

# Programming Languages and Compilers (CS 421)



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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha



## Example : test.ml

---

```
{ type result = Int of int | Float of float |  
  String of string }
```

```
let digit = ['0'-'9']
```

```
let digits = digit +
```

```
let lower_case = ['a'-'z']
```

```
let upper_case = ['A'-'Z']
```

```
let letter = upper_case | lower_case
```

```
let letters = letter +
```



# Example : test.ml

---

```
rule main = parse
```

```
  (digits)'. 'digits as f { Float (float_of_string f) }
```

```
  | digits as n           { Int (int_of_string n) }
```

```
  | letters as s         { String s }
```

```
  | _ { main lexbuf }
```

```
{ let newlexbuf = (Lexing.from_channel stdin) in
```

```
  print_newline ();
```

```
  main newlexbuf }
```



# Example

---

```
# #use "test.ml";;
```

```
...
```

```
val main : Lexing.lexbuf -> result = <fun>
```

```
val __ocaml_lex_main_rec : Lexing.lexbuf -> int ->  
  result = <fun>
```

```
hi there 234 5.2
```

```
- : result = String "hi"
```

What happened to the rest?!?



# Example

---

```
# let b = Lexing.from_channel stdin;;
```

```
# main b;;
```

```
hi 673 there
```

```
- : result = String "hi"
```

```
# main b;;
```

```
- : result = Int 673
```

```
# main b;;
```

```
- : result = String "there"
```



# Problem

---

- How to get lexer to look at more than the first token at one time?
- Answer: *action* has to tell it to -- recursive calls
  - Not what you want to sew this together with ocaml yacc
- Side Benefit: can add “state” into lexing
- Note: already used this with the \_ case



# Example

---

rule main = parse

(digits) '.' digits as f { Float

(float\_of\_string f) :: **main lexbuf**}

| digits as n { Int (int\_of\_string n) ::  
**main lexbuf** }

| letters as s { String s :: **main**  
**lexbuf**}

| eof { [] }

| \_ { **main lexbuf** }



# Example Results

---

hi there 234 5.2

- : result list = [String "hi"; String "there"; Int 234; Float 5.2]

#

Used Ctrl-d to send the end-of-file signal





# Dealing with comments

---

## First Attempt

```
let open_comment = "("*
```

```
let close_comment = "*"
```

```
rule main = parse
```

```
  (digits) '.' digits as f { Float (float_of_string  
f) :: main lexbuf }
```

```
| digits as n          { Int (int_of_string n) ::  
main lexbuf }
```

```
| letters as s        { String s :: main lexbuf }
```



# Dealing with comments

---

| **open\_comment** { comment lexbuf }

| eof { [] }

| \_ { main lexbuf }

and comment = parse

**close\_comment** { main lexbuf }

| \_ { comment lexbuf }



# Dealing with nested comments

---

```
rule main = parse ...
| open_comment      { comment 1 lexbuf }
| eof               { [] }
| _ { main lexbuf }
and comment depth = parse
  open_comment      { comment (depth+1) lexbuf }
  }
| close_comment     { if depth = 1
                      then main lexbuf
                      else comment (depth - 1) lexbuf }
| _                 { comment depth lexbuf }
```



# Dealing with nested comments

---

rule main = parse

(digits) '.' digits as f { Float (float\_of\_string f) ::  
main lexbuf }

| digits as n { Int (int\_of\_string n) :: main  
lexbuf }

| letters as s { String s :: main lexbuf }

| open\_comment { (comment 1 lexbuf ) }

| eof { [] }

| \_ { main lexbuf }



# Dealing with nested comments

---

and comment depth = parse

```
open_comment      { comment (depth+1) lexbuf  
}
```

```
| close_comment   { if depth = 1  
                    then main lexbuf  
                    else comment (depth - 1) lexbuf }
```

```
| _               { comment depth lexbuf }
```



# Types of Formal Language Descriptions

---

- Regular expressions, regular grammars
- Context-free grammars, BNF grammars, syntax diagrams
  
- Finite state automata
- Pushdown automata
- Whole family more of grammars and automata – covered in automata theory



# BNF Grammars

---

- Start with a set of characters, **a,b,c,...**
  - We call these *terminals*
- Add a set of different characters, **X,Y,Z,...**
  - We call these *nonterminals*
- One special nonterminal **S** called *start symbol*



# BNF Grammars

---

- BNF rules (aka *productions*) have form

$$\mathbf{X} ::= y$$

where  $\mathbf{X}$  is any nonterminal and  $y$  is a string of terminals and nonterminals

- BNF *grammar* is a set of BNF rules such that every nonterminal appears on the left of some rule





# Sample Grammar

---

- Terminals: 0 1 + ( )
- Nonterminals:  $\langle \text{Sum} \rangle$
- Start symbol =  $\langle \text{Sum} \rangle$
  
- $\langle \text{Sum} \rangle ::= 0$
- $\langle \text{Sum} \rangle ::= 1$
- $\langle \text{Sum} \rangle ::= \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$
- $\langle \text{Sum} \rangle ::= (\langle \text{Sum} \rangle)$
- Can be abbreviated as  
$$\langle \text{Sum} \rangle ::= 0 \mid 1 \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \mid (\langle \text{Sum} \rangle)$$



# BNF Derivations

---

- Given rules

$$\mathbf{X} ::= y\mathbf{Z}w \text{ and } \mathbf{Z} ::= v$$

we may replace  $\mathbf{Z}$  by  $v$  to say

$$\mathbf{X} \Rightarrow y\mathbf{Z}w \Rightarrow yvw$$

- Sequence of such replacements called *derivation*
- Derivation called *right-most* if always replace the right-most non-terminal



# BNF Semantics

---

- The meaning of a BNF grammar is the set of all strings consisting only of terminals that can be derived from the Start symbol



# BNF Derivations

---

- Start with the start symbol:

$\langle \text{Sum} \rangle \Rightarrow$



# BNF Derivations

---

- Pick a non-terminal

**<Sum>** =>

# BNF Derivations

- Pick a rule and substitute:

- $\langle \text{Sum} \rangle ::= \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$



# BNF Derivations

---

- Pick a non-terminal:

$\langle \text{Sum} \rangle \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

# BNF Derivations

- Pick a rule and substitute:

- $\langle \text{Sum} \rangle ::= ( \langle \text{Sum} \rangle )$

$$\langle \text{Sum} \rangle \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$$

$$\Rightarrow ( \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle$$





# BNF Derivations

---

- Pick a non-terminal:

$$\begin{aligned}\langle \text{Sum} \rangle & \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \\ & \Rightarrow ( \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle\end{aligned}$$

# BNF Derivations

- Pick a rule and substitute:

- $\langle \text{Sum} \rangle ::= \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\Rightarrow ( \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle$

$\Rightarrow ( \langle \text{Sum} \rangle + \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle$



# BNF Derivations

---

- Pick a non-terminal:

$\langle \text{Sum} \rangle \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\Rightarrow ( \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle$

$\Rightarrow ( \langle \text{Sum} \rangle + \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle$



# BNF Derivations

---

- Pick a rule and substitute:

- $\langle \text{Sum} \rangle ::= 1$

$\langle \text{Sum} \rangle \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\Rightarrow ( \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle$

$\Rightarrow ( \langle \text{Sum} \rangle + \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle$

$\Rightarrow ( \langle \text{Sum} \rangle + 1 ) + \langle \text{Sum} \rangle$



# BNF Derivations

---

- Pick a non-terminal:

$\langle \text{Sum} \rangle \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\Rightarrow ( \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle$

$\Rightarrow ( \langle \text{Sum} \rangle + \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle$

$\Rightarrow ( \langle \text{Sum} \rangle + 1 ) + \langle \text{Sum} \rangle$

# BNF Derivations

- Pick a rule and substitute:

- $\langle \text{Sum} \rangle ::= 0$

$$\langle \text{Sum} \rangle \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$$

$$\Rightarrow ( \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle$$

$$\Rightarrow ( \langle \text{Sum} \rangle + \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle$$

$$\Rightarrow ( \langle \text{Sum} \rangle + 1 ) + \langle \text{Sum} \rangle$$

$$\Rightarrow ( \langle \text{Sum} \rangle + 1 ) + 0$$



# BNF Derivations

---

- Pick a non-terminal:

$$\begin{aligned}\langle \text{Sum} \rangle & \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \\ & \Rightarrow ( \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle \\ & \Rightarrow ( \langle \text{Sum} \rangle + \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle \\ & \Rightarrow ( \langle \text{Sum} \rangle + 1 ) + \langle \text{Sum} \rangle \\ & \Rightarrow ( \langle \text{Sum} \rangle + 1 ) + 0\end{aligned}$$

# BNF Derivations

- Pick a rule and substitute

- $\langle \text{Sum} \rangle ::= 0$

$$\langle \text{Sum} \rangle \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$$

$$\Rightarrow ( \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle$$

$$\Rightarrow ( \langle \text{Sum} \rangle + \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle$$

$$\Rightarrow ( \langle \text{Sum} \rangle + 1 ) + \langle \text{Sum} \rangle$$

$$\Rightarrow ( \langle \text{Sum} \rangle + 1 ) 0$$

$$\Rightarrow ( 0 + 1 ) + 0$$





# BNF Derivations

---

- $( 0 + 1 ) + 0$  is generated by grammar

$$\begin{aligned} \langle \text{Sum} \rangle & \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \\ & \Rightarrow ( \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle \\ & \Rightarrow ( \langle \text{Sum} \rangle + \langle \text{Sum} \rangle ) + \langle \text{Sum} \rangle \\ & \Rightarrow ( \langle \text{Sum} \rangle + 1 ) + \langle \text{Sum} \rangle \\ & \Rightarrow ( \langle \text{Sum} \rangle + 1 ) + 0 \\ & \Rightarrow ( 0 + 1 ) + 0 \end{aligned}$$



# Extended BNF Grammars

---

- Alternatives: allow rules of form  $X ::= y/z$ 
  - Abbreviates  $X ::= y, X ::= z$
- Options:  $X ::= y[v]z$ 
  - Abbreviates  $X ::= yvz, X ::= yz$
- Repetition:  $X ::= y\{v\}^*z$ 
  - Can be eliminated by adding new nonterminal  $V$  and rules  $X ::= yz, X ::= yVz, V ::= v, V ::= w$



# Parse Trees

---

- Graphical representation of derivation
- Each node labeled with either non-terminal or terminal
- If node is labeled with a terminal, then it is a leaf (no sub-trees)
- If node is labeled with a non-terminal, then it has one branch for each character in the right-hand side of rule used to substitute for it



# Example

---

- Consider grammar:

$$\langle \text{exp} \rangle ::= \langle \text{factor} \rangle$$
$$| \langle \text{factor} \rangle + \langle \text{factor} \rangle$$
$$\langle \text{factor} \rangle ::= \langle \text{bin} \rangle$$
$$| \langle \text{bin} \rangle * \langle \text{exp} \rangle$$
$$\langle \text{bin} \rangle ::= 0 \mid 1$$

- Problem: Build parse tree for  $1 * 1 + 0$  as an  $\langle \text{exp} \rangle$



## Example cont.

---

- $1 * 1 + 0$ :  $\langle \text{exp} \rangle$

$\langle \text{exp} \rangle$  is the start symbol for this parse tree



## Example cont.

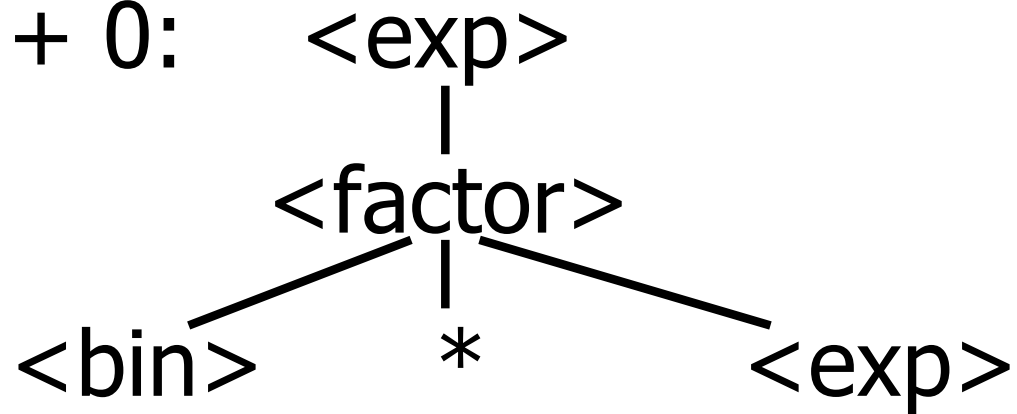
---

■  $1 * 1 + 0$ :  $\langle \text{exp} \rangle$   
|  
 $\langle \text{factor} \rangle$

Use rule:  $\langle \text{exp} \rangle ::= \langle \text{factor} \rangle$

## Example cont.

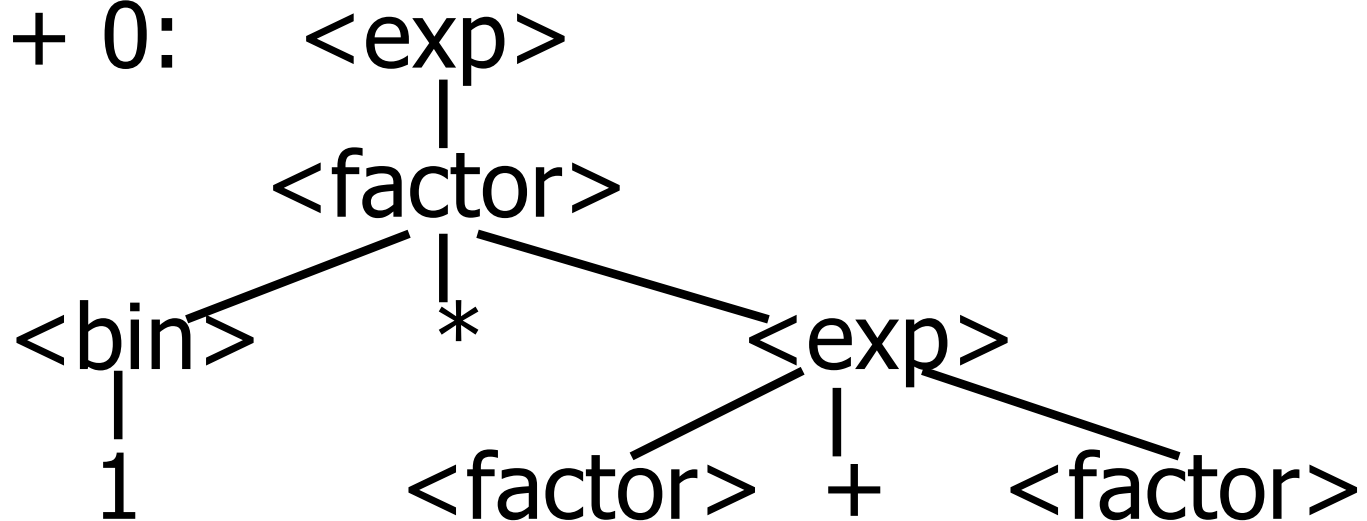
- $1 * 1 + 0$ :



Use rule:  $\langle \text{factor} \rangle ::= \langle \text{bin} \rangle * \langle \text{exp} \rangle$

## Example cont.

- 1 \* 1 + 0:

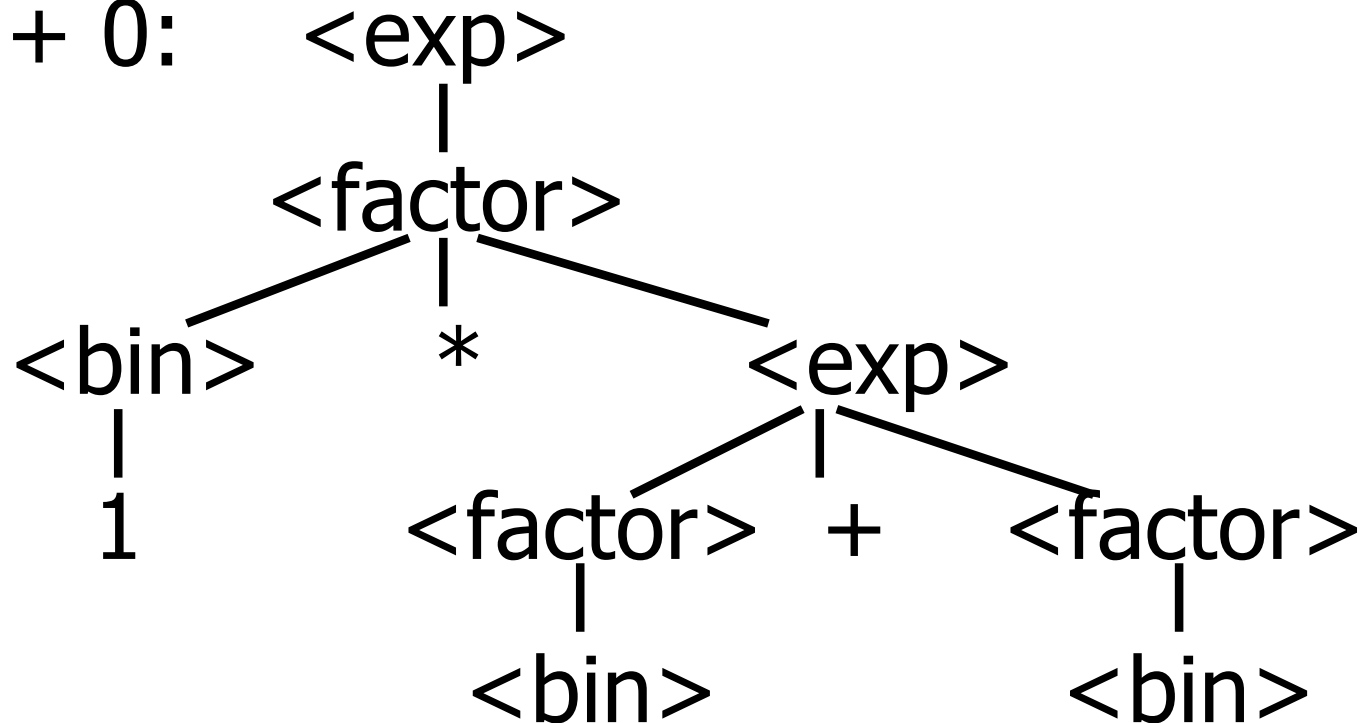


Use rules:  $\langle \text{bin} \rangle ::= 1$  and  
 $\langle \text{exp} \rangle ::= \langle \text{factor} \rangle + \langle \text{factor} \rangle$



## Example cont.

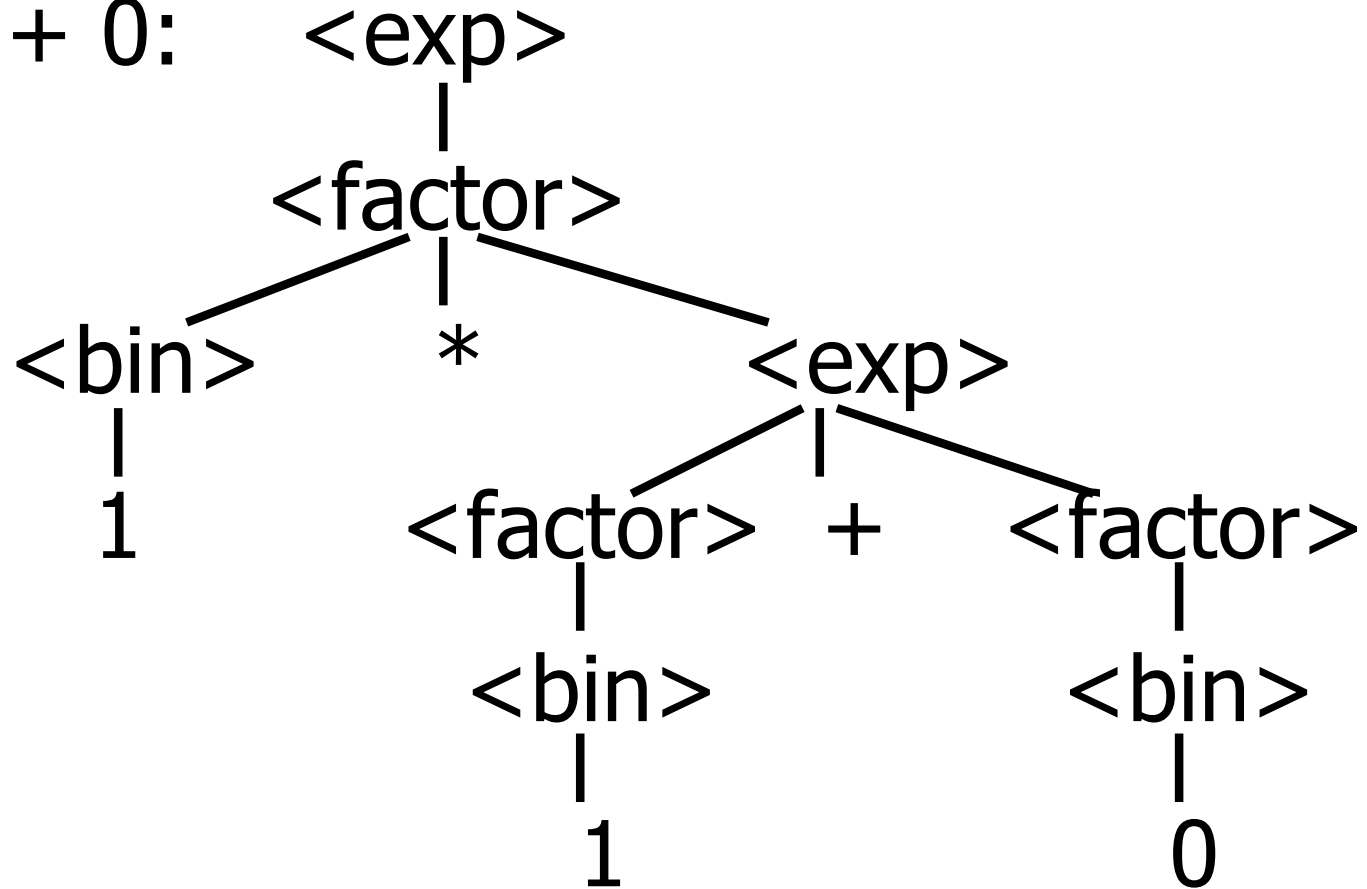
- $1 * 1 + 0$ :



Use rule:  $\langle \text{factor} \rangle ::= \langle \text{bin} \rangle$

# Example cont.

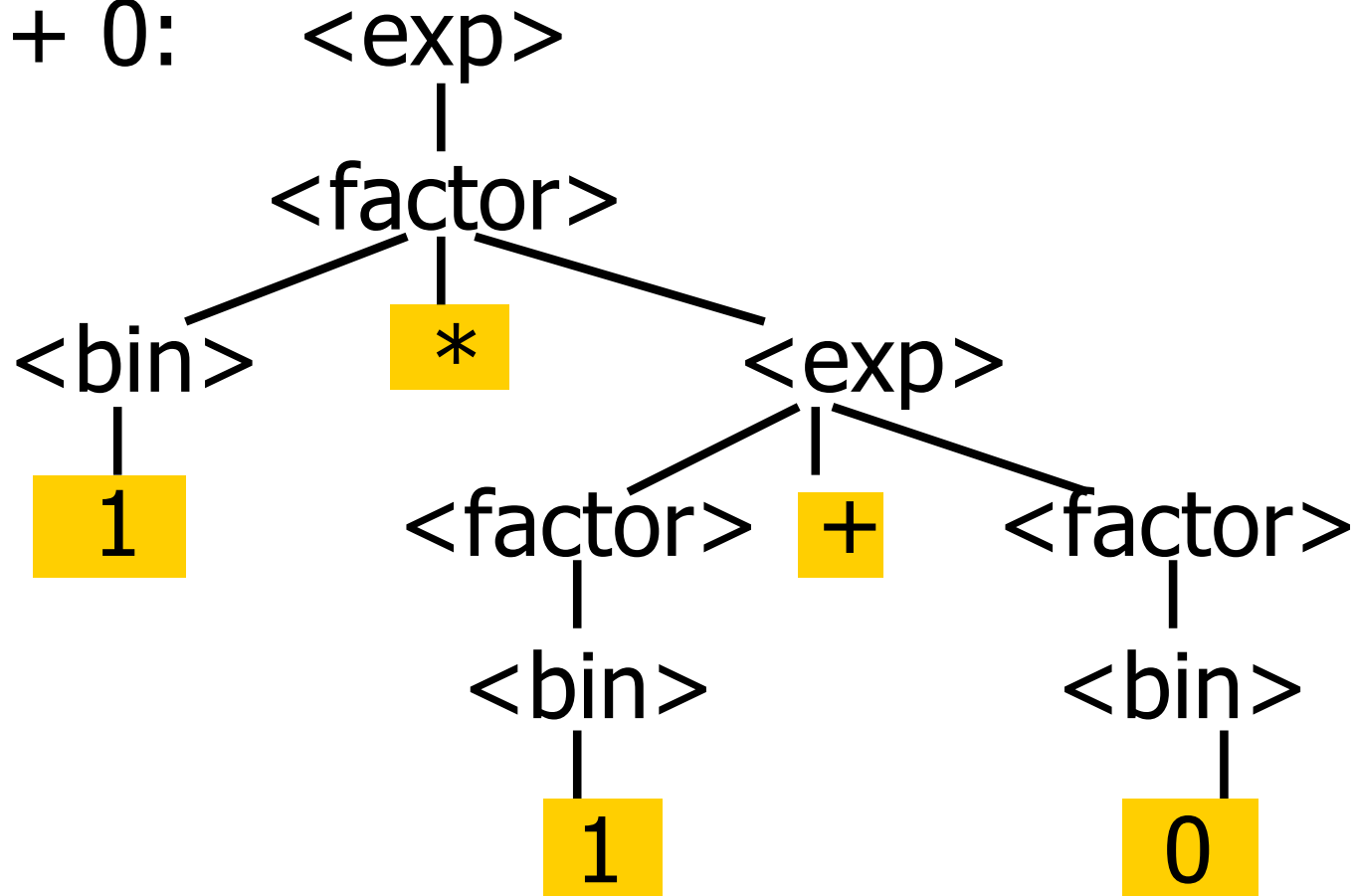
- 1 \* 1 + 0:



Use rules:  $\langle \text{bin} \rangle ::= 1 \mid 0$

## Example cont.

- 1 \* 1 + 0:



Fringe of tree is string generated by grammar



# Parse Tree Data Structures

---

- Parse trees may be represented by OCaml datatypes
- One datatype for each nonterminal
- One constructor for each rule
- Defined as mutually recursive collection of datatype declarations



# Example

---

- Recall grammar:

$\langle \text{exp} \rangle ::= \langle \text{factor} \rangle \mid \langle \text{factor} \rangle + \langle \text{factor} \rangle$

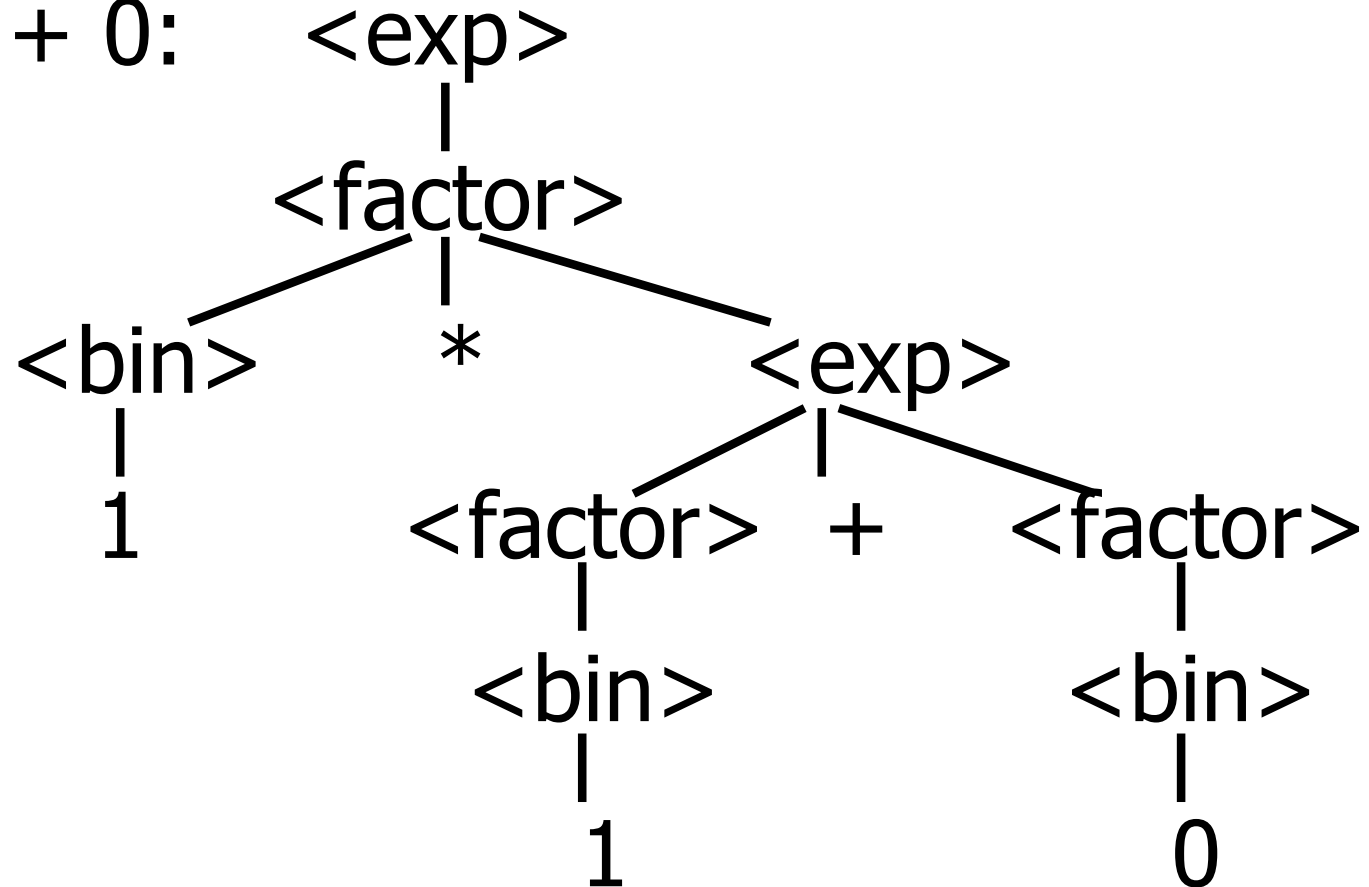
$\langle \text{factor} \rangle ::= \langle \text{bin} \rangle \mid \langle \text{bin} \rangle * \langle \text{exp} \rangle$

$\langle \text{bin} \rangle ::= 0 \mid 1$

- type  $\text{exp} = \text{Factor2Exp}$  of  $\text{factor}$ 
  - | Plus of  $\text{factor} * \text{factor}$
- and  $\text{factor} = \text{Bin2Factor}$  of  $\text{bin}$ 
  - | Mult of  $\text{bin} * \text{exp}$
- and  $\text{bin} = \text{Zero} \mid \text{One}$

# Example cont.

- $1 * 1 + 0$ :





## Example cont.

---

- Can be represented as

Factor2Exp

(Mult(One,

Plus(Bin2Factor One,

Bin2Factor Zero)))



# Ambiguous Grammars and Languages

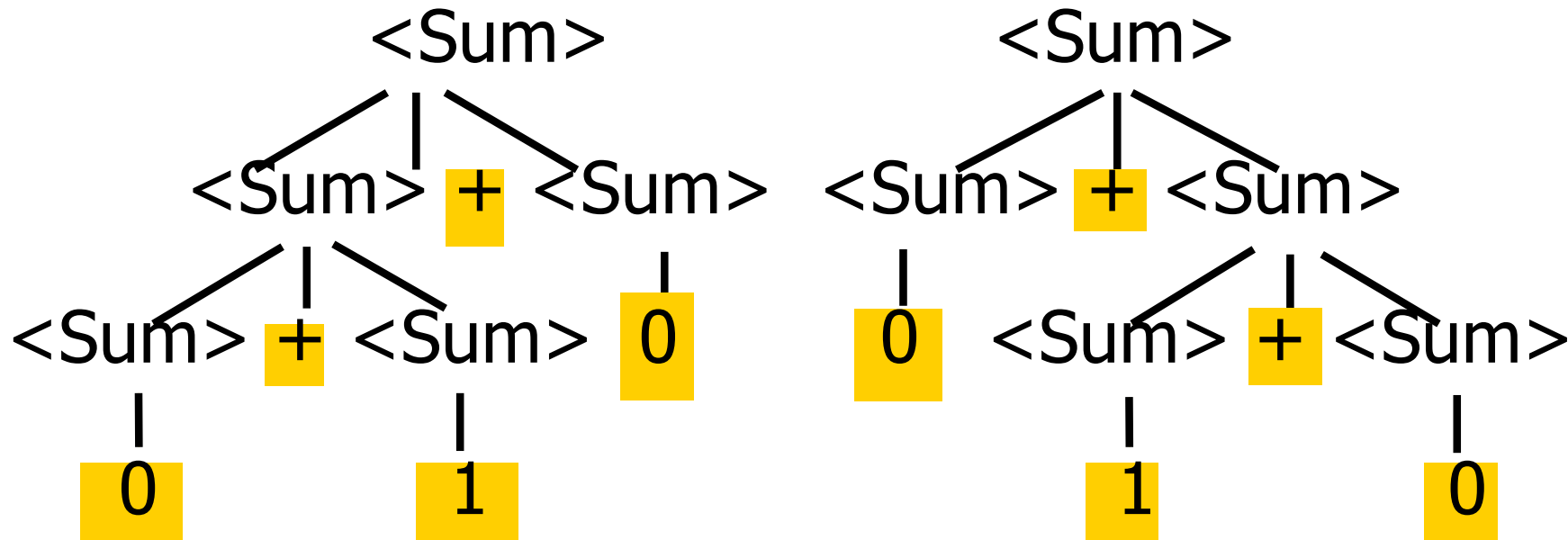
---

- A BNF grammar is *ambiguous* if its language contains strings for which there is more than one parse tree
- If all BNF's for a language are ambiguous then the language is *inherently ambiguous*



# Example: Ambiguous Grammar

■  $0 + 1 + 0$





# Two Major Sources of Ambiguity

---

- Lack of determination of operator precedence
- Lack of determination of operator associativity
- Not the only sources of ambiguity



# Disambiguating a Grammar

---

- Given ambiguous grammar  $G$ , with start symbol  $S$ , find a grammar  $G'$  with same start symbol, such that
  - language of  $G =$  language of  $G'$
- Not always possible
- No algorithm in general



# Disambiguating a Grammar

---

- Idea: Each non-terminal represents all strings having some property
- Identify these properties (often in terms of things that can't happen)
- Use these properties to inductively guarantee every string in language has a unique parse



# Steps to Grammar Disambiguation

---

- Identify the rules and a smallest use that display ambiguity
- Decide which parse to keep; why should others be thrown out?
- What syntactic restrictions on subexpressions are needed to throw out the bad (while keeping the good)?
- Add a new non-terminal and rules to describe this set of restricted subexpressions (called stratifying, or refactoring)
- **Characterize each non-terminal by a language invariant**
- Replace old rules to use new non-terminals
- Rinse and repeat



# Example

---

- Ambiguous grammar:

$$\langle \text{exp} \rangle ::= 0 \mid 1 \mid \langle \text{exp} \rangle + \langle \text{exp} \rangle \\ \mid \langle \text{exp} \rangle * \langle \text{exp} \rangle$$

- String with more than one parse:

$$0 + 1 + 0$$
$$1 * 1 + 1$$

- Source of ambiguity: associativity and precedence



# Two Major Sources of Ambiguity

---

- Lack of determination of operator precedence
- Lack of determination of operator associativity
- Not the only sources of ambiguity



# How to Enforce Associativity

---

- Have at most one recursive call per production
- When two or more recursive calls would be natural leave right-most one for right associativity, left-most one for left associativity





# Example

---

- $\langle \text{Sum} \rangle ::= 0 \mid 1 \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$   
 $\mid (\langle \text{Sum} \rangle)$
- Becomes
  - $\langle \text{Sum} \rangle ::= \langle \text{Num} \rangle \mid \langle \text{Num} \rangle + \langle \text{Sum} \rangle$
  - $\langle \text{Num} \rangle ::= 0 \mid 1 \mid (\langle \text{Sum} \rangle)$

$\langle \text{Sum} \rangle + \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$



# Operator Precedence

---

- Operators of highest precedence evaluated first (bind more tightly).
- Precedence for infix binary operators given in following table
- Needs to be reflected in grammar

# Precedence Table - Sample

	Fortran	Pascal	C/C++	Ada	SML
highest	**	*, /, div, mod	++, --	**	div, mod, /, *
	*, /	+, -	*, /, %	*, /, mod	+, -, ^
	+, -		+, -	+, -	::



# Precedence in Grammar

---

- Higher precedence translates to longer derivation chain

- Example:

$$\langle \text{exp} \rangle ::= 0 \mid 1 \mid \langle \text{exp} \rangle + \langle \text{exp} \rangle \\ \mid \langle \text{exp} \rangle * \langle \text{exp} \rangle$$

- Becomes

$$\langle \text{exp} \rangle ::= \langle \text{mult\_exp} \rangle \\ \mid \langle \text{exp} \rangle + \langle \text{mult\_exp} \rangle \\ \langle \text{mult\_exp} \rangle ::= \langle \text{id} \rangle \mid \langle \text{mult\_exp} \rangle * \langle \text{id} \rangle \\ \langle \text{id} \rangle ::= 0 \mid 1$$



# Parser Code

---

- `<grammar>.mly` defines one parsing function per entry point
- Parsing function takes a lexing function (lexer buffer to token) and a lexer buffer as arguments
- Returns semantic attribute of corresponding entry point



# Ocamlyacc Input

---

- File format:

%{

*<header>*

%}

*<declarations>*

%%

*<rules>*

%%

*<trailer>*



## Ocamlyacc *<header>*

---

- Contains arbitrary Ocaml code
- Typically used to give types and functions needed for the semantic actions of rules and to give specialized error recovery
- May be omitted
- *<footer>* similar. Possibly used to call parser



# Ocamlyacc <declarations>

---

- **%token** *symbol ... symbol*
- Declare given symbols as tokens
- **%token** <*type*> *symbol ... symbol*
- Declare given symbols as token constructors, taking an argument of type <*type*>
- **%start** *symbol ... symbol*
- Declare given symbols as entry points; functions of same names in <*grammar*>.ml





# Ocamlyacc *<declarations>*

---

- **%type** *<type> symbol ... symbol*

Specify type of attributes for given symbols.

Mandatory for start symbols

- **%left** *symbol ... symbol*

- **%right** *symbol ... symbol*

- **%nonassoc** *symbol ... symbol*

Associate precedence and associativity to given symbols. Same line, same precedence; earlier line, lower precedence (broadest scope)



# Ocamlyacc *<rules>*

---

- *nonterminal* :

*symbol ... symbol { semantic\_action }*

| ...

| *symbol ... symbol { semantic\_action }*

;

- Semantic actions are arbitrary Ocaml expressions
- Must be of same type as declared (or inferred) for *nonterminal*
- Access semantic attributes (values) of symbols by position: \$1 for first symbol, \$2 to second ...



# Example - Base types

---

```
(* File: expr.ml *)
```

```
type expr =
```

```
  Term_as_Expr of term
```

```
  | Plus_Expr of (term * expr)
```

```
  | Minus_Expr of (term * expr)
```

```
and term =
```

```
  Factor_as_Term of factor
```

```
  | Mult_Term of (factor * term)
```

```
  | Div_Term of (factor * term)
```

```
and factor =
```

```
  Id_as_Factor of string
```

```
  | Parenthesized_Expr_as_Factor of expr
```



# Example - Lexer (exprlex.mll)

---

```
{ (*open Exprparse*) }
let numeric = ['0' - '9']
let letter = ['a' - 'z' 'A' - 'Z']
rule token = parse
| "+" {Plus_token}
| "-" {Minus_token}
| "*" {Times_token}
| "/" {Divide_token}
| "(" {Left_parenthesis}
| ")" {Right_parenthesis}
| letter (letter|numeric|"_" )* as id {Id_token id}
| [' ' '\t' '\n'] {token lexbuf}
| eof {EOL}
```



# Example - Parser (exprparse.mly)

---

```
%{ open Expr
```

```
%}
```

```
%token <string> Id_token
```

```
%token Left_parenthesis Right_parenthesis
```

```
%token Times_token Divide_token
```

```
%token Plus_token Minus_token
```

```
%token EOL
```

```
%start main
```

```
%type <expr> main
```

```
%%
```



# Example - Parser (exprparse.mly)

---

expr:

term

{ Term\_as\_Expr \$1 }

| term Plus\_token expr

{ Plus\_Expr (\$1, \$3) }

| term Minus\_token expr

{ Minus\_Expr (\$1, \$3) }



# Example - Parser (exprparse.mly)

---

term:

factor

{ Factor\_as\_Term \$1 }

| factor Times\_token term

{ Mult\_Term (\$1, \$3) }

| factor Divide\_token term

{ Div\_Term (\$1, \$3) }



# Example - Parser (exprparse.mly)

---

factor:

Id\_token

{ Id\_as\_Factor \$1 }

| Left\_parenthesis expr Right\_parenthesis

{ Parenthesized\_Expr\_as\_Factor \$2 }

main:

| expr EOL

{ \$1 }





# Example - Using Parser

---

```
# #use "expr.ml";;
```

```
...
```

```
# #use "exprparse.ml";;
```

```
...
```

```
# #use "exprlex.ml";;
```

```
...
```

```
# let test s =
```

```
  let lexbuf = Lexing.from_string (s^"\n") in  
    main token lexbuf;;
```



# Example - Using Parser

---

```
# test "a + b";;
```

```
- : expr =
```

```
Plus_Expr
```

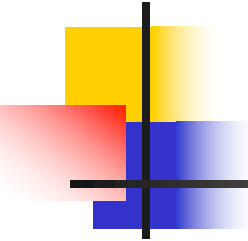
```
(Factor_as_Term (Id_as_Factor "a"),  
Term_as_Expr (Factor_as_Term  
(Id_as_Factor "b")))
```



# LR Parsing

---

- Read tokens left to right (L)
- Create a rightmost derivation (R)
- How is this possible?
- Start at the bottom (left) and work your way up
- Last step has only one non-terminal to be replaced so is right-most
- Working backwards, replace mixed strings by non-terminals
- Always proceed so that there are no non-terminals to the right of the string to be replaced

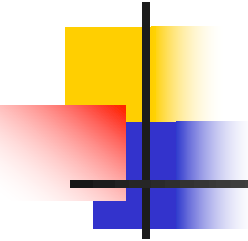


Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$   
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

---

$\langle \text{Sum} \rangle \Rightarrow$

$= \bullet (0 + 1) + 0$       shift



Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$   
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

---

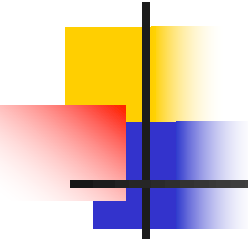
$\langle \text{Sum} \rangle \Rightarrow$

$$= (\bullet 0 + 1) + 0$$

$$= \bullet (0 + 1) + 0$$

shift

shift



Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$   
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$$\Rightarrow (0 \bullet + 1) + 0$$

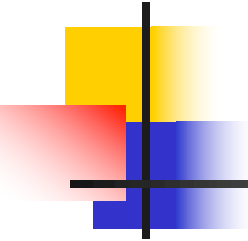
$$= (\bullet 0 + 1) + 0$$

$$= \bullet (0 + 1) + 0$$

reduce

shift

shift



Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$   
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$$= ( \langle \text{Sum} \rangle \bullet + 1 ) + 0$$

shift

$$\Rightarrow ( 0 \bullet + 1 ) + 0$$

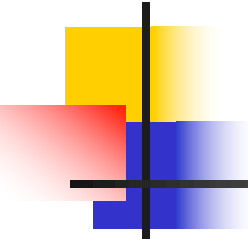
reduce

$$= ( \bullet 0 + 1 ) + 0$$

shift

$$= \bullet ( 0 + 1 ) + 0$$

shift

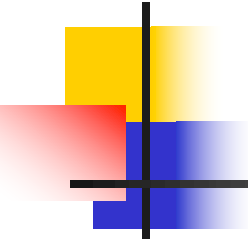


Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$   
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= (\langle \text{Sum} \rangle + \bullet 1) + 0$       shift  
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$       shift  
 $\Rightarrow (0 \bullet + 1) + 0$       reduce  
 $= (\bullet 0 + 1) + 0$       shift  
 $= \bullet (0 + 1) + 0$       shift





Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$   
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow ( \langle \text{Sum} \rangle + 1 \bullet ) + 0$	reduce
$= ( \langle \text{Sum} \rangle + \bullet 1 ) + 0$	shift
$= ( \langle \text{Sum} \rangle \bullet + 1 ) + 0$	shift
$\Rightarrow ( 0 \bullet + 1 ) + 0$	reduce
$= ( \bullet 0 + 1 ) + 0$	shift
$= \bullet ( 0 + 1 ) + 0$	shift

Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$   
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$  reduce  
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$  reduce  
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$  shift  
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$  shift  
 $\Rightarrow (0 \bullet + 1) + 0$  reduce  
 $= (\bullet 0 + 1) + 0$  shift  
 $= \bullet (0 + 1) + 0$  shift

Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$   
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= (\langle \text{Sum} \rangle \bullet) + 0$  shift  
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$  reduce  
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$  reduce  
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$  shift  
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$  shift  
 $\Rightarrow (0 \bullet + 1) + 0$  reduce  
 $= (\bullet 0 + 1) + 0$  shift  
 $= \bullet (0 + 1) + 0$  shift

Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$   
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$       reduce  
 $= (\langle \text{Sum} \rangle \bullet) + 0$       shift  
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$       reduce  
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$       reduce  
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$       shift  
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$       shift  
 $\Rightarrow (0 \bullet + 1) + 0$       reduce  
 $= (\bullet 0 + 1) + 0$       shift  
 $= \bullet (0 + 1) + 0$       shift

Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$   
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= \langle \text{Sum} \rangle \bullet + 0$  shift  
 $\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$  reduce  
 $= (\langle \text{Sum} \rangle \bullet) + 0$  shift  
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$  reduce  
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$  reduce  
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$  shift  
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$  shift  
 $\Rightarrow (0 \bullet + 1) + 0$  reduce  
 $= (\bullet 0 + 1) + 0$  shift  
 $= \bullet (0 + 1) + 0$  shift

Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$   
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= \langle \text{Sum} \rangle + \bullet 0$  shift  
 $= \langle \text{Sum} \rangle \bullet + 0$  shift  
 $\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$  reduce  
 $= (\langle \text{Sum} \rangle \bullet) + 0$  shift  
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$  reduce  
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$  reduce  
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$  shift  
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$  shift  
 $\Rightarrow (0 \bullet + 1) + 0$  reduce  
 $= (\bullet 0 + 1) + 0$  shift  
 $= \bullet (0 + 1) + 0$  shift

Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$   
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$   
 $\Rightarrow \langle \text{Sum} \rangle + 0 \bullet$  reduce  
 $= \langle \text{Sum} \rangle + \bullet 0$  shift  
 $= \langle \text{Sum} \rangle \bullet + 0$  shift  
 $\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$  reduce  
 $= (\langle \text{Sum} \rangle \bullet) + 0$  shift  
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$  reduce  
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$  reduce  
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$  shift  
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$  shift  
 $\Rightarrow (0 \bullet + 1) + 0$  reduce  
 $= (\bullet 0 + 1) + 0$  shift  
 $= \bullet (0 + 1) + 0$  shift

Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$   
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle$	$\Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$	●	reduce
	$\Rightarrow \langle \text{Sum} \rangle + 0$	●	reduce
	$= \langle \text{Sum} \rangle +$	● 0	shift
	$= \langle \text{Sum} \rangle$	● + 0	shift
	$\Rightarrow ( \langle \text{Sum} \rangle )$	● + 0	reduce
	$= ( \langle \text{Sum} \rangle$	● ) + 0	shift
	$\Rightarrow ( \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$	● ) + 0	reduce
	$\Rightarrow ( \langle \text{Sum} \rangle + 1$	● ) + 0	reduce
	$= ( \langle \text{Sum} \rangle +$	● 1 ) + 0	shift
	$= ( \langle \text{Sum} \rangle$	● + 1 ) + 0	shift
	$\Rightarrow ( 0$	● + 1 ) + 0	reduce
	$= ($	● 0 + 1 ) + 0	shift
	$=$	● ( 0 + 1 ) + 0	shift



Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$   
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle$	●	$\Rightarrow$	$\langle \text{Sum} \rangle + \langle \text{Sum} \rangle$	●	reduce	
		$\Rightarrow$	$\langle \text{Sum} \rangle + 0$	●	reduce	
		=	$\langle \text{Sum} \rangle +$	●	0	shift
		=	$\langle \text{Sum} \rangle$	●	+ 0	shift
		$\Rightarrow$	$( \langle \text{Sum} \rangle )$	●	+ 0	reduce
		=	$( \langle \text{Sum} \rangle$	●	) + 0	shift
		$\Rightarrow$	$( \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$	●	) + 0	reduce
		$\Rightarrow$	$( \langle \text{Sum} \rangle + 1$	●	) + 0	reduce
		=	$( \langle \text{Sum} \rangle +$	●	1 ) + 0	shift
		=	$( \langle \text{Sum} \rangle$	●	+ 1 ) + 0	shift
		$\Rightarrow$	$( 0$	●	+ 1 ) + 0	reduce
		=	$($	●	0 + 1 ) + 0	shift
		=	●	$( 0 + 1 ) + 0$	shift	



# Example

---

$$( 0 + 1 ) + 0$$





# Example

---

$$( 0 + 1 ) + 0$$





# Example

---

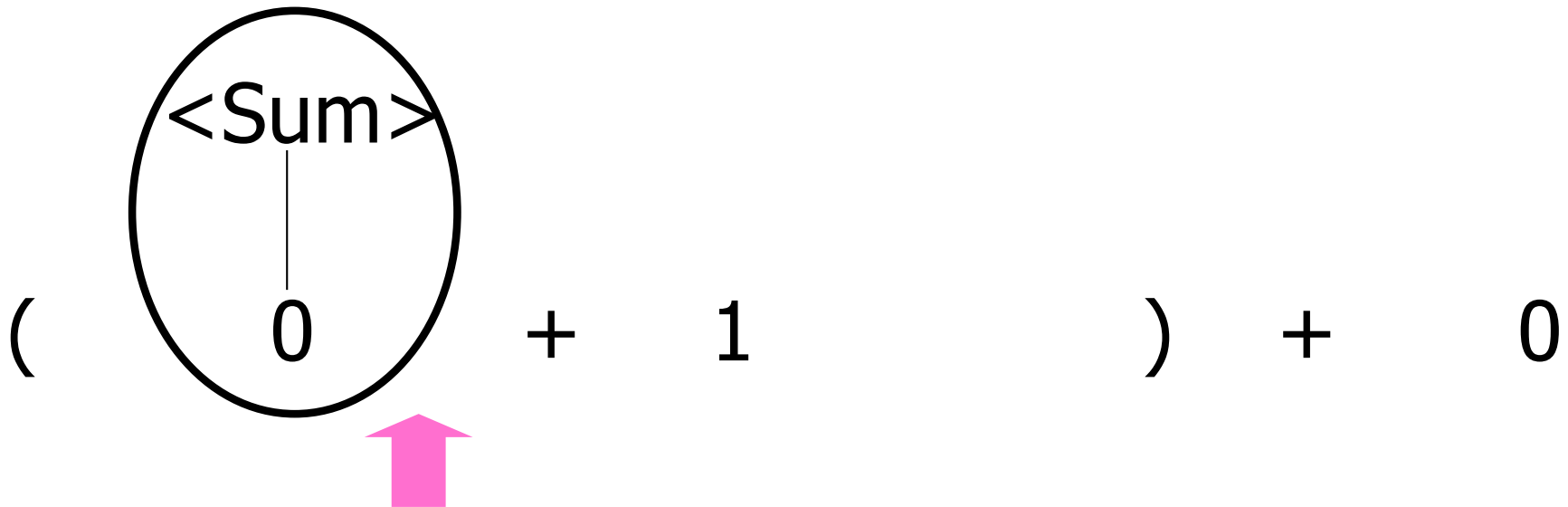
$$( 0 + 1 ) + 0$$





# Example

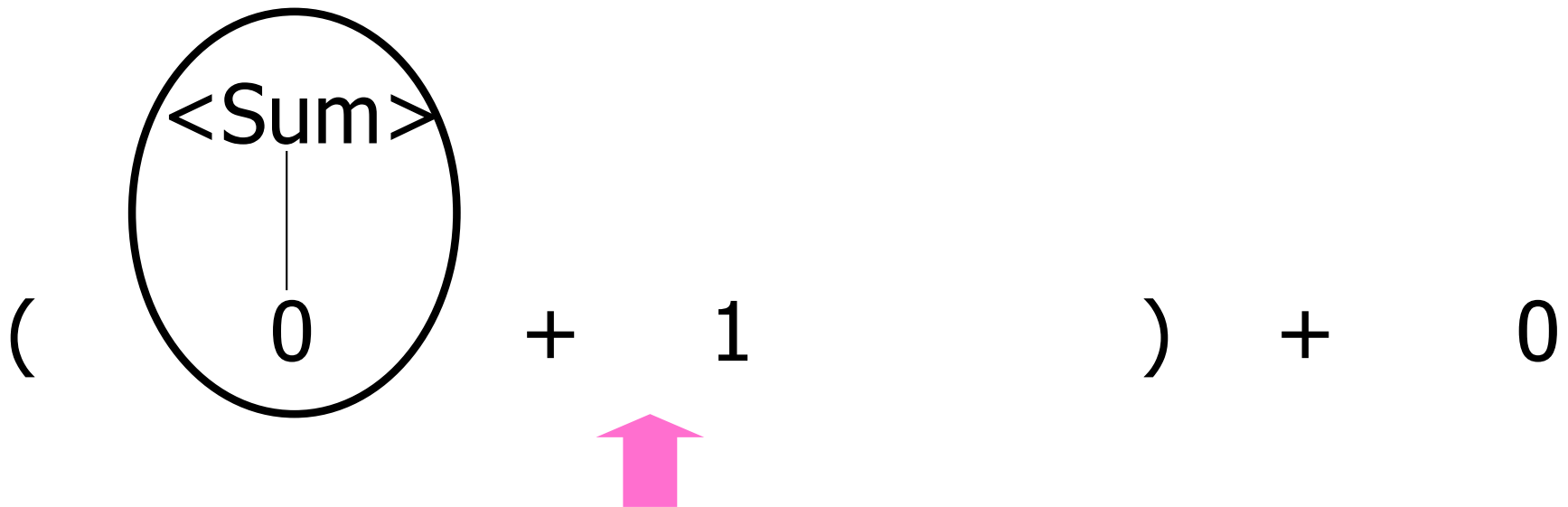
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# Example

---





# Example

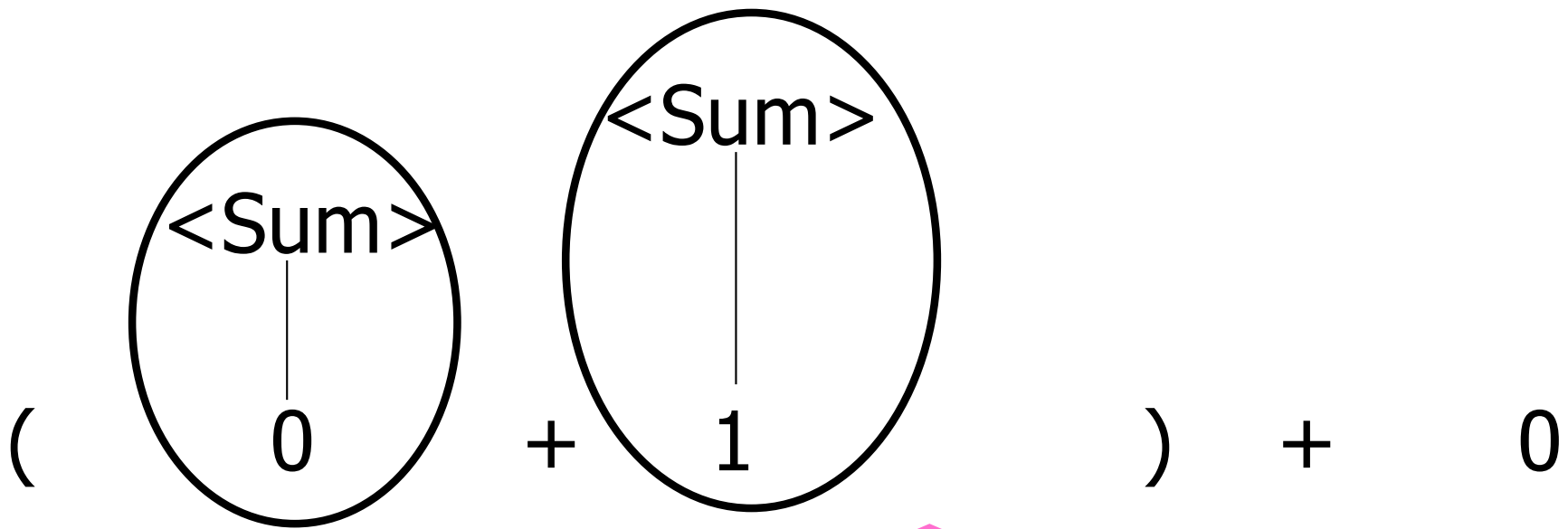
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# Example

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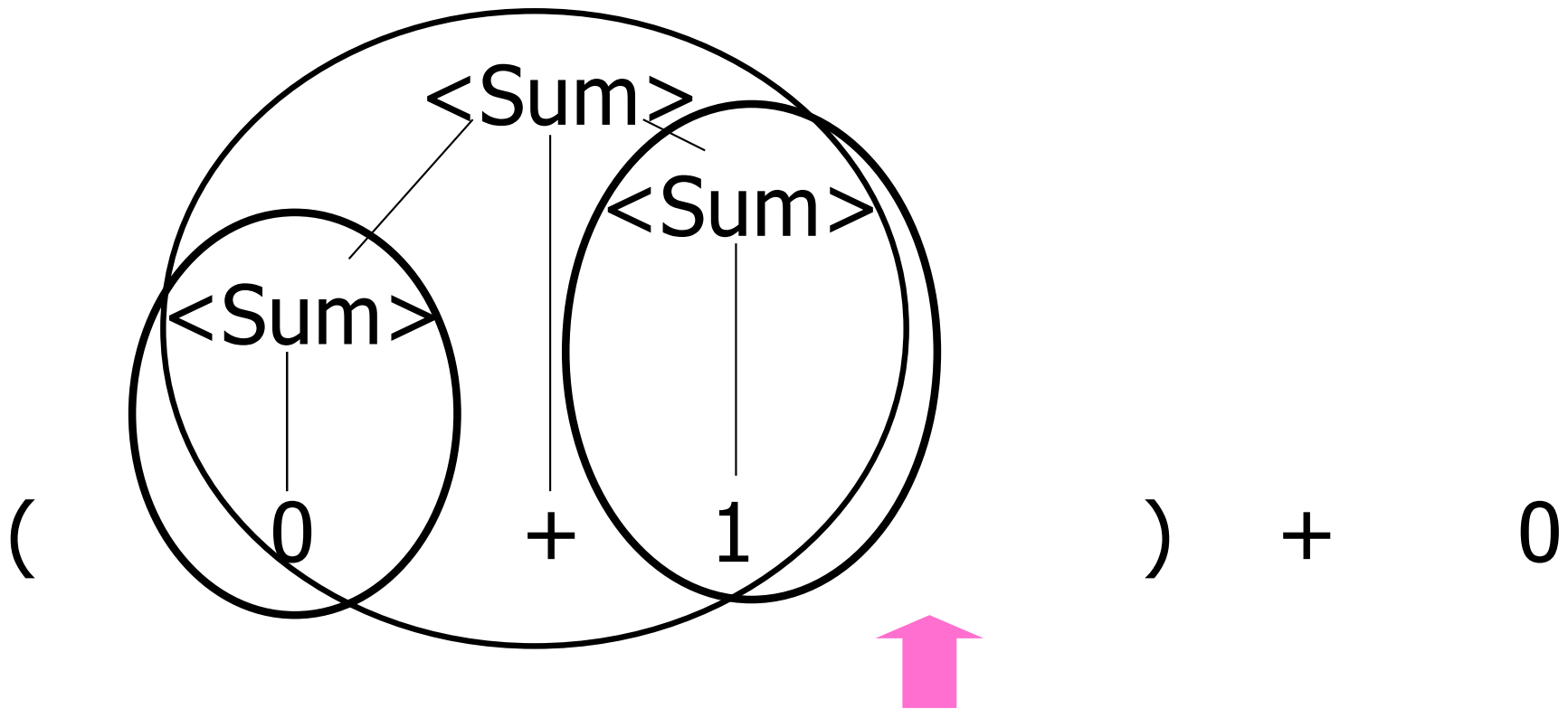






# Example

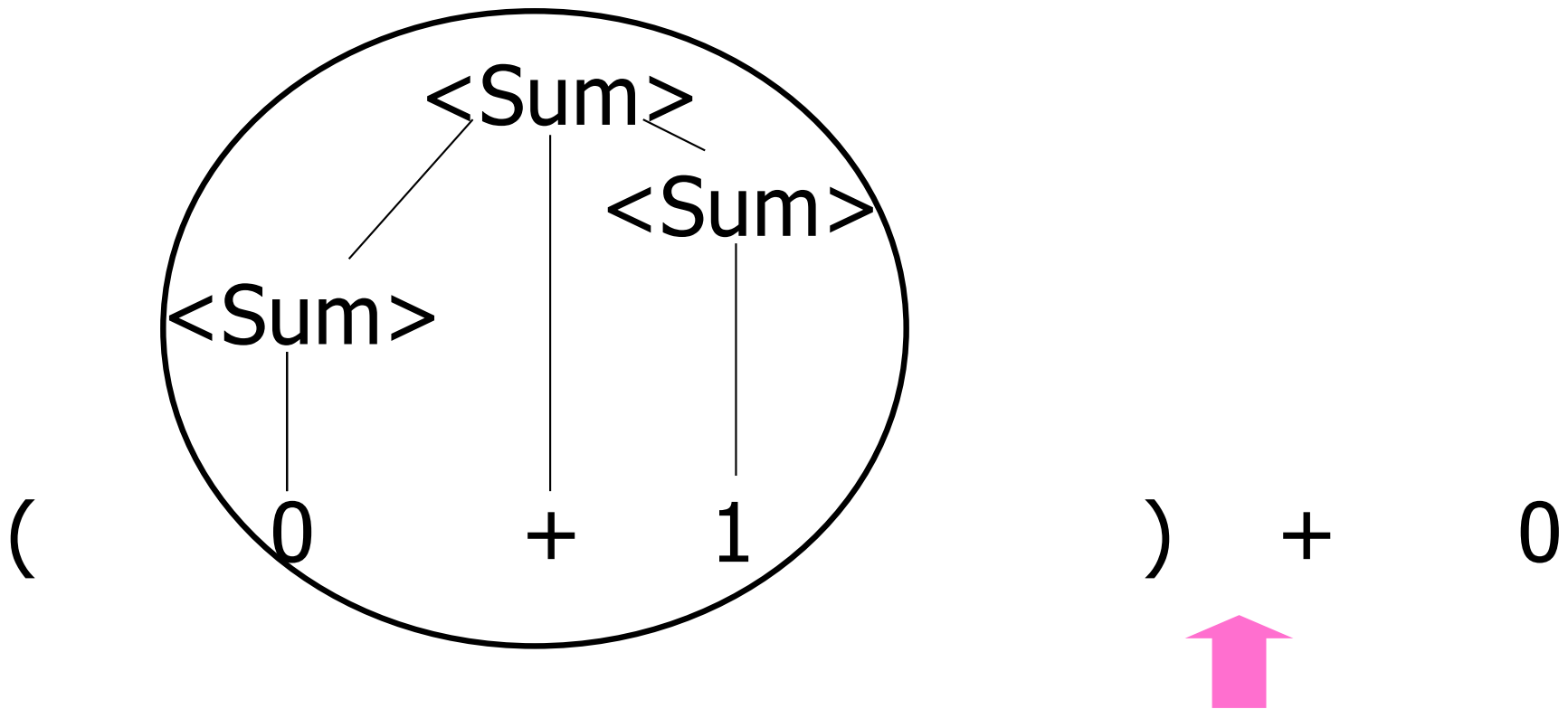
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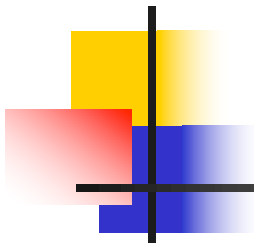




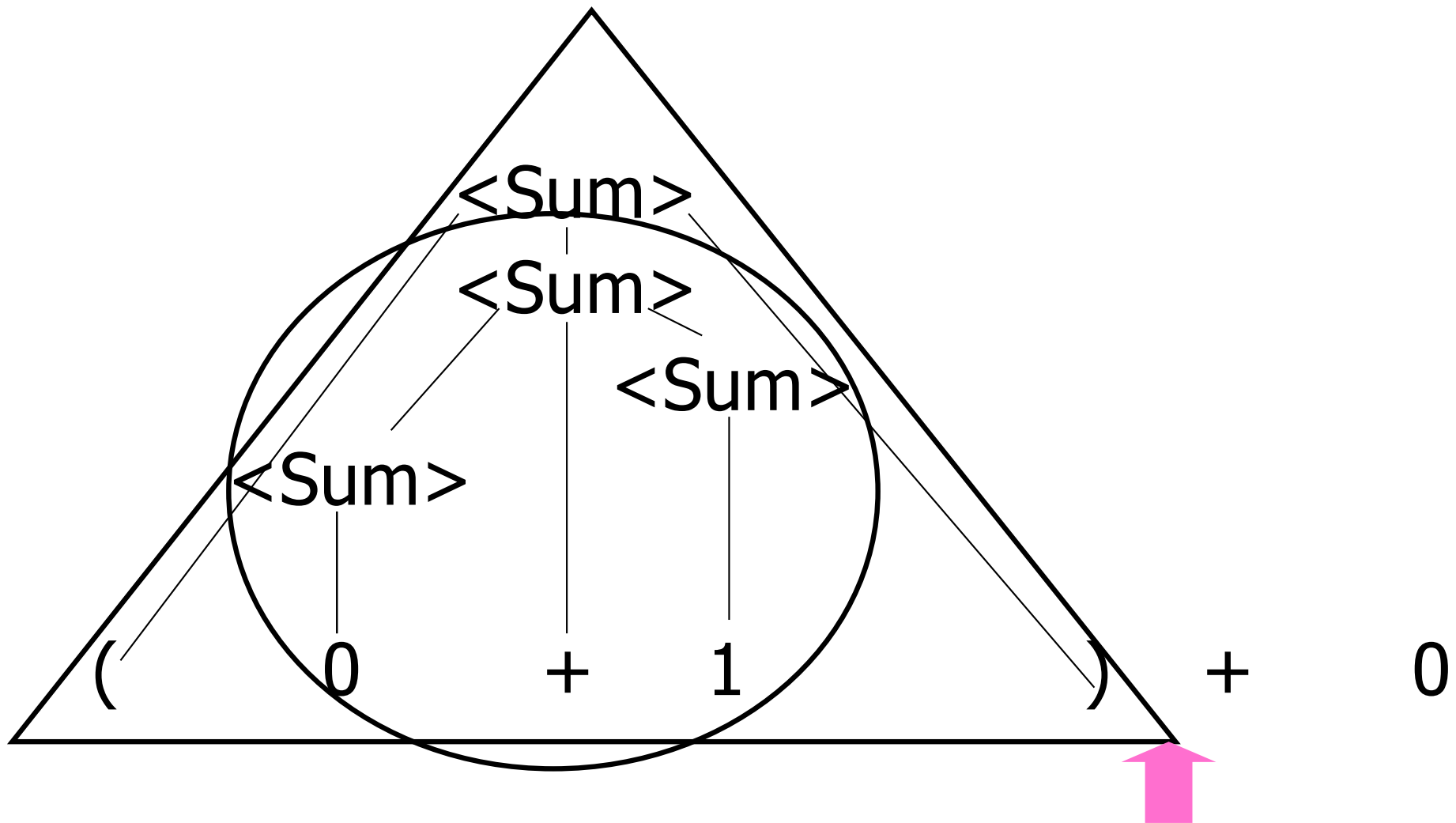
# Example

---



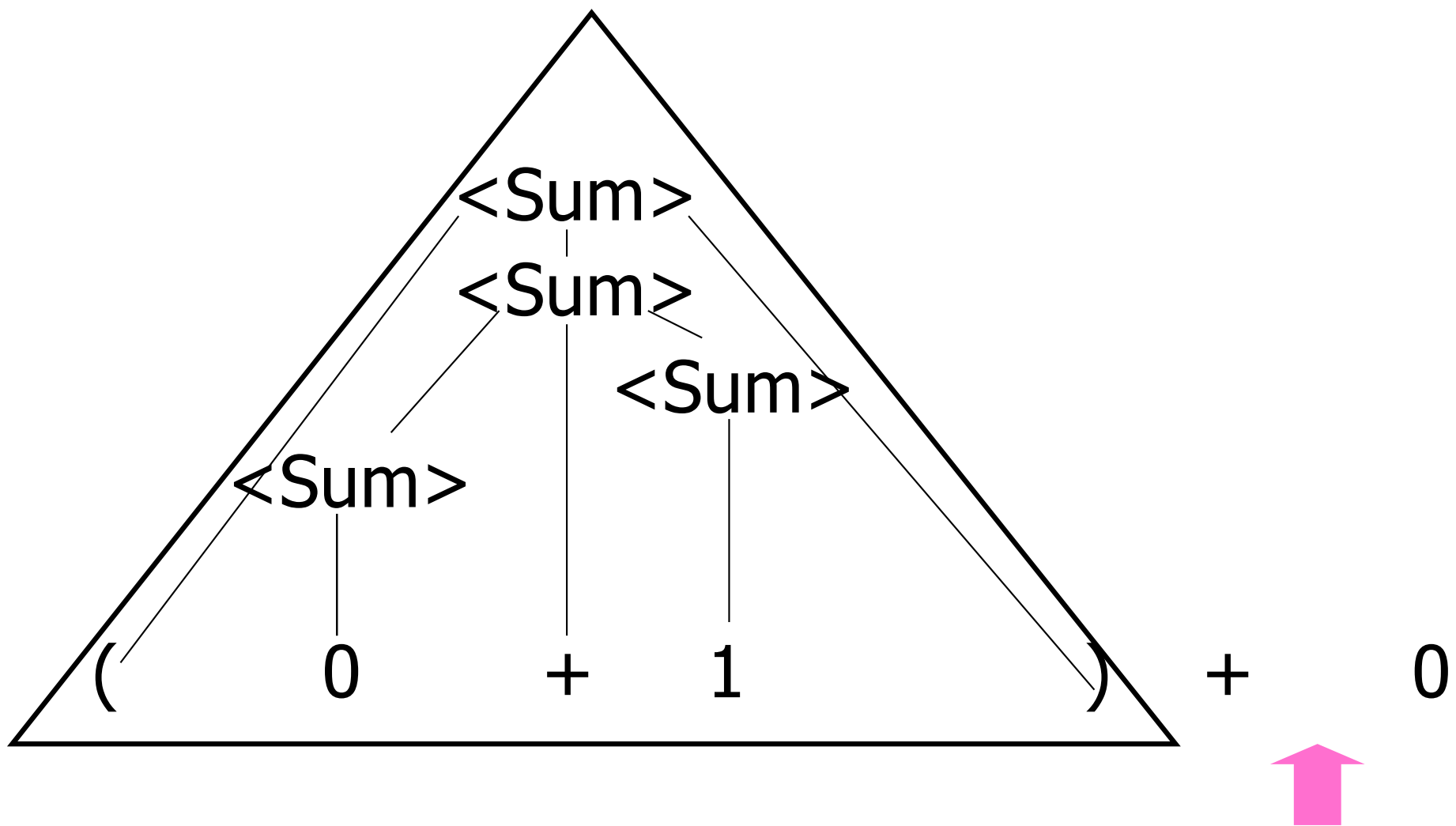


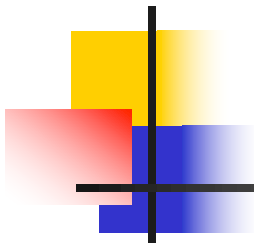
# Example



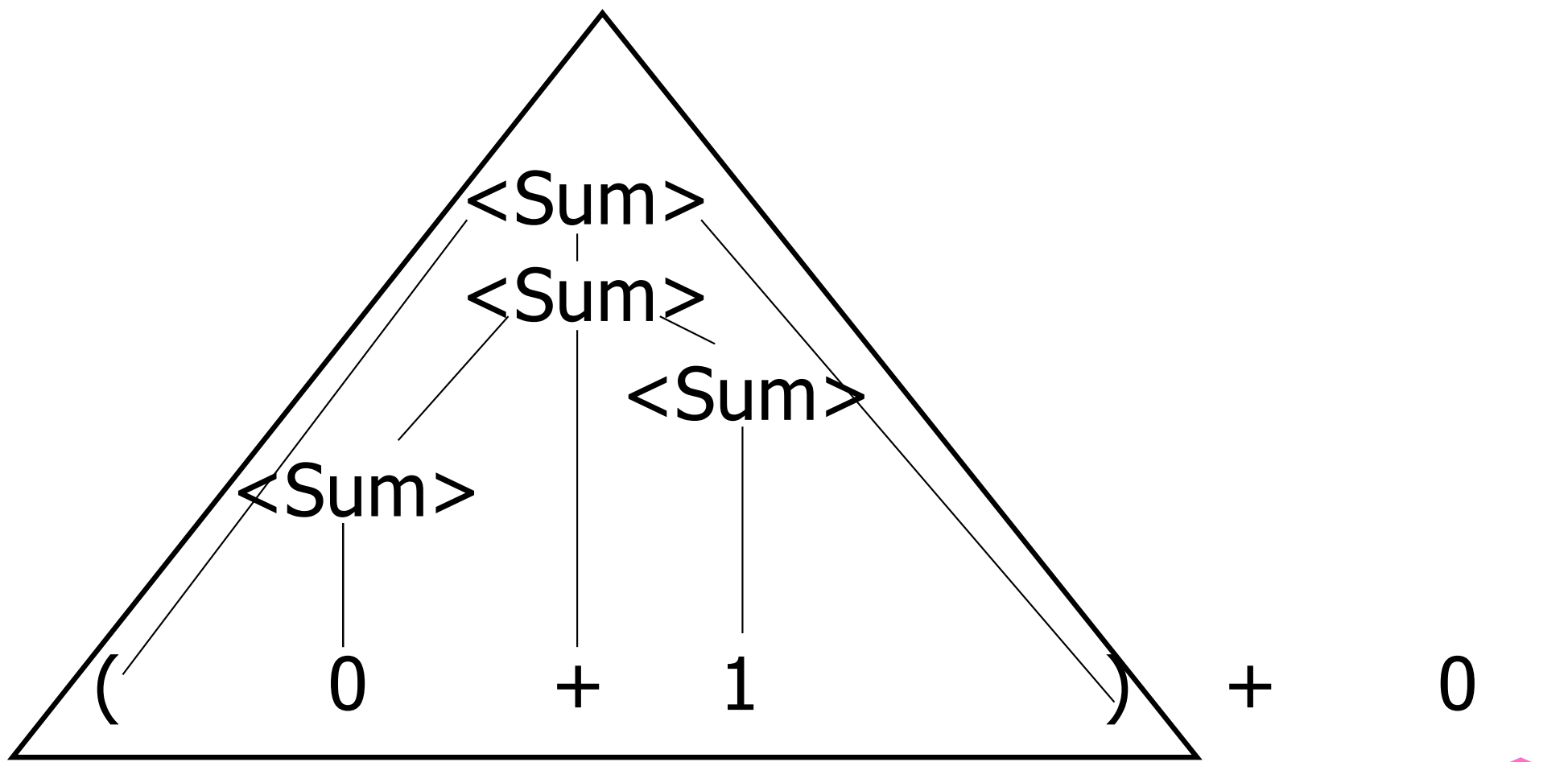


# Example



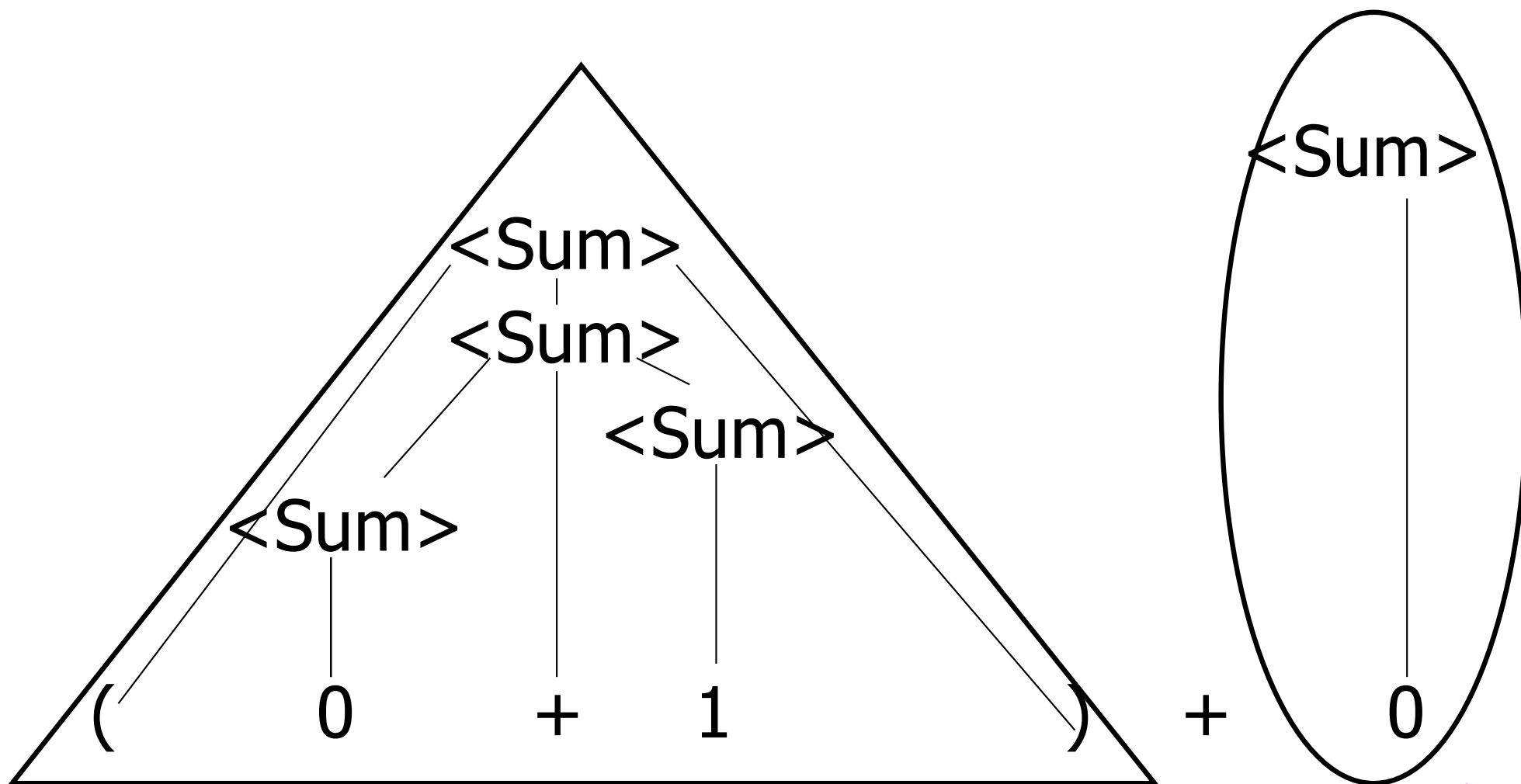


# Example



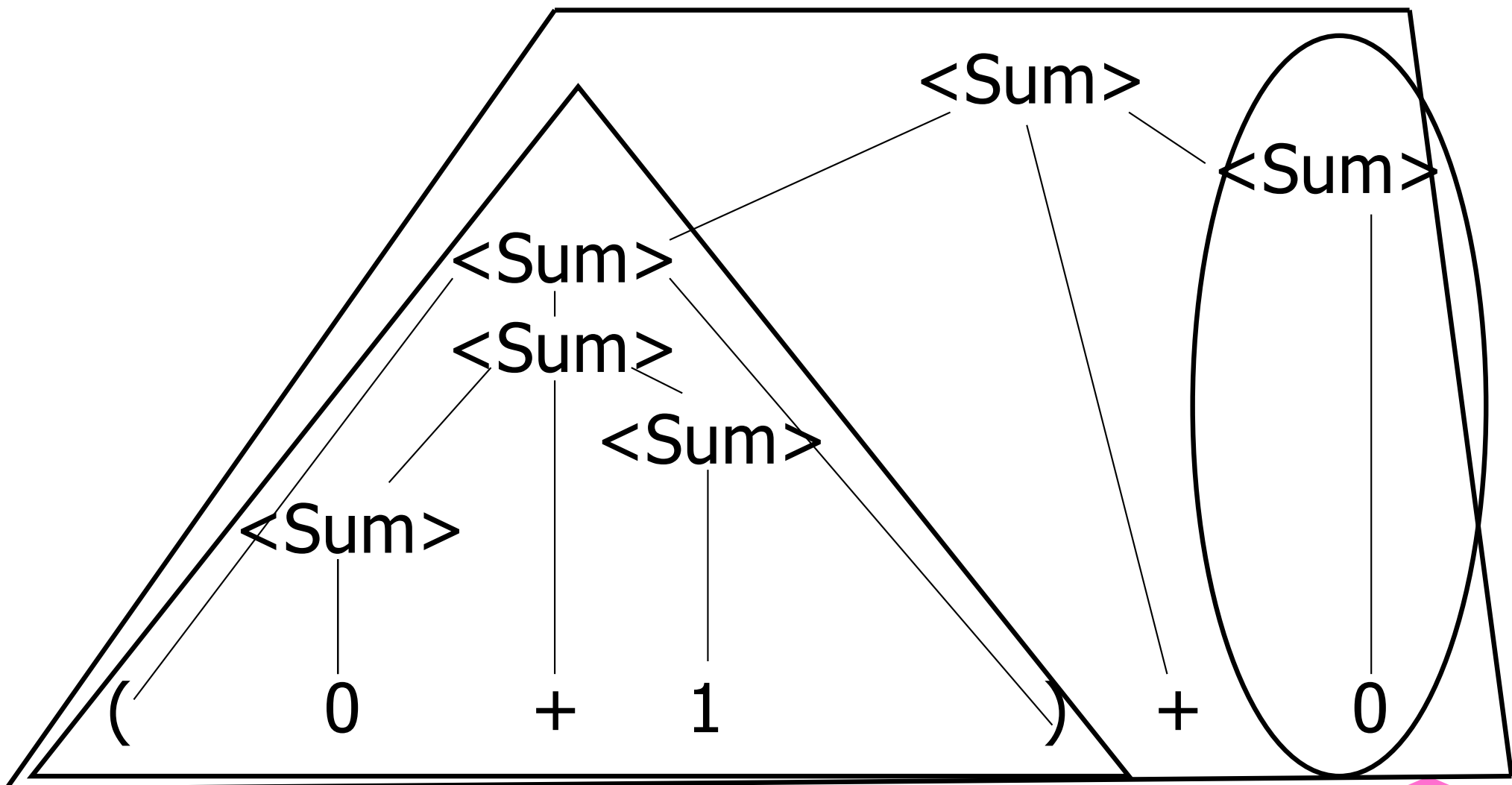


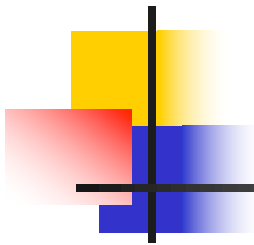
# Example



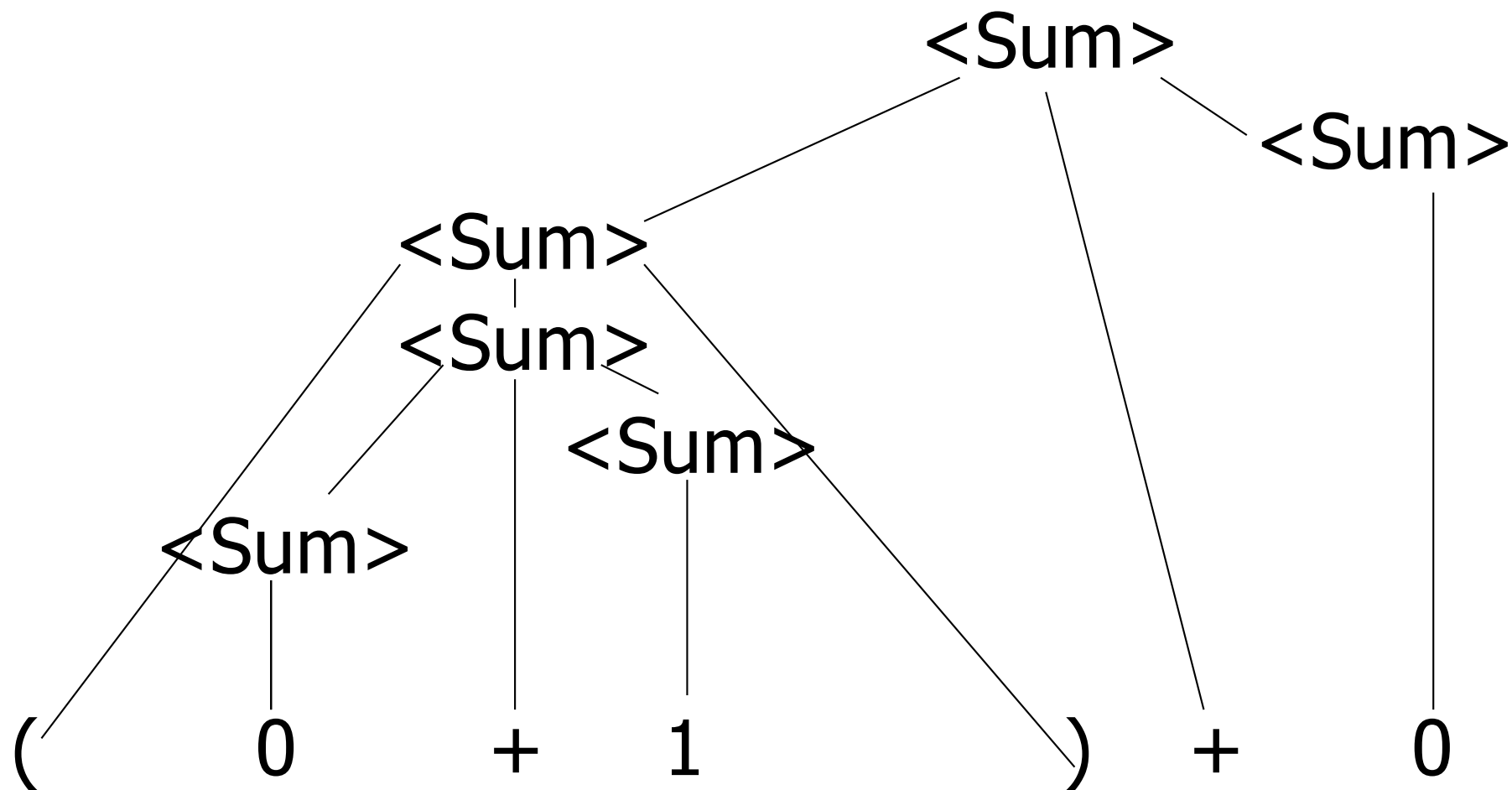


# Example





# Example







# LR Parsing Tables

---

- Build a pair of tables, Action and Goto, from the grammar
  - This is the hardest part, we omit here
  - Rows labeled by states
  - For Action, columns labeled by terminals and “end-of-tokens” marker
    - (more generally strings of terminals of fixed length)
  - For Goto, columns labeled by non-terminals



# Action and Goto Tables

---

- Given a state and the next input, Action table says either
  - **shift** and go to state  $n$ , or
  - **reduce** by production  $k$  (explained in a bit)
  - **accept** or **error**
- Given a state and a non-terminal, Goto table says
  - go to state  $m$



# LR(i) Parsing Algorithm

---

- Based on push-down automata
- Uses states and transitions (as recorded in Action and Goto tables)
- Uses a stack containing states, terminals and non-terminals



# LR(i) Parsing Algorithm

---

0. Insure token stream ends in special “end-of-tokens” symbol
1. Start in state 1 with an empty stack
2. Push **state**(1) onto stack
- 3. Look at next  $i$  tokens from token stream ( $toks$ ) (don't remove yet)
4. If top symbol on stack is **state**( $n$ ), look up action in Action table at  $(n, toks)$



# LR(i) Parsing Algorithm

---

5. If action = **shift**  $m$ ,

- a) Remove the top token from token stream and push it onto the stack
- b) Push **state**( $m$ ) onto stack
- c) Go to step 3



# LR(i) Parsing Algorithm

---

6. If action = **reduce**  $k$  where production  $k$  is  
 $E ::= u$
- a) Remove  $2 * \text{length}(u)$  symbols from stack ( $u$  and all the interleaved states)
  - b) If new top symbol on stack is **state**( $m$ ), look up new state  $p$  in  $\text{Goto}(m, E)$
  - c) Push  $E$  onto the stack, then push **state**( $p$ ) onto the stack
  - d) Go to step 3



# LR(i) Parsing Algorithm

---

7. If action = **accept**

- Stop parsing, return success

8. If action = **error**,

- Stop parsing, return failure



# Adding Synthesized Attributes

---

- Add to each **reduce** a rule for calculating the new synthesized attribute from the component attributes
- Add to each non-terminal pushed onto the stack, the attribute calculated for it
- When performing a **reduce**,
  - gather the recorded attributes from each non-terminal popped from stack
  - Compute new attribute for non-terminal pushed onto stack

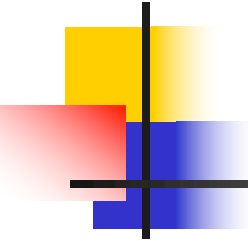




# Shift-Reduce Conflicts

---

- **Problem:** can't decide whether the action for a state and input character should be **shift** or **reduce**
- Caused by ambiguity in grammar
- Usually caused by lack of associativity or precedence information in grammar



Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$   
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

---

$\bullet 0 + 1 + 0$                       shift  
 $\rightarrow 0 \bullet + 1 + 0$                       reduce  
 $\rightarrow \langle \text{Sum} \rangle \bullet + 1 + 0$                       shift  
 $\rightarrow \langle \text{Sum} \rangle + \bullet 1 + 0$                       shift  
 $\rightarrow \langle \text{Sum} \rangle + 1 \bullet + 0$                       reduce  
 $\rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet + 0$



# Example - cont

---

- **Problem:** shift or reduce?
- You can shift-shift-reduce-reduce or reduce-shift-shift-reduce
- Shift first - right associative
- Reduce first- left associative



# Reduce - Reduce Conflicts

---

- **Problem:** can't decide between two different rules to reduce by
- Again caused by ambiguity in grammar
- **Symptom:** RHS of one production suffix of another
- Requires examining grammar and rewriting it
- Harder to solve than shift-reduce errors



# Example

---

■  $S ::= A \mid aB$       $A ::= abc$       $B ::= bc$

● abc     shift

a ● bc     shift

ab ● c     shift

abc ●

■ Problem: reduce by  $B ::= bc$  then by  $S ::= A$  or by  $A ::= abc$  then  $S ::= aB$ ?