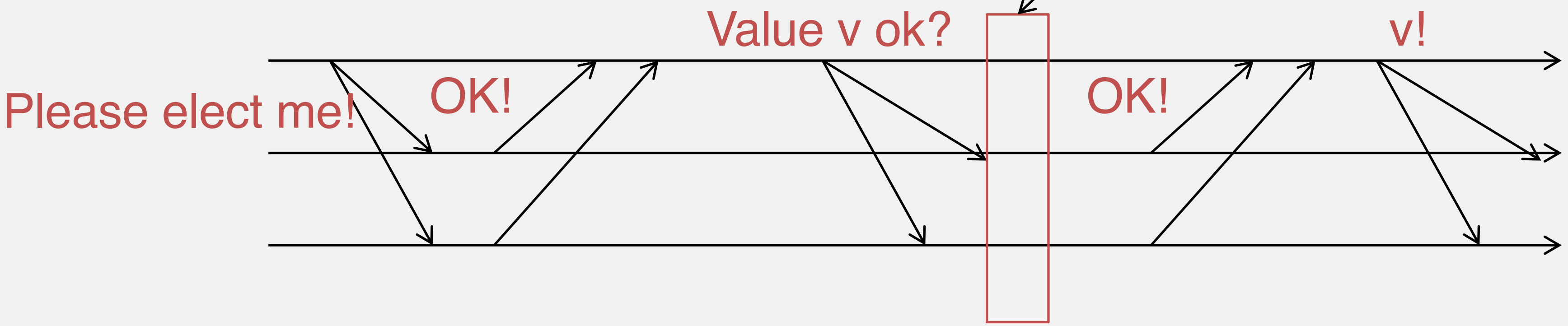
Which is the point of No-Return?

- That is, when is consensus reached in the system



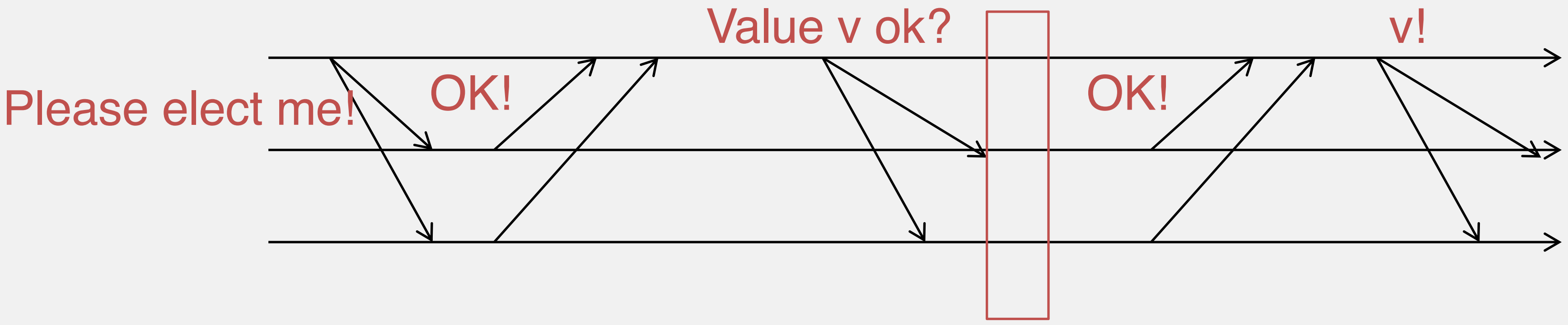
Which is the point of No-Return?

- If/when a majority of processes hear proposed value and accept it (i.e., are about to/have respond(ed) with an OK!)
- Processes *may not know it yet*, but a decision has been made for the group
  - Even leader does not know it yet
- What if leader fails after that?
  - Keep having rounds until some round completes



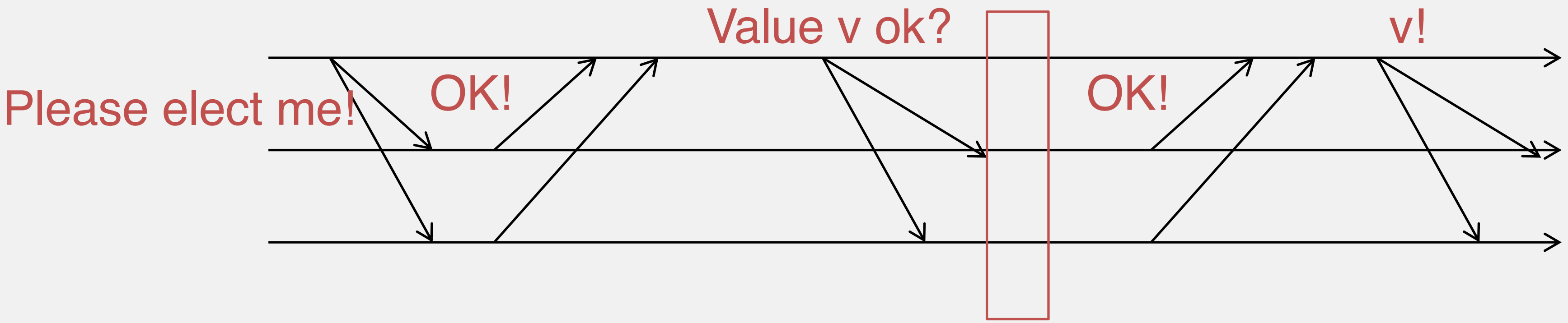
Safety

- If some round has a majority (i.e., quorum) hearing proposed value v' and accepting it, then subsequently at each round either: 1) the round chooses v' as decision or 2) the round fails
- Proof:
  - Potential leader waits for majority of OKs in Phase 1
  - At least one will contain v' (because two majorities or quorums always intersect)
  - It will choose to send out v' in Phase 2
- Success requires a majority, and any two majority sets intersect



What could go Wrong?

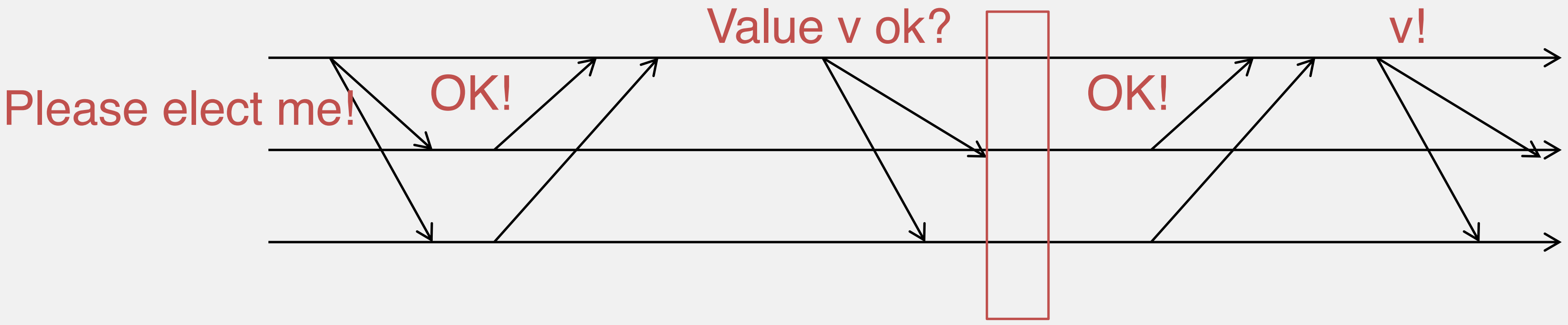
- Process fails
  - Majority does not include it
  - When process restarts, it uses log to retrieve a past decision (if any) and past-seen ballot ids. Tries to know of past decisions.
- Leader fails
  - Start another round
- Messages dropped
  - If too flaky, just start another round
- Note that anyone can start a round any time
- Protocol may never end – tough luck, buddy!
  - Impossibility result not violated
  - If things go well sometime in the future, consensus reached



What could go Wrong?

- A lot more!

- This is a highly simplified view of Paxos.
- See Lamport's original paper: http://research.microsoft.com/en-us/um/people/lamport/pubs/paxos-simple.pdf



Optional Exercises/Questions to Test your Own Knowledge

1. Why does Paxos provide safety?
2. Why does Paxos not provide liveness?
3. Someone implements Paxos but it has a bug – everywhere there was a quorum (>N/2), the new implementation only requires > N/3 processes. Is this new algorithm safe?
4. Paxos appears to be structured in "rounds", which appears to indicate that it is intended for synchronous systems. Why does Paxos still work in an asynchronous system?
5. Assuming no failures, what is the point of no return in Paxos?
6. What could go wrong in Paxos?

Summary

- Paxos protocol: widely used implementation of a safe, eventually-live consensus protocol for asynchronous systems
  - Paxos (or variants) used in Apache Zookeeper, Google's Chubby system, Active Disk Paxos, and many other cloud computing systems
- Other similar protocols: Raft