
HW 8 – Lambda Calculus and Hoare Logic

CS 421 – Summer 2013

Revision 1.1

Assigned July 18, 2013

Due July 30, 2013, 11:59 PM

Extension 24 hours (20% penalty)

1 Change Log

1.0 Initial Release.

1.1 Fixed a typo in problem 2, and changed the due date.

2 Turn-In Procedure

Your answers to the following questions are to be submitted electronically via the `handin` script as though an MP.

3 Objectives and Background

The purpose of this HW is to test your understanding of

- α -conversion and β -reduction
- evaluation schemes for the lambda calculus
- Hoare logic proofs

Another purpose of HWs is to provide you with experience answering non-programming written questions of the kind you may experience on the midterms and final. We won't have covered all of the material on this homework when it's released, but it's also a bit longer than most, so please try to start early.

4 Problems

1. (18 pts) Consider the term $A = \lambda x. \lambda y. (\lambda x. x) y$. For each term below, indicate whether it can be α -converted to A , whether it is α -equivalent to A , and whether it is $\alpha\beta$ -equivalent to A .

	α -convertible	α -equivalent	$\alpha\beta$ -equivalent
$\lambda y. \lambda y. (\lambda x. x) y$			
$\lambda y. (\lambda x. x) y$			
$\lambda x. \lambda y. y$			
$\lambda y. \lambda y. (\lambda y. y) y$			
$\lambda x. (\lambda y. y x) (\lambda x. \lambda z. z)$			
$(\lambda x. \lambda y. \lambda z. x z) (\lambda w. w)$			

2. (15 pts) Reduce the following term, using α -conversion and β -reduction, as far as possible with each of the specified evaluation strategies. Assume that application is left-associative. Show all intermediate steps.

$$(\lambda x. (\lambda y. x) x) ((\lambda y. \lambda z. y y) (\lambda x. x))$$

1. (5pts) eager evaluation
2. (5pts) lazy evaluation
3. (5pts) unrestricted $\alpha\beta$ -reduction (i.e., you can apply α -conversion or β -reduction anywhere you like in the term)

3. (30 pts) Prove the following Hoare triple:

$$\{a > 0 \wedge b \geq 0\}$$

`x := 1; y := b; while y > 0 do (x := x * a; y := y - 1) od`

$$\{x = a^b\}$$