Mutual Exclusion

CS425 / ECE428 – DISTRIBUTED SYSTEMS – FALL 2021

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Why Mutual Exclusion?

Bank's Servers in the Cloud: Think of two simultaneous deposits of $10,000 into your bank account, each from one ATM.

- Both ATMs read initial amount of $1000 concurrently from the bank's cloud server
- Both ATMs add $10,000 to this amount (locally at the ATM)
- Both write the final amount to the server
- What's wrong?

The ATMs need mutually exclusive access to your account entry at the server (or, to executing the code that modifies the account entry)
Mutual Exclusion

Critical section problem: Piece of code (at all clients) for which we need to ensure there is at most one client executing it at any point of time.

Solutions:
- Semaphores, mutexes, etc. in single-node operating systems
- Message-passing-based protocols in distributed systems:
  - `enter()` the critical section
  - `AccessResource()` in the critical section
  - `exit()` the critical section

Distributed mutual exclusion requirements:
- **Safety** – At most one process may execute in CS at any time
- **Liveness** – Every request for a CS is eventually granted
- **Ordering** (desirable) – Requests are granted in the order they were made
Single-process mutex

```python
await lock.acquire()

# do critical section

lock.release()
```
Distributed Mutual Exclusion: Performance Evaluation Criteria

**Bandwidth**: the total number of messages sent in each entry and exit operation.

**Client delay**: the delay incurred by a process at each entry and exit operation (when no other process is in, or waiting)
- (We will prefer mostly the entry operation.)

**Synchronization delay**: the time interval between one process exiting the critical section and the next process entering it (when there is only one process waiting)

These translate into throughput — the rate at which the processes can access the critical section, i.e., $x$ processes per second.

(These definitions more correct than the ones in the textbook)
Assumptions/System Model

For all the algorithms studied, we make the following assumptions:

- Each pair of processes is connected by reliable channels (such as TCP).
- Messages are eventually delivered to recipients' input buffer in FIFO order.
- Processes do not fail (why?)
Central server Mutex

# client
send(server, "request")
message = await receive(server)
assert message = "granted"
# do critical section
send(server, "release")

# server
async def process_message(message):
    if message == "request":
        await lock.acquire()
        send(message.sender, "granted")
        lock.release()
    else:
        assert message == "release"
Central server Mutex

# message loop
while True:
    message = receive_message()
    if message == None:
        break
    asyncio.create_task(
        process_message(message))

# server
async def process_message(message):
    if message == "request":
        await lock.acquire()
        send(message.sender, "granted")
    else:
        assert message == "release"
        lock.release()
def process_message(message):
    if message == "request:
        if held == False:
            held = True
            send(message.sender, "granted")
        else:
            queue.append(message)
    else:
        assert message == "release"
        if queue.empty():
            held = False
            else:
                message = queue.pop(0)
                send(message.sender, "granted")
Refresher - Mutexes

To synchronize access of multiple threads to common data structures
  Allows two operations:
    lock()
      while true: // each iteration atomic
        if lock not in use:
          label lock in use
          break
    unlock()
      label lock not in use
How are mutexes used?

mutex L = UNLOCKED;

ATM1:
lock(L); // enter
// critical section
obtain bank amount;
add in deposit;
update bank amount;
unlock(L); // exit

extern mutex L;

ATM2
lock(L); // enter
// critical section
obtain bank amount;
add in deposit;
update bank amount;
unlock(L); // exit

One Use: Mutual Exclusion – Bank ATM example
1. Centralized Control of Mutual Exclusion

Central coordinator arbitrates CS access
- elected (next lecture)

Upon receiving *request* message from *p*
- If not in critical section, send *grant* to *p*
  - Mark self as in CS
- Else, add *p*'s request to the *queue*

Upon receiving *release* message
- Send *grant* message to first process in the *queue*
- If *queue* empty
  - Mark self as not in CS

Client enter CS
- Send *request* message
- Wait for *grant* message

Client exit CS
- Send *release* message

Guarantees
- Safety
- Liveness
- Ordering

Bandwidth: 2 msg / enter,
1 msg / exit

Client delay: 1 RTT (request/grant)

Synchronization delay: 2 one-way latencies
Server code:

Lock L;
handle_message() {
    if (message = request) {
        lock(L);
        send(grant)
    } else
    if (message = release) {
        unlock(L);
    }
}
2. Token Ring Approach

Processes are organized in a logical ring: pi has a communication channel to pi+1 mod (n).

Operations:
- Only the process holding the token can enter the CS.
- To enter the critical section, wait passively for the token. When in CS, hold on to the token.
- To exit the CS, the process sends the token onto its neighbor.
- If a process does not want to enter the CS when it receives the token, it forwards the token to the next neighbor.
2. Token Ring Approach

Features:
- Safety & liveness are guaranteed, but ordering is not.
- Bandwidth: 1 message per exit
- Client delay: 0 to N message transmissions.
- Synchronization delay between one process's exit from the CS and the next process's entry is between 1 and N-1 message transmissions.
Multicast-based solution

state = \{ \text{wanted, held, free} \}

enter:
- \text{state} = \text{wanted}
- \text{multicast(“request”) to all other nodes}
- \text{wait for reply from each node}
- \text{state} = \text{held}

exit:
- \text{state} = \text{free}
- \text{reply to all processes in queue}

When receiving a request message from \( p_i \)

If \text{state} = \text{free}
- \text{Send reply to } p_i

Else
- \text{Add } p_i \text{ to queue}

Properties
- Safety?
- Liveness?
Ricart-Agrawala

\[ \text{state} = \{ \text{wanted, held, free} \} \]

**enter:**
- \( \text{state} = \text{wanted} \)
- multicast(“request”, \( T, p_i \)) to all other nodes
  - \( T \): Lamport timestamp, used to break cycles
- wait for reply from each node
- \( \text{state} = \text{held} \)

**exit:**
- \( \text{state} = \text{free} \)
- reply to all processes in queue

When receiving a request message from \( p_i \) with timestamp \( T \)

- If \( \text{state} = \text{free} \) OR
  - \( \text{state} = \text{wanted} \) and \( T_{ip_i} < T_{jp_j} \)
    - Send reply to \( p_i \)
- else
  - Add \( p_i \) to queue
Ricart-Agrawala Diagram
Ricart-Agrawala Safety

If $P_i$ and $P_j$ both want to enter critical section with timestamps $T_i$ and $T_j$
  ◦ Assume wolog $T_i, P_i < T_j, P_j$

**Case 1:** $P_j$’s request arrives at $P_i$ when $P_i$ is in wanted state
  ◦ $P_i$ queues request until after critical section

**Case 2:** $P_j$’s request arrives when $P_i$ is in free state
  ◦ Then $P_i$ must have finished CS already
  ◦ Otherwise $P_j$’s request happened before (Lamport) $P_i$’s
    ◦ Then $T_i > T_j$ (contradiction)
Ricart-Agrawala Properties

Corollary 1:
- If $T_i < T_j$ then $P_i$ enters critical section before $P_j$

Corollary 2: Liveness
- When $P_i$ wants to enter with timestamp $T_i$, it only needs to wait for processes with lower timestamps
- When each process receives request, all future requests will have higher timestamps

Corollary 3: Ordering
- Requests are satisfied in causal order
- ~ best you can do!
Ricart-Agrawala Performance

Bandwidth
- 1 multicast message (request)
- N-1 unicast messages (replies)
- O(N) unicast messages if implemented using B-multicast

Client delay
- O(1) if propagation delay dominates
- O(N) if transmission delay dominates

Synchronization delay
- O(1)

Delay components
- Transmission delay: time to write message to wire
- Propagation delay: time for signals to travel to destination

Transmission delay, 100 byte msg:
- 1 Gbps wired: 800 ns; WiFi: 1-3 us

Propagation delay:
- Same data center: 100-200 us
- Same city: 1-10 ms
- Inter-city: 10-200 ms
Analysis: Ricart & Agrawala

Safety, liveness, and ordering (causal) are guaranteed
  ◦ Why?

Bandwidth: 2(N-1) messages per entry operation
  ◦ N-1 unicasts for the multicast request + N-1 replies
  ◦ N messages if the underlying network supports multicast
  ◦ N-1 unicast messages per exit operation
    ◦ 1 multicast if the underlying network supports multicast

Client delay: one round-trip time

Synchronization delay: one message transmission time
Maekawa’s algorithm

- First solution with a sublinear $O(\sqrt{N})$ message complexity.

- “Close to” Ricart-Agrawala’s solution, but each process is required to obtain permission from only a subset of peers
Maekawa’s algorithm

With each process $i$, associate a subset $S_i$. Divide the set of processes into subsets that satisfy the following two conditions:

- $i \in S_i$
- $\forall i,j : 0 \leq i,j \leq n-1 \mid S_i \cap S_j \neq \emptyset$

Main idea. Each process $i$ is required to receive permission from $S_i$ only. Correctness requires that multiple processes will never receive permission from all members of their respective subsets.
Maekawa’s algorithm

Example. Let there be seven processes 0, 1, 2, 3, 4, 5, 6

\[
\begin{align*}
S_0 &= \{0, 1, 2\} \\
S_1 &= \{1, 3, 5\} \\
S_2 &= \{2, 4, 5\} \\
S_3 &= \{0, 3, 4\} \\
S_4 &= \{1, 4, 6\} \\
S_5 &= \{0, 5, 6\} \\
S_6 &= \{2, 3, 6\}
\end{align*}
\]
Maekawa’s algorithm

Version 1 \{Life of process I\}

1. Send timestamped request to each process in Si.

2. Request received \(\rightarrow\) send ack to process with the lowest timestamp. Thereafter, "lock" (i.e. commit) yourself to that process, and keep others waiting.

3. Enter CS if you receive an ack from each member in Si.

4. To exit CS, send release to every process in Si.

5. Release received \(\rightarrow\) unlock yourself. Then send ack to the next process with the lowest timestamp.

\[
\begin{align*}
S_0 &= \{0, 1, 2\} \\
S_1 &= \{1, 3, 5\} \\
S_2 &= \{2, 4, 5\} \\
S_3 &= \{0, 3, 4\} \\
S_4 &= \{1, 4, 6\} \\
S_5 &= \{0, 5, 6\} \\
S_6 &= \{2, 3, 6\}
\end{align*}
\]
Maekawa’s algorithm-version 1

Safety. At most one process can enter its critical section at any time.

Let i and j attempt to enter their Critical Sections

\( S_i \cap S_j \neq \emptyset \) implies there is a process \( k \in S_i \cap S_j \)

Process k will never send ack to both.

So it will act as the arbitrator and establishes ME1

\[
\begin{align*}
S_0 &= \{0, 1, 2\} \\
S_1 &= \{1, 3, 5\} \\
S_2 &= \{2, 4, 5\} \\
S_3 &= \{0, 3, 4\} \\
S_4 &= \{1, 4, 6\} \\
S_5 &= \{0, 5, 6\} \\
S_6 &= \{2, 3, 6\}
\end{align*}
\]
Maekawa’s algorithm-version 1

Liveness. *Unfortunately deadlock is possible!* Assume 0, 1, 2 want to enter their critical sections.

\[
S_0 = \{0, 1, 2\} \\
S_1 = \{1, 3, 5\} \\
S_2 = \{2, 4, 5\} \\
S_3 = \{0, 3, 4\} \\
S_4 = \{1, 4, 6\} \\
S_5 = \{0, 5, 6\} \\
S_6 = \{2, 3, 6\}
\]

From \(S_0 = \{0, 1, 2\}\), 0, 2 send \textit{ack} to 0, but 1 sends \textit{ack} to 1;

From \(S_1 = \{1, 3, 5\}\), 1, 3 send \textit{ack} to 1, but 5 sends \textit{ack} to 2;

From \(S_2 = \{2, 4, 5\}\), 4, 5 send \textit{ack} to 2, but 2 sends \textit{ack} to 0;

Now, 0 waits for 1 \textit{(to send a release)}, 1 waits for 2 \textit{(to send a release)}, and 2 waits for 0 \textit{(to send a release)}. So deadlock is possible!
Maekawa’s algorithm-Version 2

Avoiding deadlock

If processes always receive messages in increasing order of timestamp, then deadlock “could be” avoided. But this is too strong an assumption.

Version 2 uses three additional messages:

- failed
- inquire
- relinquish

\[
\begin{align*}
S_0 &= \{0, 1, 2\} \\
S_1 &= \{1, 3, 5\} \\
S_2 &= \{2, 4, 5\} \\
S_3 &= \{0, 3, 4\} \\
S_4 &= \{1, 4, 6\} \\
S_5 &= \{0, 5, 6\} \\
S_6 &= \{2, 3, 6\}
\end{align*}
\]
Maekawa’s algorithm-Version 2

New features in version 2

- Send **ack** and set **lock** as usual.

- If **lock** is **set** and a request with a larger timestamp arrives, send **failed** (*you have no chance*). If the incoming request has a lower timestamp, then send **inquire** (*are you in CS?*) to the locked process.

- Receive **inquire** and at least one **failed** message → send **relinquish**. The recipient resets the lock.
Maekawa’s algorithm-Version 2
Let $K = |S_i|$. Let each process be a member of $D$ subsets. When $N = 7$, $K = D = 3$. When $K=D$, $N = K(K-1)+1$. So $K = O(\sqrt{N})$

- The message complexity of Version 1 is $3\sqrt{N}$. Maekawa’s analysis of Version 2 reveals a complexity of $7\sqrt{N}$
Matrix construction
Summary

Mutual exclusion
- Coordinator-based token
- Token ring
- Ricart and Agrawala's timestamp algo.
- Maekawa's algo.