
HW 1 Solution – Natural Semantics and Typing

CS 422 – Spring 2016
Revision 1.0

Assigned January 27, 2016
Due February 3, 2016, 8:00pm
Extension 24 hours (10% penalty)

1 Change Log

1.0 Initial Release.

2 Turn-In Procedure

Answer the following problems, save your work as a PDF named `hw1-submission.pdf` (either scanned if hand-written, or converted from a program), and submit your PDF by first adding it to your svn repository (`svn add hw1-submission.pdf`) and commit it (`svn commit -m "submitting hw1"`). Your file contains your name, netid and solution. It should be named `hw1-submission.pdf` and committed in your `assignments/hw1` directory.

3 Objectives

The purpose of this HW is to familiarize you with Natural Semantics and its similarity and dissimilarity to a type system for the first simple imperative programming language discussed in class.

4 Background

In both problems below, you will be working with the language SIMPL1 discussed in class:

$$\begin{aligned} I &\in \text{Identifiers} \\ N &\in \text{Numerals} \\ E &::= N \mid I \mid E + E \mid E * E \mid E - E \\ B &::= \text{true} \mid \text{false} \mid B \& B \mid B \text{ or } B \mid \text{not } B \mid E < E \mid E = E \\ C &::= \text{skip} \mid C; C \mid \{C\} \mid I ::= E \mid \text{if } B \text{ then } C \text{ else } C \text{ fi} \mid \text{while } B \text{ do } C \text{ od} \end{aligned}$$

Assuming a set Values of final results of expressions (in this case you can assume integers), and $m, m' : \text{Identifiers} \rightarrow \text{Values}$, in class we gave a Natural Semantics for the SIMPL1 as follows:

Constants:

Identifiers: $(I, m) \Downarrow m(I)$ if $m(I)$ exists Numerals are values: $(N, m) \Downarrow N$
Booleans: $(\text{true}, m) \Downarrow \text{true}$ $(\text{false}, m) \Downarrow \text{false}$

Arithmetic Expressions:

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \oplus V = N}{(E \oplus E', m) \Downarrow N} \text{ where } \oplus \in \{+, *, -\} \text{ and } U, V \in \text{Values}$$

Arithmetic Relations:

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \sim V = b}{(E \sim E', m) \Downarrow b} \text{ where } \sim \in \{=, <\}$$

Boolean Expressions:

$$\begin{array}{ccc} \frac{(B, m) \Downarrow \text{false}}{(B \& B', m) \Downarrow \text{false}} & \frac{(B, m) \Downarrow \text{false} \quad (B', m) \Downarrow b}{(B \text{ or } B', m) \Downarrow b} & \frac{(B, m) \Downarrow \text{false}}{(\text{not } B, m) \Downarrow \text{true}} \\ \frac{(B, m) \Downarrow \text{true} \quad (B', m) \Downarrow b}{(B \& B', m) \Downarrow b} & \frac{(B, m) \Downarrow \text{true}}{(B \text{ or } B', m) \Downarrow \text{true}} & \frac{(B, m) \Downarrow \text{true}}{(\text{not } B, m) \Downarrow \text{false}} \end{array}$$

Commands:

$$\begin{array}{lll} \text{Assignment: } \frac{(E, m) \Downarrow V}{(I ::= E, m) \Downarrow m[I \leftarrow V]} & \text{where } m[I \leftarrow V](J) = \begin{cases} V & \text{if } J = I \\ m(J) & \text{otherwise} \end{cases} & \\ \text{Skip: } (\text{skip}, m) \Downarrow m & \text{Sequencing: } \frac{(C, m) \Downarrow m' \quad (C', m') \Downarrow m''}{(C; C', m) \Downarrow m''} & \text{Block: } \frac{(C, m) \Downarrow m'}{(\{C\}, m) \Downarrow m'} \\ \text{If-true: } \frac{(B, m) \Downarrow \text{true} \quad (C, m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'} & \text{If-false: } \frac{(B, m) \Downarrow \text{false} \quad (C', m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'} & \\ \text{While-false: } \frac{(B, m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m} & & \\ \text{While-true: } \frac{(B, m) \Downarrow \text{true} \quad (C, m) \Downarrow m' \quad (\text{while } B \text{ do } C \text{ od}, m') \Downarrow m''}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m''} & & \end{array}$$

Using $\text{Types} = \{\text{int}, \text{bool}, \text{cmd}\}$ and $\Gamma, \Gamma' : \text{Identifiers} \rightarrow \text{Types}$ we also gave a Typing System for SIMPL1 as follows:

Constants:

$$\begin{array}{ll} \text{Identifiers: } \Gamma \vdash I : \Gamma(I) & \text{if } \Gamma(I) \text{ exists} \quad \text{Numerals: } \Gamma \vdash N : \text{int} \\ \text{Booleans: } \Gamma \vdash \text{true} : \text{bool} & \Gamma \vdash \text{false} : \text{bool} \end{array}$$

Arithmetic Expressions:

$$\frac{\Gamma \vdash E : \text{int} \quad \Gamma \vdash E' : \text{int}}{\Gamma \vdash E \oplus E' : \text{int}} \text{ where } \oplus \in \{+, *, -\}$$

Arithmetic Relations:

$$\frac{\Gamma \vdash E : \text{int} \quad \Gamma \vdash E' : \text{int}}{\Gamma \vdash E \sim E' : \text{bool}} \text{ where } \sim \in \{=, <\}$$

$$\frac{\Gamma \vdash B : \text{bool} \quad \Gamma \vdash B' : \text{bool}}{\Gamma \vdash B \& B' : \text{bool}} \quad \frac{\Gamma \vdash B : \text{bool} \quad \Gamma \vdash B' : \text{bool}}{\Gamma \vdash B \text{ or } B' : \text{bool}} \quad \frac{\Gamma \vdash B : \text{bool}}{\Gamma \vdash \text{not } B : \text{bool}}$$

Skip: $\Gamma \vdash \text{skip} : \text{cmd}$	Assignment: $\frac{\Gamma \vdash E : \text{int}}{\Gamma \vdash I ::= E : \text{cmd}}$	
Block: $\frac{\Gamma \vdash C : \text{cmd}}{\Gamma \vdash \{C\} : \text{cmd}}$	Sequencing: $\frac{\Gamma \vdash C' : \text{cmd} \quad \Gamma \vdash C'' : \text{cmd}}{\Gamma \vdash C; C'' : \text{cmd}}$	
IfThenElse: $\frac{\Gamma \vdash B : \text{int} \quad \Gamma \vdash C : \text{cmd} \quad \Gamma \vdash C' : \text{cmd}}{\text{if } B \text{ then } C \text{ else } C' \text{ fi} : \text{cmd}}$	While: $\frac{\Gamma \vdash B : \text{bool} \quad \Gamma \vdash C : \text{cmd}}{\Gamma \vdash \text{while } B \text{ do } C \text{ od} : \text{cmd}}$	

1. Give a Natural Semantics in derivation in SIMPL1 for

Solution:

2. Give a type derivation in SIMPL1 for the typing judgment

Solution:

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