Name:

CS 421 — Small Step Semantics Activity

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The Rules

$$< \texttt{skip}\,, \sigma > \to < E, \sigma >$$
$$< u := t, \sigma > \to < E, \sigma[u := \sigma(t)] >$$

$$\frac{< S_1, \sigma > \to < S_2, \tau >}{< S_1; S, \sigma > \to < S_2; S, \tau >}$$
$$E; S \equiv S$$

$$< \texttt{if } B \texttt{ then } S_1 \texttt{ else } S_2 \texttt{ fi}\,, \sigma > \to < S_1, \sigma > \text{ where } \sigma \models B$$
$$< \texttt{if } B \texttt{ then } S_1 \texttt{ else } S_2 \texttt{ fi}\,, \sigma > \to < S_2, \sigma > \text{ where } \sigma \models \neg B$$

$$< \texttt{while } B \texttt{ do } S_1 \texttt{ od}\,, \sigma > \to < S_1; \texttt{while } B \texttt{ do } S_1 \texttt{ od}\,, \sigma > \text{ where } \sigma \models B$$
$$< \texttt{while } B \texttt{ do } S_1 \texttt{ od}\,, \sigma > \to < E, \sigma > \text{ where } \sigma \models \neg B$$

Reductions

Reduce the following programs according to the semantic rules given.

Problem 1)

```
< if x>y then m:=x else skip fi; if x<y then m:=y else skip fi;, {x:=10,y:=30}>
```

Problem 2)

```
<n := 0; while x > 1 do x:=x/2; n:=n+1 od, {x:=8}>
```

Problem 3)

(Don't spend too much time on this one.)

```
<p:=1; n:=3; while n>1 do p:=p*x od, {x:=3}>
```

Make your own rules!

Problem 4)

Write a rule to explain the `when B S` statement. It executes S only if B is true.

Problem 5)

Write a rule for `do S while B od`. It is like while, but executes S at least one time.

Church Rosser

Problem 6) Consider this semantic rule:

$$x_1 \circ x_2 \circ \cdots \circ x_i \circ x_{i+1} \circ \cdots x_n \rightarrow x_1 \circ x_2 \circ \cdots \circ (x_i * x_{i+1}) \circ \cdots x_n$$

Does it have the Church-Rosser property? Try to prove it.

Problem 7) Consider this semantic rule:

$$x_1 \circ x_2 \circ \cdots \circ x_i \circ x_{i+1} \circ \cdots x_n \rightarrow x_1 \circ x_2 \circ \cdots \circ (x_i - x_{i+1}) \circ \cdots x_n$$

Does it have the Church-Rosser property? Try to prove it.