
MP 11 – Proof Systems

CS 421 – Spring 2009

Revision 1.0

Assigned Friday, April 8, 2009

Due Wednesday, April 15, in class

Extension 48 hours (20% penalty)

Total points 50

1 Change Log

1.0 Initial Release.

2 Overview

After completing this MP, you should have a better understanding of proof systems.

3 Collaboration

Collaboration is NOT allowed on this assignment.

4 Instructions

Fill in the blanks in the following proof trees. Write your answers in the blanks on the page. Submit in hard-copy at the beginning of class.

5 Problems (50 pts)

1. (10 pts) Fill in the blanks in the derivation of the type judgment $\emptyset \vdash \text{fun } x \rightarrow (+ x) 1 : \text{int} \rightarrow \text{int}$.

$$\begin{array}{c}
 \frac{\frac{\frac{\emptyset[x : \text{int}] \vdash + : \text{int} \rightarrow \text{int} \rightarrow \text{int}}{\emptyset[x : \text{int}] \vdash + x : \text{int} \rightarrow \text{int}} \quad \frac{\frac{\emptyset[x : \text{int}] \vdash x : \text{int}}{\emptyset[x : \text{int}] \vdash 1 : \text{int}}}{\emptyset[x : \text{int}] \vdash (+ x) 1 : \text{int}} \\
 \hline
 \emptyset \vdash \text{fun } x \rightarrow (+ x) 1 : \text{int} \rightarrow \text{int}
 \end{array}$$

2. (15 pts) Fill in the proof tree for the type judgment $\emptyset \vdash \text{fun } f \rightarrow f (+ 1) : ((\text{int} \rightarrow \text{int}) \rightarrow \text{int}) \rightarrow \text{int}$.

You can use the abbreviation Γ_f for $\emptyset[f : (\text{int} \rightarrow \text{int}) \rightarrow \text{int}]$.

$$\begin{array}{c}
 \frac{\frac{\frac{\Gamma_f \vdash f : (\text{int} \rightarrow \text{int}) \rightarrow \text{int}}{\Gamma_f \vdash f (+ 1) : \text{int}} \quad \frac{\frac{\frac{\Gamma_f \vdash + : \text{int} \rightarrow \text{int} \rightarrow \text{int}}{\Gamma_f \vdash (+ 1) : \text{int} \rightarrow \text{int}} \quad \frac{\Gamma_f \vdash 1 : \text{int}}{\Gamma_f \vdash 1 : \text{int}}}{\Gamma_f \vdash f (+ 1) : \text{int}}}{\emptyset \vdash \text{fun } f \rightarrow f (+ 1) : ((\text{int} \rightarrow \text{int}) \rightarrow \text{int}) \rightarrow \text{int}}
 \end{array}$$

3. (10 pts) Fill in the blanks in the derivation of the evaluation judgment $(\text{fun } x \rightarrow x + 1) 3 \Downarrow 4$.

$$\begin{array}{c}
 \frac{\frac{\frac{\text{fun } x \rightarrow x + 1 \Downarrow \text{fun } x \rightarrow x + 1}{} \quad \frac{3 \Downarrow 3}{} \quad \frac{\frac{3 \Downarrow 3}{3 + 1 \Downarrow 4} \quad \frac{1 \Downarrow 1}{}{} }{\text{fun } x \rightarrow x + 1 \Downarrow \text{fun } x \rightarrow x + 1}}{\text{fun } x \rightarrow x + 1 \Downarrow \text{fun } x \rightarrow x + 1}} \\
 \hline
 (\text{fun } x \rightarrow x + 1) 3 \Downarrow 4
 \end{array}$$

4. (15 pts) The proof tree for the evaluation judgment $(\text{fun } f \rightarrow f (f \ 2))(\text{fun } y \rightarrow y + 1) \Downarrow 4$ begins with:

$$\frac{\frac{}{(\text{fun } f \rightarrow f (f \ 2)) \Downarrow (\text{fun } f \rightarrow f (f \ 2))} \quad \frac{}{(\text{fun } y \rightarrow y + 1) \Downarrow (\text{fun } y \rightarrow y + 1)} \quad \frac{}{(\text{fun } y \rightarrow y + 1)((\text{fun } y \rightarrow y + 1) \ 2) \Downarrow 4} \quad ?}{(\text{fun } f \rightarrow f (f \ 2))(\text{fun } y \rightarrow y + 1) \Downarrow 4}$$

Write the proof tree that should be filled in for ? to complete the derivation. You can break it into sections if it gets too wide to fit on the page.

$$\frac{\frac{}{\text{fun } y \rightarrow y + 1 \Downarrow \text{fun } y \rightarrow y + 1} \quad * \quad \frac{\frac{}{3 \Downarrow 3} \quad \frac{}{1 \Downarrow 1}}{3 + 1 \Downarrow 4}}{(\text{fun } y \rightarrow y + 1)((\text{fun } y \rightarrow y + 1) \ 2) \Downarrow 4}$$

where * is the proof tree

$$\frac{\frac{}{\text{fun } y \rightarrow y + 1 \Downarrow \text{fun } y \rightarrow y + 1} \quad \frac{}{2 \Downarrow 2} \quad \frac{\frac{}{2 \Downarrow 2} \quad \frac{}{1 \Downarrow 1}}{2 + 1 \Downarrow 3}}{(\text{fun } y \rightarrow y + 1) \ 2 \Downarrow 3}$$