MP 11 – Proof Systems

CS 421 – Spring 2009

Revision 1.0

Assigned Friday, April 8, 2009
Due Wednesday, April 15, in class
Extension 48 hours (20% penalty)
Total points 50

1 Change Log

1.1 Fixed due date.

1.0 Initial Release.

2 Overview

After completing this MP, you should have a better understanding of proof systems.

3 Collaboration

Collaboration is NOT allowed on this assignment.

4 Instructions

Fill in the blanks in the following proof trees. Write your answers in the blanks on the page. Submit in hard-copy at the beginning of class.

5 Problems (50 pts)

1. (10 pts) Fill in the blanks in the derivation of the type judgment $\emptyset \vdash \texttt{fun x} \rightarrow (+\ \texttt{x})\ \texttt{1} : \texttt{int} \rightarrow \texttt{int}$.

$$\dfrac{\dfrac{\dfrac{}{\emptyset[\texttt{x} : \texttt{int}] \vdash + : \_\_\_\_\_} \quad \dfrac{}{\emptyset[\texttt{x} : \texttt{int}] \vdash \texttt{x} : \_\_\_\_\_}}{\emptyset[\texttt{x} : \texttt{int}] \vdash + \ \texttt{x} : \_\_\_\_\_} \quad \dfrac{}{\_\_\_\_\_ \vdash \texttt{1} : \texttt{int}}}{\emptyset[\texttt{x} : \texttt{int}] \vdash (+\ \texttt{x})\ \texttt{1} : \_\_\_\_\_}}{\emptyset \vdash \texttt{fun x} \rightarrow (+\ \texttt{x})\ \texttt{1} : \texttt{int} \rightarrow \texttt{int}}$$

2. (15 pts) Fill in the proof tree for the type judgment $\emptyset \vdash \texttt{fun f} \rightarrow \texttt{f}\ (+\ \texttt{1}) : ((\texttt{int} \rightarrow \texttt{int}) \rightarrow \texttt{int}) \rightarrow \texttt{int}$.
   You can use the abbreviation Γ_f for $\emptyset[\texttt{f} : (\texttt{int} \rightarrow \texttt{int}) \rightarrow \texttt{int}]$.

$$\dfrac{\dfrac{\dfrac{}{\_\_\_\_\_} \quad \dfrac{}{\_\_\_\_\_}}{\_\_\_\_\_} \quad \dfrac{}{\_\_\_\_\_}}{\emptyset \vdash \texttt{fun f} \rightarrow \texttt{f}\ (+\ \texttt{1}) : ((\texttt{int} \rightarrow \texttt{int}) \rightarrow \texttt{int}) \rightarrow \texttt{int}}$$

3. (10 pts) Fill in the blanks in the derivation of the evaluation judgment $(\texttt{fun x} \rightarrow \texttt{x} + \texttt{1})\ \texttt{3} \Downarrow \texttt{4}$.

$$\dfrac{\dfrac{}{\_\_\_\_\_ \Downarrow \_\_\_\_\_} \quad \dfrac{}{\texttt{3} \Downarrow \texttt{3}} \quad \dfrac{\dfrac{}{\texttt{3} \Downarrow \texttt{3}} \quad \dfrac{}{\texttt{1} \Downarrow \texttt{1}}}{\_\_\_\_\_ \Downarrow \texttt{4}}}{(\texttt{fun x} \rightarrow \texttt{x} + \texttt{1})\ \texttt{3} \Downarrow \texttt{4}}$$

4. (15 pts) The proof tree for the evaluation judgment $(\texttt{fun f} \to \texttt{f (f 2)})(\texttt{fun y} \to \texttt{y} + \texttt{1}) \Downarrow \texttt{4}$ begins with:

$$\dfrac{\dfrac{}{(\texttt{fun f} \to \texttt{f (f 2)}) \Downarrow (\texttt{fun f} \to \texttt{f (f 2)})} \quad \dfrac{}{(\texttt{fun y} \to \texttt{y} + \texttt{1}) \Downarrow (\texttt{fun y} \to \texttt{y} + \texttt{1})} \quad \dfrac{?}{(\texttt{fun y} \to \texttt{y} + \texttt{1})((\texttt{fun y} \to \texttt{y} + \texttt{1})\ \texttt{2}) \Downarrow \texttt{4}}}{(\texttt{fun f} \to \texttt{f (f 2)})(\texttt{fun y} \to \texttt{y} + \texttt{1}) \Downarrow \texttt{4}}$$

Write the proof tree that should be filled in for ? to complete the derivation. You can break it into sections if it gets too wide to fit on the page.