Programming Languages and Compilers (CS 421)

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Objective

- Finish the discussion on lexing
- Ocamllex (takes .mll files and generates .ml)
- Context-Free Grammars and BNF

General Input for .mll Files

{ *header* }
let *ident* = *regexp* ...
rule *entrypoint* [*arg1*... *argn*] = parse
regexp { *action* }
| ...
| *regexp* { *action* }
and *entrypoint* [*arg1*... *argn*] = parse ...and
...
{ *trailer* }

Ocamllex Input

- *header* and *trailer* contain arbitrary ocaml code put at top an bottom of *<filename>*.ml
- let *ident* = *regexp* ... Introduces *ident* for use in later regular expressions

Ocamllex Input

- *<filename>*.ml contains one lexing function per *entrypoint*
  - Name of function is name given for *entrypoint*
  - Each entry point becomes an Ocaml function that takes $n+1$ arguments, the extra implicit last argument being of type Lexing.lexbuf
- *arg1* ... *argn* are for use in *action*

Ocamllex Regular Expression

- Single quoted characters for letters: 'a'
- _: (underscore) matches any letter
- Eof: special "end_of_file" marker
- Concatenation same as usual
- "*string*": concatenation of sequence of characters
- $e_1 \mid e_2$: choice - what was $e_1 \vee e_2$

Ocamllex Regular Expression

- $[c_1 - c_2]$: choice of any character between first and second inclusive, as determined by character codes
- $[\hat{}c_1 - c_2]$: choice of any character NOT in set
- $e*$: same as before
- $e+$: same as $e\ e*$
- $e?$: option - was $e_1 \vee \varepsilon$

Ocamllex Regular Expression

- $e_1 \# e_2$: the characters in e_1 but not in e_2; e_1 and e_2 must describe just sets of characters
- *ident*: abbreviation for earlier reg exp in let *ident* = *regexp*
- e_1 as *id*: binds the result of e_1 to *id* to be used in the associated *action*

Ocamllex Manual

- More details can be found at

http://caml.inria.fr/pub/docs/manual-ocaml/lexyacc.html

Example : test.mll

```
{ type result = Int of int | Float of float |
  String of string }
let digit = ['0'-'9']
let digits = digit +
let lower_case = ['a'-'z']
let upper_case = ['A'-'Z']
let letter = upper_case | lower_case
let letters = letter +
```


Example : test.mll

```
rule main = parse
  (digits)'.'digits as f  { Float (float_of_string f) }
| digits as n             { Int (int_of_string n) }
| letters as s            { String s}
| _ { main lexbuf }
{ let newlexbuf = (Lexing.from_channel stdin) in
print_string "Ready to lex.";
print_newline ();
main newlexbuf  }
```


Example

```
#use "test.ml";;
...
val main : Lexing.lexbuf -> result = <fun>
val __ocaml_lex_main_rec : Lexing.lexbuf -> int ->
  result = <fun>
Ready to lex.
hi there 234 5.2
- : result = String "hi"
```

What happened to the rest?!?

Example

```
# let b = Lexing.from_channel stdin;;
# main b;;
hi 673 there
- : result = String "hi"
# main b;;
- : result = Int 673
# main b;;
- : result = String "there"
```


Problem

- How to get lexer to look at more than the first token at one time?
- Answer: *action* has to tell it to -- recursive calls
- Side Benefit: can add “state” into lexing
- Note: already used this with the _ case

Example

```
rule main = parse
   (digits) '.' digits as f { Float
   (float_of_string f) :: main lexbuf}
  | digits as n          { Int (int_of_string n) ::
    main lexbuf }
  | letters as s         { String s :: main
    lexbuf}
  | eof                   { [] }
  | _                     { main lexbuf }
```


Example Results

```
Ready to lex.
hi there 234 5.2
- : result list = [String "hi"; String "there"; Int
  234; Float 5.2]
#
```

Used Ctrl-d to send the end-of-file signal

Dealing with comments

First Attempt

```
let open_comment = "(*"
let close_comment = "*)"
rule main = parse
   (digits) '.' digits as f { Float (float_of_string
   f) :: main lexbuf}
  | digits as n          { Int (int_of_string n) ::
    main lexbuf }
  | letters as s         { String s :: main lexbuf}
```


Dealing with comments

```
  | open_comment        { comment  lexbuf}
  | eof                 { [] }
  | _ { main lexbuf }
and comment = parse
    close_comment       { main lexbuf }
  | _                   { comment lexbuf }
```

Dealing with nested comments

```
rule main = parse …
 | open_comment        { comment 1 lexbuf}
 | eof                 { [] }
 | _ { main lexbuf }
and comment depth = parse
   open_comment        { comment (depth+1) lexbuf
   }
 | close_comment       { if depth = 1
                       then main lexbuf
                       else comment (depth - 1) lexbuf }
 | _                   { comment depth lexbuf }
```


Dealing with nested comments

```
rule main = parse
   (digits) '.' digits as f { Float (float_of_string f) ::
   main lexbuf}
 | digits as n         { Int (int_of_string n) :: main
   lexbuf }
 | letters as s        { String s :: main lexbuf}
 | open_comment        { (comment 1 lexbuf}
 | eof                 { [] }
 | _ { main lexbuf }
```


Dealing with nested comments

```
and comment depth = parse
   open_comment        { comment (depth+1) lexbuf
   }
 | close_comment       { if depth = 1
                       then main lexbuf
                       else comment (depth - 1) lexbuf }
 | _                   { comment depth lexbuf }
```


Types of Formal Language Descriptions

- Regular expressions, regular grammars
- Context-free grammars, BNF grammars, syntax diagrams
- Finite state automata

- Whole family more of grammars and automata – covered in automata theory

Sample Grammar

- Language: Parenthesized sums of 0's and 1's

- <Sum> ::= 0
- <Sum >::= 1
- <Sum> ::= <Sum> + <Sum>
- <Sum> ::= (<Sum>)

BNF Grammars

- Start with a set of characters, **a,b,c,…**
  - We call these *terminals*
- Add a set of different characters, **X,Y,Z,…**
  - We call these *nonterminals*
- One special nonterminal **S** called *start symbol*

BNF Grammars

- BNF rules (aka *productions*) have form

  X ::= *y*

  where **X** is any nonterminal and *y* is a string of terminals and nonterminals
- BNF *grammar* is a set of BNF rules such that every nonterminal appears on the left of some rule

Sample Grammar

- Terminals: 0 1 + ( )
- Nonterminals: <Sum>
- Start symbol = <Sum>

- <Sum> ::= 0
- <Sum >::= 1
- <Sum> ::= <Sum> + <Sum>
- <Sum> ::= (<Sum>)
- Can be abbreviated as

<Sum> ::= 0 | 1
| <Sum> + <Sum> | (<Sum>)

BNF Deriviations

- Given rules

  X::= *y* **Z** *w* and **Z** ::= *v*

we may replace **Z** by *v* to say

X => *y* **Z** *w* => *yvw*

- Sequence of such replacements called *derivation*
- Derivation called *right-most* if always replace the right-most non-terminal

BNF Derivations

- Start with the start symbol:

<Sum> =>

BNF Derivations

- Pick a non-terminal

<Sum> =>

BNF Derivations

- Pick a rule and substitute:
  - <Sum> ::= <Sum> + <Sum>

<Sum> => <Sum> + <Sum >

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >

BNF Derivations

- Pick a rule and substitute:
  - <Sum> ::= (<Sum>)

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>

BNF Derivations

- Pick a rule and substitute:
  - <Sum> ::= <Sum> + <Sum>

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>

BNF Derivations

- Pick a rule and substitute:
  - <Sum >::= 1

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>

BNF Derivations

- Pick a rule and substitute:
  - <Sum >::= 0

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) + 0

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) + 0

BNF Derivations

- Pick a rule and substitute
  - <Sum> ::= 0

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) 0
=> (0 + 1) + 0

BNF Derivations

- (0 + 1) + 0 is generated by grammar

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) + 0
=> (0 + 1) + 0

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

<Sum> =>

BNF Semantics

- The meaning of a BNF grammar is the set of all strings consisting only of terminals that can be derived from the Start symbol

Regular Grammars

- Subclass of BNF
- Only rules of form
  <nonterminal>::=<terminal><nonterminal> or
  <nonterminal>::=<terminal> or
  <nonterminal>::=ε
- Defines same class of languages as regular expressions
- Important for writing lexers (programs that convert strings of characters into strings of tokens)

Example

- Regular grammar:
  <Balanced> ::= ε
  <Balanced> ::= 0 <OneAndMore>
  <Balanced> ::= 1 <ZeroAndMore>
  <OneAndMore> ::= 1 <Balanced>
  <ZeroAndMore> ::= 0 <Balanced>
- Generates even length strings where every initial substring of even length has same number of 0's as 1's

Extended BNF Grammars

- Alternatives: allow rules of from X ::= *y* | *z*
  - Abbreviates X ::= *y*, X ::= *z*
- Options: X ::= *y* [*v*] *z*
  - Abbreviates X ::= *yvz*, X ::= *yz*
- Repetition: X ::= *y* { *v* }* *z*
  - Can be eliminated by adding new nonterminal V and rules

  X ::= *yz* X ::= yVz V ::= *v* V ::= *v* V

Parse Trees

- Graphical representation of derivation
- Each node labeled with either non-terminal or terminal
- If node is labeled with a terminal, then it is a leaf (no sub-trees)
- If node is labeled with a non-terminal, then it has one branch for each character in the right-hand side of rule used to substitute for it

Example

- Consider grammar:
  <exp> ::= <factor>
  | <factor> + <factor>
  <factor> ::= <bin>
  | <bin> * <exp>
  <bin> ::= 0 | 1
- Problem: Build parse tree for 1 * 1 + 0 as an <exp>

Example cont.

- 1 * 1 + 0: <exp>

<exp> is the start symbol for this parse tree

Example cont.

- 1 * 1 + 0: <exp>
  <factor>

Use rule: <exp> ::= <factor>

Example cont.

- 1 * 1 + 0: <exp>
  <factor>
  <bin> * <exp>

Use rule: <factor> ::= <bin> * <exp>

Example cont.

- 1 * 1 + 0: <exp>
  <factor>
  <bin> * <exp>
  1 <factor> + <factor>

Use rules: <bin> ::= 1 and
<exp> ::= <factor> + <factor>

Example cont.

- 1 * 1 + 0: <exp>
  <factor>
  <bin> * <exp>
  1 <factor> + <factor>
  <bin> <bin>

Use rule: <factor> ::= <bin>

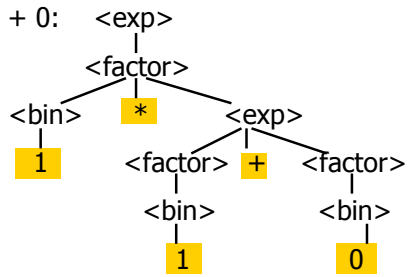
Example cont.

- 1 * 1 + 0: <exp>
  <factor>
  <bin> * <exp>
  1 <factor> + <factor>
  <bin> <bin>
  1 0

Use rules: <bin> ::= 1 | 0

Example cont.

- 1 * 1 + 0:



Fringe of tree is string generated by grammar

Your Turn: 1 * 0 + 0 * 1

Parse Tree Data Structures

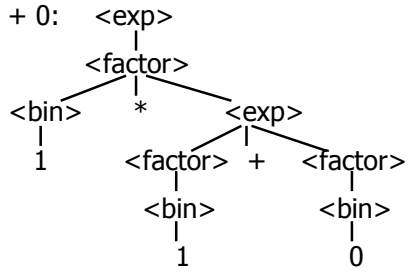
- Parse trees may be represented by OCaml datatypes
- One datatype for each nonterminal
- One constructor for each rule
- Defined as mutually recursive collection of datatype declarations

Example

- Recall grammar:
  <exp> ::= <factor> | <factor> + <factor>
  <factor> ::= <bin> | <bin> * <exp>
  <bin> ::= 0 | 1
- type exp = Factor2Exp of factor
  | Plus of factor * factor
  and factor = Bin2Factor of bin
  | Mult of bin * exp
  and bin = Zero | One

Example cont.

- 1 * 1 + 0:



Example cont.

- Can be represented as

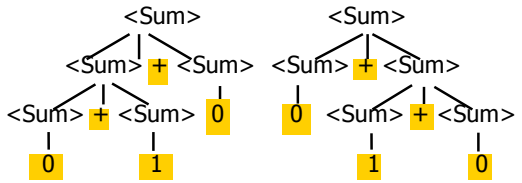
```
Factor2Exp
(Mult(One,
        Plus(Bin2Factor One,
                Bin2Factor Zero)))
```

Ambiguous Grammars and Languages

- A BNF grammar is *ambiguous* if its language contains strings for which there is more than one parse tree
- If all BNF' s for a language are ambiguous then the language is *inherently ambiguous*

Example: Ambiguous Grammar

- 0 + 1 + 0



Example

- What is the result for:

3 + 4 * 5 + 6

Example

- What is the result for:

3 + 4 * 5 + 6

- Possible answers:
  - 41 = ((3 + 4) * 5) + 6
  - 47 = 3 + (4 * (5 + 6))
  - 29 = (3 + (4 * 5)) + 6 = 3 + ((4 * 5) + 6)
  - 77 = (3 + 4) * (5 + 6)

Example

- What is the value of:

7 – 5 – 2

Example

- What is the value of:

7 – 5 – 2

- Possible answers:
  - In Pascal, C++, SML assoc. left

    7 – 5 – 2 = (7 – 5) – 2 = 0
  - In APL, associate to right

    7 – 5 – 2 = 7 – (5 – 2) = 4

Two Major Sources of Ambiguity

- Lack of determination of operator precedence
- Lack of determination of operator associativity
- Not the only sources of ambiguity

Disambiguating a Grammar

- Given ambiguous grammar G, with start symbol S, find a grammar G′ with same start symbol, such that

  language of G = language of G′
- Not always possible
- No algorithm in general

Disambiguating a Grammar

- Idea: Each non-terminal represents all strings having some property
- Identify these properties (often in terms of things that can't happen)
- Use these properties to inductively guarantee every string in language has a unique parse

Example

- Ambiguous grammar:

  <exp> ::= 0 | 1 | <exp> + <exp>
  | <exp> * <exp>
- String with more then one parse:

  0 + 1 + 0

  1 * 1 + 1
- Source of ambiguity: associativity and precedence

How to Enforce Associativity

- Have at most one recursive call per production
- When two or more recursive calls would be natural leave right-most one for right associativity, left-most one for left associativity

Example

- <Sum> ::= 0 | 1 | <Sum> + <Sum>
  | (<Sum>)
- Becomes
  - <Sum> ::= <Num> | <Num> + <Sum>
  - <Num> ::= 0 | 1 | (<Sum>)

Operator Precedence

- Operators of highest precedence evaluated first (bind more tightly).
- Precedence for infix binary operators given in following table
- Needs to be reflected in grammar

Precedence Table - Sample

	Fortan	Pascal	C/C++	Ada	SML
highest	**	*, /, div, mod	++, --	**	div, mod, /, *
	*, /	+, -	*, /, %	*, /, mod	+, -, ^
	+, -		+, -	+, -	::

First Example Again

- In any above language,

  3 + 4 * 5 + 6 = 29
- In APL, all infix operators have same precedence
  - Thus we still don't know what the value is (handled by associativity)
- How do we handle precedence in grammar?

Predence in Grammar

- Higher precedence translates to longer derivation chain
- Example:

<exp> ::= 0 | 1 | <exp> + <exp>
| <exp> * <exp>

- Becomes

<exp> ::= <mult_exp>
| <exp> + <mult_exp>
<mult_exp> ::= <id> | <mult_exp> * <id>
<id> ::= 0 | 1