HW 12 – Proof Systems

CS 421 – Spring 2009

Revision 1.1

Assigned Thursday, April 16, 2009
Due Wednesday, April 22, in class
Extension 48 hours (20% penalty)
Total points 50

1 Change Log

1.1 Hints and proof tree layouts added.

1.0 Initial Release.

2 Overview

After completing this MP, you should have a better understanding of proof systems for OCaml typing and operational semantics.

3 Collaboration

Collaboration is NOT allowed on this assignment.

4 Instructions

Submit in hard-copy at the beginning of class.

5 Problems (50 pts)

For problems 1-4, use the proof tree layouts given at the end.

1. (10 pts) Using the typing rules for $\mathtt{T}_{\mathtt{OCaml}}$, give the derivation tree for the judgment

$$\emptyset \vdash \mathtt{let\ rec\ x} = \mathtt{fun\ y} \rightarrow \mathtt{x(y)} + \mathtt{1\ in\ x(1)} : \mathtt{int}$$

2. (10 pts) Using the typing rules for $\texttt{T}_{\texttt{OCaml}}$, give the derivation tree for the judgment

$$\emptyset \vdash \texttt{let f} = \texttt{fun x} \rightarrow \texttt{x in (f f)1} : \texttt{int}$$

3. (10 pts) Using the evaluation rules for $\texttt{OS}_{\texttt{clo}}$, give the derivation tree for the judgment

$$\emptyset, (\texttt{fun f} \rightarrow \texttt{f (f 2)})(\texttt{fun y} \rightarrow \texttt{y} + \texttt{1}) \Downarrow \texttt{4}$$

4. (10 pts) Using the evaluation rules for $\texttt{OS}_{\texttt{state}}$, give the derivation tree for the judgment

$$\emptyset, \emptyset \vdash (\texttt{fun x} \rightarrow \texttt{x} := \ !\texttt{x} + \texttt{1})(\texttt{ref 0}) \Downarrow (), \{\ell \mapsto \texttt{1}\}$$

5. (10 pts) Recall that $\texttt{OS}_{\texttt{subst}}$ and $\texttt{OS}_{\texttt{clo}}$ are evaluation models for the same language where the former uses substitution and the latter uses closures. $\texttt{OS}_{\texttt{state}}$ extends $\texttt{OS}_{\texttt{clo}}$ with state to handle references, dereferencing and assignment. Let $\texttt{OS}_{\texttt{ss}}$ be the set of evaluation rules for the same language that still has state, but uses substitution instead of closures. Give the definition of the Application rule in $\texttt{OS}_{\texttt{ss}}$. (Technically, this requires that locations are considered as expressions in order for a substitution to be well-defined. You can assume that this extension has been made.)

   Below are the definitions of the (δ) and (Abstr) rules for your reference.

$$(\delta) \qquad \frac{\sigma \vdash e_1 \Downarrow v_1, \sigma_1 \qquad \sigma_1 \vdash e_2 \Downarrow v_2, \sigma_2 \qquad v = v_1 \oplus v_2}{\sigma \vdash e_1 \oplus e_2 \Downarrow v, \sigma_2}$$

$$(\text{Abstr}) \qquad \sigma \vdash (\texttt{fun x} \rightarrow e) \Downarrow (\texttt{fun x} \rightarrow e), \sigma$$

   Fill in the blanks in the (App) rule below.

$$(\text{App}) \qquad \frac{\_\_ \vdash e_1 \Downarrow \_\_\_\_\_\_\_\_\_\_, \_\_ \qquad \_\_ \vdash e_2 \Downarrow \_\_, \_\_ \qquad \_\_ \vdash \_\_\_\_\_\_\_\_\_\_ \Downarrow \_\_, \_\_}{\sigma \vdash e_1 e_2 \Downarrow v, \_\_\_\_}$$

1)

{x:α→int, y:α}⊢x: α→int   {x:α→int, y:α}⊢y: α

{x:α→int, y:α}⊢x(y): int   {x:α→int, y:α}⊢1: int

{_______________} ⊢x(y)+1: ___   {____________}⊢x: { ______________}⊢1: int

{____________}⊢ _______________: __________   {_______________}⊢x(1): ______

∅⊢let rec x = fun y →x(y)+1 in x(1) : int

2)

{___________}⊢__ :   {__________}⊢__: _________

_____⊢x: ____   {______________}⊢______:_____________   {____________}⊢1: int

_____⊢fun x →x :__________   {_______________}⊢_____: _____

∅⊢let f = fun x →x in (f f)1 : int

3)

______________________ , y ⇓ 3   ______________________ , 1 ⇓ 1

C = ______________________ , y+1 ⇓ 4

______________________ , y ⇓ 2   ______________________ , 1 ⇓ 1

η, f ⇓ <(fun y → y+1), ∅>   η, 2 ⇓ 2   ______________________ , ____ ⇓ 3

B = ________________ , ______ ⇓ ______

__ , __ ⇓ ____________   **B**   **C**

∅,(fun f → f(f 2)) ⇓ <(fun f → f(f 2)), ∅>   ∅,(fun y → y+1) ⇓ <(fun y → y+1), ∅>   ________________ , _____ ⇓ 4

∅,(fun f → f(f 2))(fun y → y+1) ⇓ 4

To reduce notational clutter, use η as an abbreviation for the environment {f: <(fun y → y+1), ∅>}.

4)

$\_\_\_\_ , \_\_\_\_ \vdash x \Downarrow \_\_ , \_\_\_\_$ $\quad$ $\{\ell \to 0\}(\ell) = 0$

$\_\_\_\_\_ , \_\_\_\_\_ \vdash \_\_\_\_ \Downarrow \_\_ , \_\_\_\_\_$ $\quad$ $\_\_\_\_\_ , \_\_\_\_\_ \vdash \_\_\_\_ \Downarrow \_\_ , \_\_\_\_\_$

$\_\_\_\_\_ , \_\_\_\_ \vdash x \Downarrow \_\_ , \_\_\_\_\_$ $\quad$ $\_\_\_\_\_ , \_\_\_\_\_ \vdash \_\_\_\_ \Downarrow \_\_ , \_\_\_\_\_$

D = $\_\_\_\_\_ , \_\_\_\_\_ \vdash x := !x+1 \Downarrow \_\_ , \_\_\_\_\_$

$\emptyset, \emptyset \vdash 0 \Downarrow 0, \emptyset$

$\emptyset, \emptyset \vdash (\text{fun } x \to x := !x+1) \Downarrow \langle(\text{fun } x \to x := !x+1), \emptyset\rangle, \emptyset$ $\quad$ $\emptyset, \emptyset \vdash \text{ref } 0 \Downarrow \_\_ , \_\_\_\_$ $\quad$ **D**

$\emptyset, \emptyset \vdash (\text{fun } x \to x := !x+1)(\text{ref } 0) \Downarrow (), \{\ell \to 1\}$