

Programming Languages and Compilers (CS 421)



Elsa L Gunter

2112 SC, UIUC

<http://courses.engr.illinois.edu/cs421>

Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha



Terminology: Review

- A function is in **Direct Style** when it returns its result back to the caller.
- A function is in **Continuation Passing Style** when it, and every function call in it, passes its result to another function.
- A **Tail Call** occurs when a function returns the result of another function call without any more computations (eg tail recursion)
- Instead of returning the result to the caller, we pass it forward to another function giving the computation after the call.



CPS Transformation

- Step 1: Add continuation argument to any function definition:
 - $\text{let } f \text{ arg} = e \Rightarrow \text{let } f \text{ arg } k = e$
 - Idea: Every function takes an extra parameter saying where the result goes
- Step 2: A simple expression in tail position should be passed to a continuation instead of returned:
 - $\text{return } a \Rightarrow k \ a$
 - Assuming a is a constant or variable.
 - “Simple” = “No available function calls.”



CPS Transformation

- Step 3: Pass the current continuation to every function call in tail position
 - $\text{return } f \text{ arg} \Rightarrow f \text{ arg } k$
 - The function “isn’t going to return,” so we need to tell it where to put the result.



CPS Transformation

- Step 4: Each function call not in tail position needs to be converted to take a new continuation (containing the old continuation as appropriate)
 - $\text{return op (f arg)} \Rightarrow \text{f arg (fun r -> k(op r))}$
 - op represents a primitive operation

 - $\text{return g(f arg)} \Rightarrow \text{f arg (fun r-> g r k)}$



Example

Before:

```
let rec add_list lst =  
  match lst with  
  | [] -> 0  
  | 0 :: xs -> add_list xs  
  | x :: xs -> (+) x  
    (add_list xs);;
```

After:

```
let rec add_listk lst k =  
  (* rule 1 *)  
  match lst with  
  | [] -> k 0 (* rule 2 *)  
  | 0 :: xs -> add_listk xs k  
    (* rule 3 *)  
  | x :: xs -> add_listk xs  
    (fun r -> k ((+) x r));;  
  (* rule 4 *)
```



Example

Before:

```
let rec mem (y,lst) =  
  match lst with  
  [ ] -> false  
  | x :: xs ->  
    if (x = y)  
    then true  
    else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =  
  (* rule 1 *)
```



Example

Before:

```
let rec mem (y,lst) =  
  match lst with  
  [ ] -> false  
  | x :: xs ->  
    if (x = y)  
    then true  
    else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =  
  (* rule 1 *)  
  k false (* rule 2 *)  
  
  k true (* rule 2 *)
```




Example

Before:

```
let rec mem (y,lst) =  
  match lst with  
  [ ] -> false  
| x :: xs ->  
  if (x = y)  
  then true  
  else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =  
  (* rule 1 *)  
  
  k false (* rule 2 *)  
  
  k true (* rule 2 *)  
  memk (y, xs) k (* rule 3 *)
```



Example

Before:

```
let rec mem (y,lst) =  
  match lst with  
  [ ] -> false  
| x :: xs ->  
  if (x = y)  
  then true  
  else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =  
  (* rule 1 *)  
  k false (* rule 2 *)  
  
  eqk (x, y)  
  (fun b -> b (* rule 4 *))  
  k true (* rule 2 *)  
  memk (y, xs) (* rule 3 *)
```



Example

Before:

```
let rec mem (y,lst) =  
  match lst with  
  [ ] -> false  
| x :: xs ->  
  if (x = y)  
  then true  
  else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =  
  (* rule 1 *)  
  k false (* rule 2 *)  
  
  eqk (x, y)  
  (fun b -> if b (* rule 4 *)  
  then k true (* rule 2 *)  
  else memk (y, xs) (* rule 3 *))
```



Example

Before:

```
let rec mem (y,lst) =
```

```
match lst with
```

```
  [ ] -> false
```

```
| x :: xs ->
```

```
  if (x = y)
```

```
    then true
```

```
    else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =  
    (* rule 1 *)
```

```
match lst with
```

```
| [ ] -> k false (* rule 2 *)
```

```
| x :: xs ->
```

```
  eqk (x, y)
```

```
    (fun b -> if b (* rule 4 *))
```

```
  then k true (* rule 2 *)
```

```
    else memk (y, xs) k (* rule 3 *)
```



Example

Before:

```
let rec mem (y,lst) =  
  match lst with  
  [ ] -> false  
| x :: xs ->  
  if (x = y)  
  then true  
  else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =  
  (* rule 1 *)  
  match lst with  
  | [ ] -> k false (* rule 2 *)  
  | x :: xs ->  
    eqk (x, y)  
    (fun b -> if b (* rule 4 *)  
  then k true (* rule 2 *)  
    else memk (y, xs) k (* rule 3 *)
```



Data type in Ocaml: lists

- Frequently used lists in recursive program
- Matched over two structural cases
 - `[]` - the empty list
 - `(x :: xs)` a non-empty list
- Covers all possible lists
- `type 'a list = [] | (::) of 'a * 'a list`
 - Not quite legitimate declaration because of special syntax

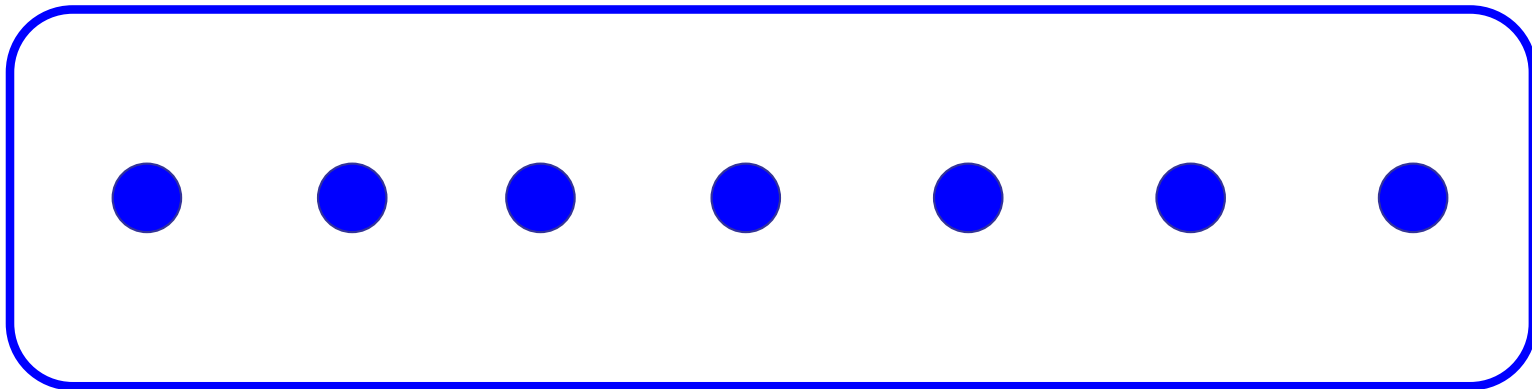


Variants - Syntax (slightly simplified)

- $\text{type } name = C_1 [\text{of } ty_1] \mid \dots \mid C_n [\text{of } ty_n]$
- Introduce a type called *name*
- $(\text{fun } x \rightarrow C_i x) : ty_1 \rightarrow name$
- C_i is called a *constructor*; if the optional type argument is omitted, it is called a *constant*
- Constructors are the basis of almost all pattern matching

Enumeration Types as Variants

An enumeration type is a collection of distinct values



In C and Ocaml they have an order structure;
order by order of input



Enumeration Types as Variants

```
# type weekday = Monday | Tuesday | Wednesday  
| Thursday | Friday | Saturday | Sunday;;
```

```
type weekday =
```

```
Monday
```

```
| Tuesday
```

```
| Wednesday
```

```
| Thursday
```

```
| Friday
```

```
| Saturday
```

```
| Sunday
```



Functions over Enumerations

```
# let day_after day = match day with
```

```
  Monday -> Tuesday
```

```
| Tuesday -> Wednesday
```

```
| Wednesday -> Thursday
```

```
| Thursday -> Friday
```

```
| Friday -> Saturday
```

```
| Saturday -> Sunday
```

```
| Sunday -> Monday;;
```

```
val day_after : weekday -> weekday = <fun>
```



Functions over Enumerations

```
# let rec days_later n day =  
  match n with 0 -> day  
  | _ -> if n > 0  
         then day_after (days_later (n - 1) day)  
         else days_later (n + 7) day;;  
val days_later : int -> weekday -> weekday  
= <fun>
```



Functions over Enumerations

```
# days_later 2 Tuesday;;
```

```
- : weekday = Thursday
```

```
# days_later (-1) Wednesday;;
```

```
- : weekday = Tuesday
```

```
# days_later (-4) Monday;;
```

```
- : weekday = Thursday
```



Problem:

```
# type weekday = Monday | Tuesday |  
Wednesday  
| Thursday | Friday | Saturday | Sunday;;  
■ Write function is_weekend : weekday -> bool  
let is_weekend day =
```



Problem:

```
# type weekday = Monday | Tuesday |  
Wednesday  
| Thursday | Friday | Saturday | Sunday;;  
■ Write function is_weekend : weekday -> bool  
let is_weekend day =  
    match day with Saturday -> true  
    | Sunday -> true  
    | _ -> false
```



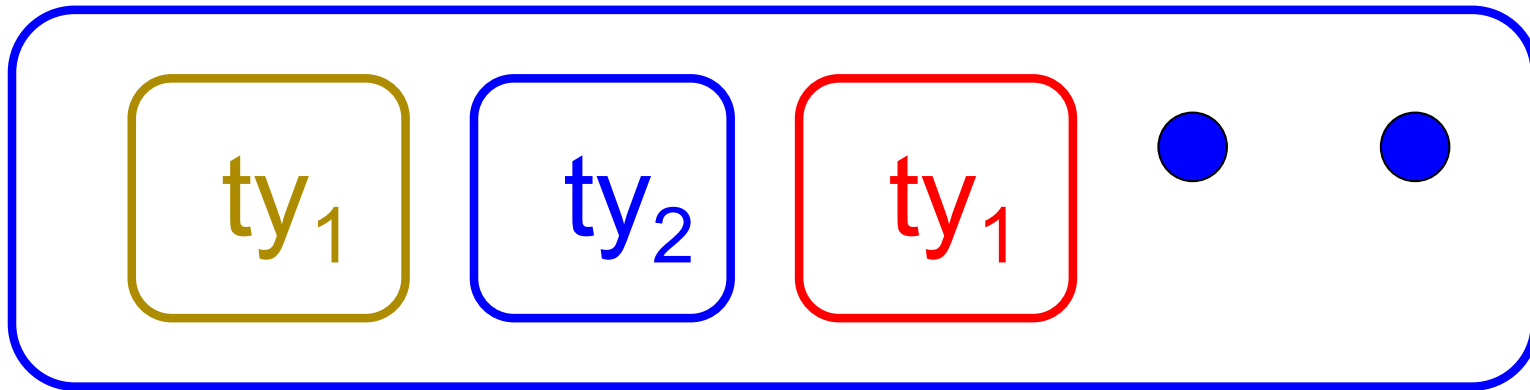
Example Enumeration Types

```
# type bin_op = IntPlusOp | IntMinusOp  
              | EqOp | CommaOp | ConsOp
```

```
# type mon_op = HdOp | TlOp | FstOp  
              | SndOp
```

Disjoint Union Types

- Disjoint union of types, with some possibly occurring more than once



- We can also add in some new singleton elements



Disjoint Union Types

```
# type id = DriversLicense of int
  | SocialSecurity of int | Name of string;;
type id = DriversLicense of int | SocialSecurity
  of int | Name of string
# let check_id id = match id with
  DriversLicense num ->
    not (List.mem num [13570; 99999])
  | SocialSecurity num -> num < 900000000
  | Name str -> not (str = "John Doe");;
val check_id : id -> bool = <fun>
```



Problem

- Create a type to represent the currencies for US, UK, Europe and Japan



Problem

- Create a type to represent the currencies for US, UK, Europe and Japan

type currency =

 Dollar of int

| Pound of int

| Euro of int

| Yen of int



Example Disjoint Union Type

```
# type const =  
  BoolConst of bool  
| IntConst of int  
| FloatConst of float  
| StringConst of string  
| NilConst  
| UnitConst
```



Example Disjoint Union Type

```
# type const = BoolConst of bool  
  | IntConst of int | FloatConst of float  
  | StringConst of string | NilConst  
  | UnitConst
```

- How to represent 7 as a const?
- Answer: `IntConst 7`



Polymorphism in Variants

- The type `'a option` gives us something to represent non-existence or failure

```
# type 'a option = Some of 'a | None;;
```

```
type 'a option = Some of 'a | None
```

- Used to encode partial functions
- Often can replace the raising of an exception



Functions producing option

```
# let rec first p list =  
  match list with [ ] -> None  
  | (x::xs) -> if p x then Some x else first p xs;;  
val first : ('a -> bool) -> 'a list -> 'a option = <fun>  
# first (fun x -> x > 3) [1;3;4;2;5];;  
- : int option = Some 4  
# first (fun x -> x > 5) [1;3;4;2;5];;  
- : int option = None
```



Functions over option

```
# let result_ok r =  
  match r with None -> false  
  | Some _ -> true;;  
val result_ok : 'a option -> bool = <fun>  
# result_ok (first (fun x -> x > 3) [1;3;4;2;5]);;  
- : bool = true  
# result_ok (first (fun x -> x > 5) [1;3;4;2;5]);;  
- : bool = false
```




Problem

- Write a `hd` and `tl` on lists that doesn't raise an exception and works at all types of lists.



Problem

- Write a `hd` and `tl` on lists that doesn't raise an exception and works at all types of lists.
- `let hd list =`
 - `match list with [] -> None`
 - `| (x::xs) -> Some x`
- `let tl list =`
 - `match list with [] -> None`
 - `| (x::xs) -> Some xs`



Mapping over Variants

```
# let optionMap f opt =  
  match opt with None -> None  
  | Some x -> Some (f x);;  
val optionMap : ('a -> 'b) -> 'a option -> 'b  
  option = <fun>  
# optionMap  
  (fun x -> x - 2)  
  (first (fun x -> x > 3) [1;3;4;2;5]);;  
- : int option = Some 2
```

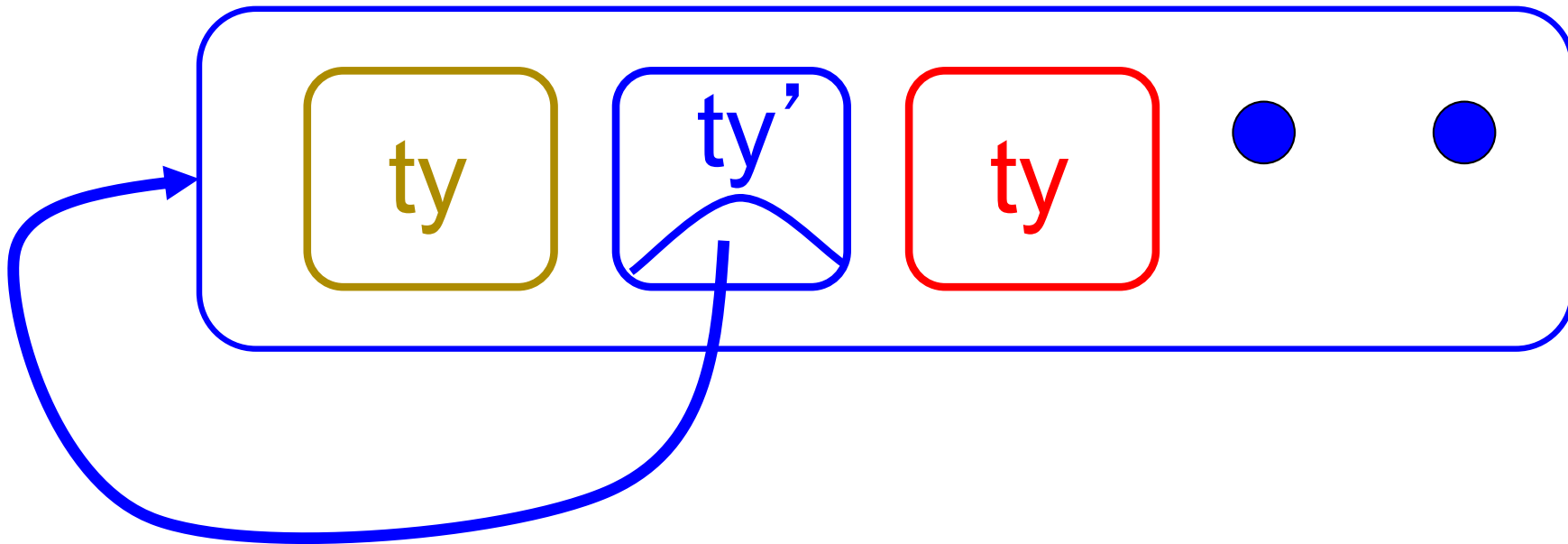


Folding over Variants

```
# let optionFold someFun noneVal opt =  
  match opt with None -> noneVal  
  | Some x -> someFun x;;  
val optionFold : ('a -> 'b) -> 'b -> 'a option ->  
  'b = <fun>  
# let optionMap f opt =  
  optionFold (fun x -> Some (f x)) None opt;;  
val optionMap : ('a -> 'b) -> 'a option -> 'b  
  option = <fun>
```

Recursive Types

- The type being defined may be a component of itself





Recursive Data Types

```
# type int_Bin_Tree =  
  Leaf of int | Node of (int_Bin_Tree *  
    int_Bin_Tree);;
```

```
type int_Bin_Tree = Leaf of int | Node of  
  (int_Bin_Tree * int_Bin_Tree)
```



Recursive Data Type Values

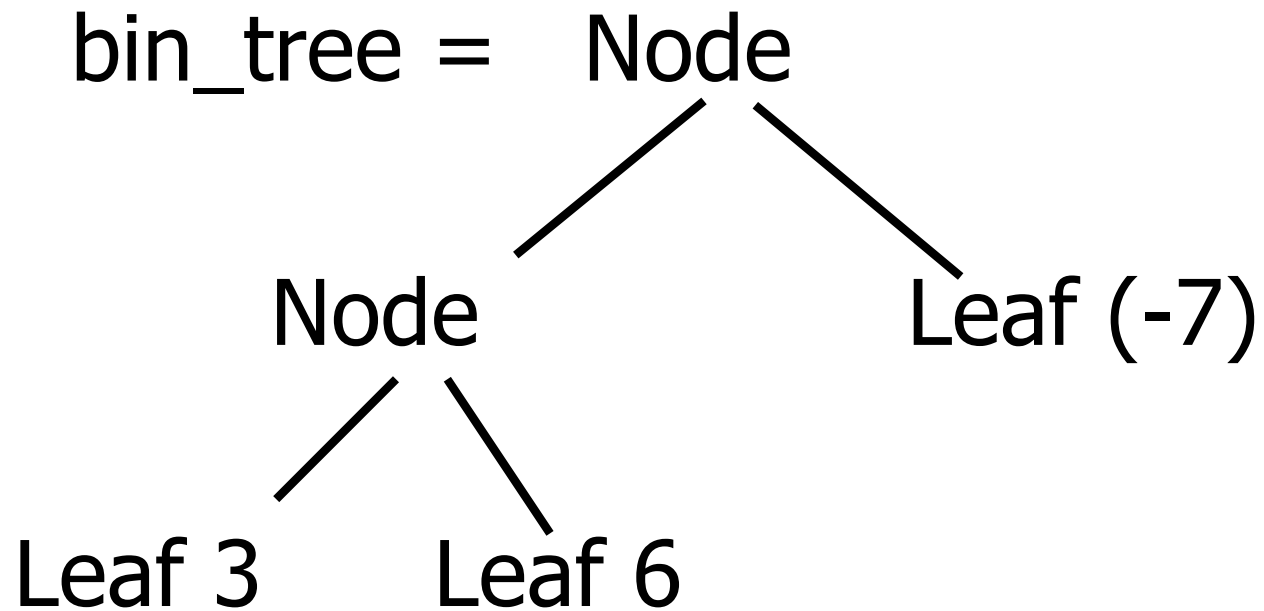
```
# let bin_tree =
```

```
Node(Node(Leaf 3, Leaf 6),Leaf (-7));;
```

```
val bin_tree : int_Bin_Tree = Node (Node  
  (Leaf 3, Leaf 6), Leaf (-7))
```



Recursive Data Type Values





Recursive Functions

```
# let rec first_leaf_value tree =  
  match tree with (Leaf n) -> n  
  | Node (left_tree, right_tree) ->  
    first_leaf_value left_tree;;  
val first_leaf_value : int_Bin_Tree -> int =  
  <fun>  
# let left = first_leaf_value bin_tree;;  
val left : int = 3
```



Recursive Data Types

```
# type exp =  
  VarExp of string  
| ConstExp of const  
| MonOpAppExp of mon_op * exp  
| BinOpAppExp of bin_op * exp * exp  
| IfExp of exp * exp * exp  
| AppExp of exp * exp  
| FunExp of string * exp
```



Recursive Data Types

```
# type bin_op = IntPlusOp | IntMinusOp  
              | EqOp | CommaOp | ConsOp | ...
```

```
# type const = BoolConst of bool | IntConst of int |  
...
```

```
# type exp = VarExp of string | ConstExp of const  
           | BinOpAppExp of bin_op * exp * exp | ...
```

■ How to represent 6 as an exp?



Recursive Data Types

```
# type bin_op = IntPlusOp | IntMinusOp
                | EqOp | CommaOp | ConsOp | ...
# type const = BoolConst of bool | IntConst of int |
...
# type exp = VarExp of string | ConstExp of const
            | BinOpAppExp of bin_op * exp * exp | ...
```

- How to represent 6 as an exp?
- Answer: ConstExp (IntConst 6)



Recursive Data Types

```
# type bin_op = IntPlusOp | IntMinusOp
              | EqOp | CommaOp | ConsOp | ...
# type const = BoolConst of bool | IntConst of int |
...
# type exp = VarExp of string | ConstExp of const
            | BinOpAppExp of bin_op * exp * exp | ...
```

- How to represent (6, 3) as an exp?



Recursive Data Types

```
# type bin_op = IntPlusOp | IntMinusOp
                | EqOp | CommaOp | ConsOp | ...
# type const = BoolConst of bool | IntConst of int |
...
# type exp = VarExp of string | ConstExp of const
            | BinOpAppExp of bin_op * exp * exp | ...
```

- How to represent (6, 3) as an exp?
- BinOpAppExp (CommaOp, ConstExp (IntConst 6),
ConstExp (IntConst 3))



Recursive Data Types

```
# type bin_op = IntPlusOp | IntMinusOp
              | EqOp | CommaOp | ConsOp | ...
# type const = BoolConst of bool | IntConst of int |
...
# type exp = VarExp of string | ConstExp of const
           | BinOpAppExp of bin_op * exp * exp | ...
```

- How to represent [(6, 3)] as an exp?
- BinOpAppExp (ConsOp, BinOpAppExp (CommaOp, ConstExp (IntConst 6), ConstExp (IntConst 3)), ConstExp NilConst))));;



Problem

```
type int_Bin_Tree = Leaf of int  
| Node of (int_Bin_Tree * int_Bin_Tree);;
```

- Write `sum_tree : int_Bin_Tree -> int`
- Adds all ints in tree

```
let rec sum_tree t =
```




Problem

```
type int_Bin_Tree = Leaf of int
```

```
| Node of (int_Bin_Tree * int_Bin_Tree);;
```

- Write `sum_tree : int_Bin_Tree -> int`
- Adds all ints in tree

```
let rec sum_tree t =
```

```
  match t with Leaf n -> n
```

```
  | Node(t1,t2) -> sum_tree t1 + sum_tree t2
```



Recursion over Recursive Data Types

```
# type exp = VarExp of string | ConstExp of const  
| BinOpAppExp of bin_op * exp * exp  
| FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?



Recursion over Recursive Data Types

```
# type exp = VarExp of string | ConstExp of const  
| BinOpAppExp of bin_op * exp * exp  
| FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?

```
# let rec varCnt exp =  
  match exp with VarExp x ->  
  | ConstExp c ->  
  | BinOpAppExp (b, e1, e2) ->  
  | FunExp (x,e) ->  
  | AppExp (e1, e2) ->
```

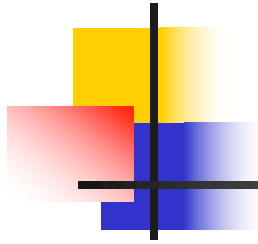


Recursion over Recursive Data Types

```
# type exp = VarExp of string | ConstExp of const  
| BinOpAppExp of bin_op * exp * exp  
| FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?

```
# let rec varCnt exp =  
  match exp with VarExp x -> 1  
  | ConstExp c -> 0  
  | BinOpAppExp (b, e1, e2) -> varCnt e1 + varCnt e2  
  | FunExp (x,e) -> 1 + varCnt e  
  | AppExp (e1, e2) -> varCnt e1 + varCnt e2
```



Your turn now

Try Problem 3 on MP5



Mapping over Recursive Types

```
# let rec ibtreeMap f tree =  
  match tree with (Leaf n) -> Leaf (f n)  
  | Node (left_tree, right_tree) ->  
    Node (ibtreeMap f left_tree,  
          ibtreeMap f right_tree);;  
val ibtreeMap : (int -> int) -> int_Bin_Tree ->  
  int_Bin_Tree = <fun>
```



Mapping over Recursive Types

```
# ibtreeMap ((+) 2) bin_tree;;
```

```
- : int_Bin_Tree = Node (Node (Leaf 5, Leaf  
8), Leaf (-5))
```



Folding over Recursive Types

```
# let rec ibtreeFoldRight leafFun nodeFun tree =  
  match tree with Leaf n -> leafFun n  
  | Node (left_tree, right_tree) ->  
    nodeFun  
      (ibtreeFoldRight leafFun nodeFun left_tree)  
      (ibtreeFoldRight leafFun nodeFun right_tree);;  
val ibtreeFoldRight : (int -> 'a) -> ('a -> 'a -> 'a) ->  
  int_Bin_Tree -> 'a = <fun>
```




Folding over Recursive Types

```
# let tree_sum =  
    ibtreeFoldRight (fun x -> x) (+);;  
val tree_sum : int_Bin_Tree -> int = <fun>  
# tree_sum bin_tree;;  
- : int = 2
```

600 minutes



Mutually Recursive Types

```
# type 'a tree = TreeLeaf of 'a
```

```
  | TreeNode of 'a treeList
```

```
and 'a treeList = Last of 'a tree
```

```
  | More of ('a tree * 'a treeList);;
```

```
type 'a tree = TreeLeaf of 'a | TreeNode of 'a  
treeList
```

```
and 'a treeList = Last of 'a tree | More of ('a  
tree * 'a treeList)
```



Mutually Recursive Types - Values

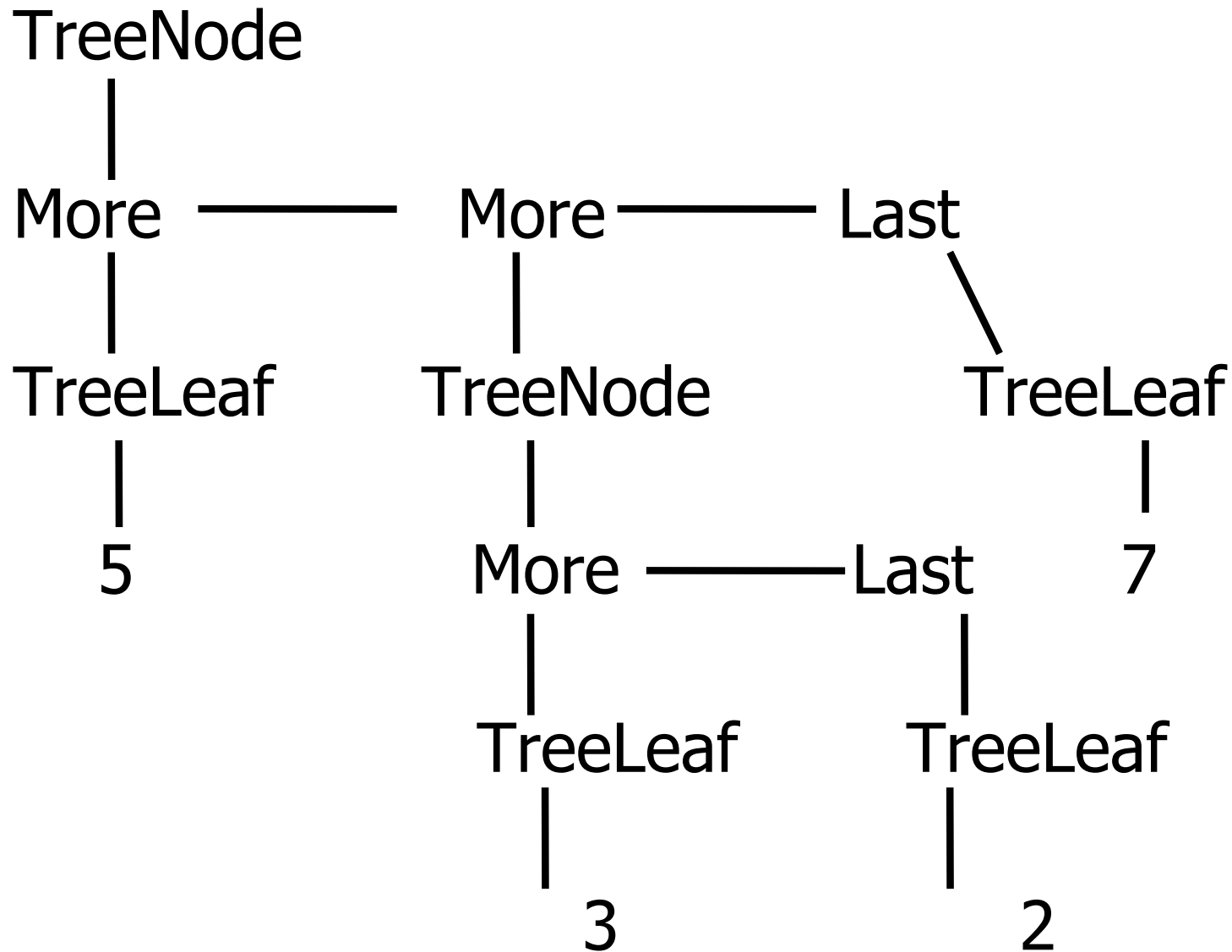
```
# let tree =  
  TreeNode  
    (More (TreeLeaf 5,  
          (More (TreeNode  
                (More (TreeLeaf 3,  
                      Last (TreeLeaf 2))),  
                      Last (TreeLeaf 7))))));;
```



Mutually Recursive Types - Values

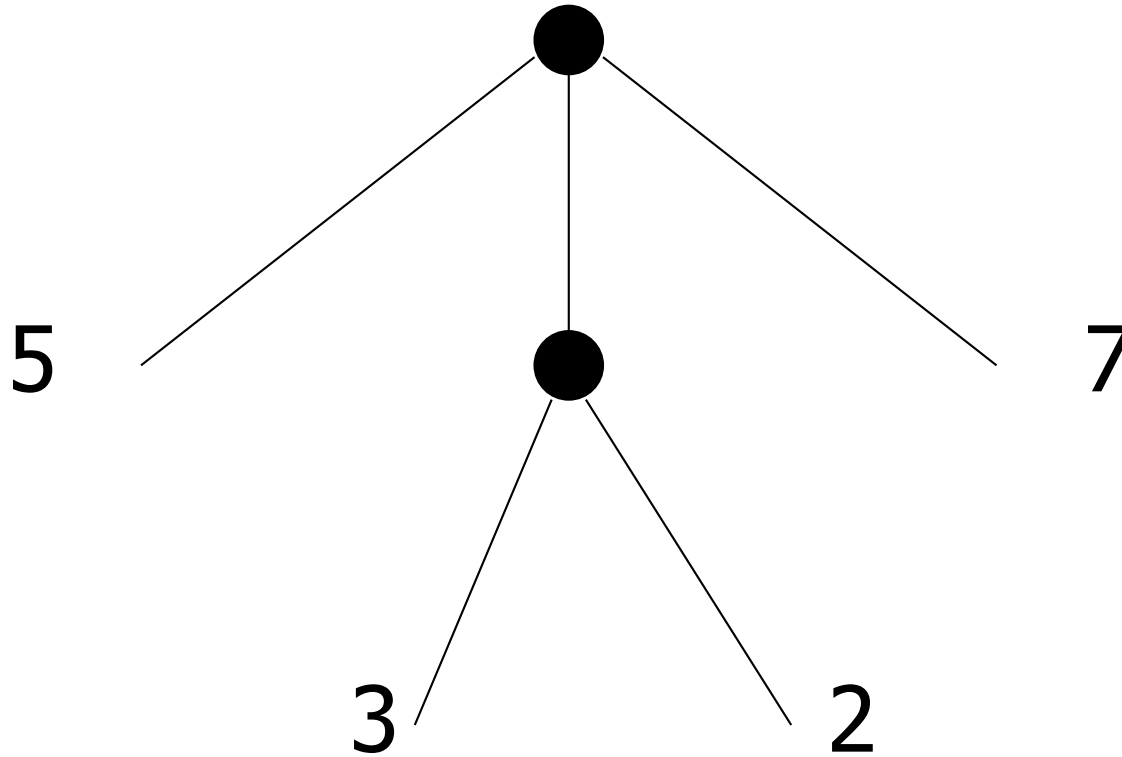
```
val tree : int tree =  
  TreeNode  
    (More  
      (TreeLeaf 5,  
        More  
          (TreeNode (More (TreeLeaf 3, Last  
            (TreeLeaf 2))), Last (TreeLeaf 7))))))
```

Mutually Recursive Types - Values



Mutually Recursive Types - Values

A more conventional picture





Mutually Recursive Functions

```
# let rec fringe tree =  
    match tree with (TreeLeaf x) -> [x]  
    | (TreeNode list) -> list_fringe list  
and list_fringe tree_list =  
    match tree_list with (Last tree) -> fringe tree  
    | (More (tree,list)) ->  
    (fringe tree) @ (list_fringe list);;
```

```
val fringe : 'a tree -> 'a list = <fun>
```

```
val list_fringe : 'a treeList -> 'a list = <fun>
```




Mutually Recursive Functions

```
# fringe tree;;
```

```
- : int list = [5; 3; 2; 7]
```



Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
Define tree_size
```



Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList  
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size

```
let rec tree_size t =
```

```
    match t with TreeLeaf _ ->
```

```
    | TreeNode ts ->
```



Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList  
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size

```
let rec tree_size t =
```

```
    match t with TreeLeaf _ -> 1
```

```
    | TreeNode ts -> treeList_size ts
```



Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList  
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define `tree_size` and `treeList_size`

```
let rec tree_size t =
```

```
    match t with TreeLeaf _ -> 1
```

```
    | TreeNode ts -> treeList_size ts
```

```
and treeList_size ts =
```



Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList  
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define `tree_size` and `treeList_size`

```
let rec tree_size t =
```

```
    match t with TreeLeaf _ -> 1
```

```
    | TreeNode ts -> treeList_size ts
```

```
and treeList_size ts =
```

```
    match ts with Last t ->
```

```
    | More t ts' ->
```



Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
Define tree_size and treeList_size
let rec tree_size t =
  match t with TreeLeaf _ -> 1
  | TreeNode ts -> treeList_size ts
and treeList_size ts =
  match ts with Last t -> tree_size t
  | More t ts' -> tree_size t + treeList_size ts'
```



Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;

Define tree_size and treeList_size

let rec tree_size t =
  match t with TreeLeaf _ -> 1
  | TreeNode ts -> treeList_size ts
and treeList_size ts =
  match ts with Last t -> tree_size t
  | More t ts' -> tree_size t + treeList_size ts'
```




Nested Recursive Types

```
# type 'a labeled_tree =
```

```
  TreeNode of ('a * 'a labeled_tree  
  list);;
```

```
type 'a labeled_tree = TreeNode of ('a  
  * 'a labeled_tree list)
```



Nested Recursive Type Values

```
# let ltree =  
  TreeNode(5,  
    [TreeNode (3, []);  
      TreeNode (2, [TreeNode (1, []);  
                          TreeNode (7, [])]);  
      TreeNode (5, [])]);;
```

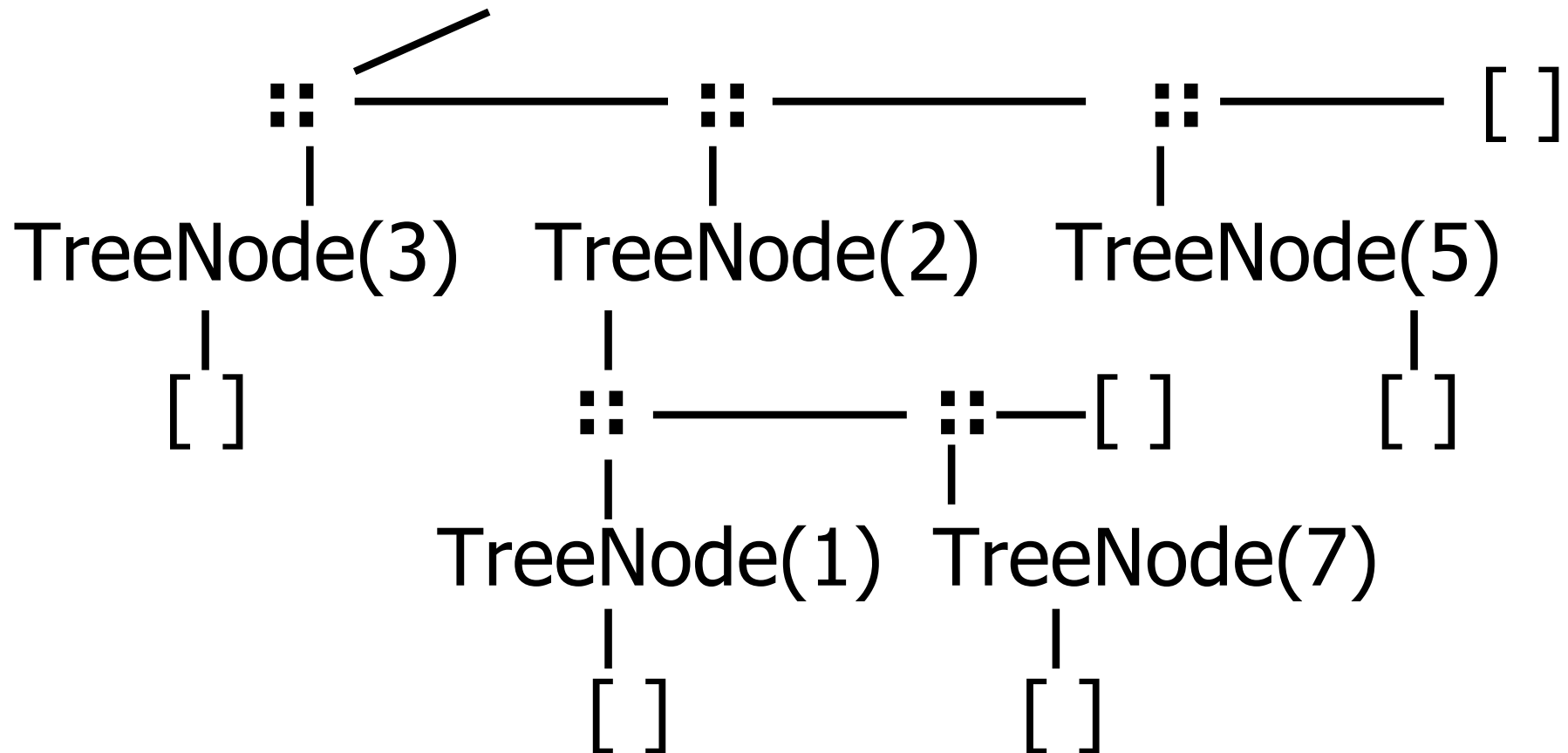


Nested Recursive Type Values

```
val ltree : int labeled_tree =  
  TreeNode  
    (5,  
     [TreeNode (3, []); TreeNode (2,  
      [TreeNode (1, []); TreeNode (7, [])]);  
     TreeNode (5, [])])
```

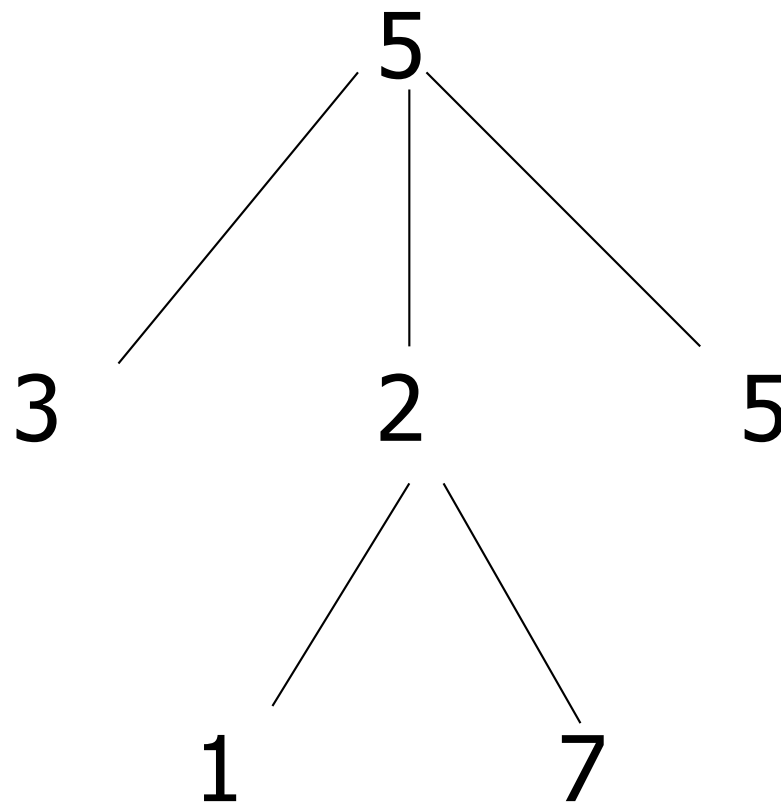
Nested Recursive Type Values

Ltree = TreeNode(5)





Nested Recursive Type Values





Mutually Recursive Functions

```
# let rec flatten_tree labtree =  
  match labtree with TreeNode (x,treelist)  
    -> x::flatten_tree_list treelist  
and flatten_tree_list treelist =  
  match treelist with [] -> []  
  | labtree::labtrees  
    -> flatten_tree labtree  
    @ flatten_tree_list labtrees;;
```



Mutually Recursive Functions

```
val flatten_tree : 'a labeled_tree -> 'a list =  
  <fun>
```

```
val flatten_tree_list : 'a labeled_tree list -> 'a  
  list = <fun>
```

```
# flatten_tree ltree;;
```

```
- : int list = [5; 3; 2; 1; 7; 5]
```

- Nested recursive types lead to mutually recursive functions



Why Data Types?

- Data types play a key role in:
 - *Data abstraction* in the design of programs
 - *Type checking* in the analysis of programs
 - *Compile-time code generation* in the translation and execution of programs
 - Data layout (how many words; which are data and which are pointers) dictated by type



Terminology

- Type: A **type** t defines a set of possible data values
 - E.g. **short** in C is $\{x \mid 2^{15} - 1 \geq x \geq -2^{15}\}$
 - A value in this set is said to have type t
- Type system: rules of a language assigning types to expressions



Types as Specifications

- Types describe properties
- Different type systems describe different properties, eg
 - Data is read-write versus read-only
 - Operation has authority to access data
 - Data came from “right” source
 - Operation might or could not raise an exception
- Common type systems focus on types describing same data layout and access methods



Sound Type System

- If an expression is assigned type t , and it evaluates to a value v , then v is in the set of values defined by t
- SML, OCAML, Scheme and Ada have sound type systems
- Most implementations of C and C++ do not



Strongly Typed Language

- When no application of an operator to arguments can lead to a run-time type error, language is *strongly typed*
 - Eg: `1 + 2.3;;`
- Depends on definition of “type error”



Strongly Typed Language

- C++ claimed to be “strongly typed”, but
 - Union types allow creating a value at one type and using it at another
 - Type coercions may cause unexpected (undesirable) effects
 - No array bounds check (in fact, no runtime checks at all)
- SML, OCAML “strongly typed” but still must do dynamic array bounds checks, runtime type case analysis, and other checks



Static vs Dynamic Types

- *Static type*: type assigned to an expression at compile time
- *Dynamic type*: type assigned to a storage location at run time
- *Statically typed language*: static type assigned to every expression at compile time
- *Dynamically typed language*: type of an expression determined at run time



Type Checking

- When is $op(arg1, \dots, argn)$ allowed?
- *Type checking* assures that operations are applied to the right number of arguments of the right types
 - Right type may mean same type as was specified, or may mean that there is a predefined implicit coercion that will be applied
- Used to resolve overloaded operations



Type Checking

- Type checking may be done *statically* at compile time or *dynamically* at run time
- Dynamically typed (aka untyped) languages (eg LISP, Prolog) do only dynamic type checking
- Statically typed languages can do most type checking statically



Dynamic Type Checking

- Performed at run-time before each operation is applied
- Types of variables and operations left unspecified until run-time
 - Same variable may be used at different types



Dynamic Type Checking

- Data object must contain type information
- Errors aren't detected until violating application is executed (maybe years after the code was written)



Static Type Checking

- Performed after parsing, before code generation
- Type of every variable and signature of every operator must be known at compile time



Static Type Checking

- Can eliminate need to store type information in data object if no dynamic type checking is needed
- Catches many programming errors at earliest point
- Can't check types that depend on dynamically computed values
 - Eg: array bounds



Static Type Checking

- Typically places restrictions on languages
 - Garbage collection
 - References instead of pointers
 - All variables initialized when created
 - Variable only used at one type
 - Union types allow for work-arounds, but effectively introduce dynamic type checks