Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Structural Recursion

- Functions on recursive datatypes (eg lists) tend to be recursive
- Recursion over recursive datatypes generally by structural recursion
  - Recursive calls made to components of structure of the same recursive type
  - Base cases of recursive types stop the recursion of the function

Functions Over Lists

```
# let rec double_up list =
    match list
    with [ ] -> [ ]  (* pattern before ->,
                        expression after *)
      | (x :: xs) -> (x :: x :: double_up xs);;
val double_up : 'a list -> 'a list = <fun>
# let fib5_2 = double_up fib5;;
val fib5_2 : int list = [8; 8; 5; 5; 3; 3; 2; 2; 1;
  1; 1; 1]
```


Functions Over Lists

```
# let silly = double_up ["hi"; "there"];;
val silly : string list = ["hi"; "hi"; "there"; "there"]
# let rec poor_rev list =
  match list
  with [] -> []
    | (x::xs) -> poor_rev xs @ [x];;
val poor_rev : 'a list -> 'a list = <fun>
# poor_rev silly;;
- : string list = ["there"; "there"; "hi"; "hi"]
```


Your turn: doubleList : int list -> int list

- Write a function that takes a list of int and returns a list of the same length, where each element has been multiplied by 2

```
let rec doubleList list =
  match list with [] -> []
  | (first :: rest) -> (2 * first) :: (doubleList rest)
```


Your turn: doubleList : int list -> int list

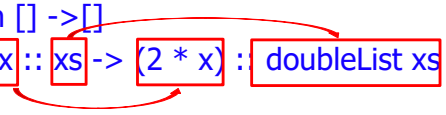
- Write a function that takes a list of int and returns a list of the same length, where each element has been multiplied by 2

```
let rec doubleList list =
  match list
    with [] ->[]
      |  x :: xs -> (2 * x) :: doubleList xs
```

Your turn: doubleList : int list -> int list

- Write a function that takes a list of int and returns a list of the same length, where each element has been multiplied by 2

```
let rec doubleList list =
  match list
    with [] ->[]
     | x :: xs -> (2 * x) :: doubleList xs
```



Same Length

- How can we efficiently answer if two lists have the same length?

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```
let rec same_length list1 list2 =
    match list1 with [] ->



    | (x::xs) ->
```


Same Length

- How can we efficiently answer if two lists have the same length?

```
let rec same_length list1 list2 =
    match list1 with [] ->
      (match list2 with [] -> true
        | (y::ys) -> false)
    | (x::xs) ->
```


Same Length

- How can we efficiently answer if two lists have the same length?

```
let rec same_length list1 list2 =
    match list1 with [] ->
      (match list2 with [] ->
        | (y::ys) ->       )
    | (x::xs) ->
```


Same Length

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```
let rec same_length list1 list2 =
    match list1 with [] ->
      (match list2 with [] -> true
        | (y::ys) -> false)
    | (x::xs) ->
      (match list2 with [] ->
        | (y::ys) ->                    )
```

Same Length

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```
let rec same_length list1 list2 =
    match list1 with [] ->
      (match list2 with [] -> true
        | (y::ys) -> false)
    | (x::xs) ->
      (match list2 with [] -> false
        | (y::ys) ->                    )
```


Same Length

- How can we efficiently answer if two lists have the same length?

```
let rec same_length list1 list2 =
    match list1 with [] ->
      (match list2 with [] -> true
        | (y::ys) -> false)
    | (x::xs) ->
      (match list2 with [] -> false
        | (y::ys) -> same_length xs ys)
```


Folding Recursion

- Another common form "folds" an operation over the elements of the structure

```
# let rec multList list = match list
  with [ ] -> 1
  | x::xs -> x * multList xs;;
val multList : int list -> int = <fun>
# multList [2;4;6];;
- : int = 48
```

- Computes (2 * (4 * (6 * 1)))

Folding Recursion : Length Example

```
# let rec length list = match list
  with [ ] -> 0   (* Nil case *)
  | a :: bs -> 1 + length bs;;  (* Cons case *)
val length : 'a list -> int = <fun>
# length [5; 4; 3; 2];;
- : int = 4
```

- Nil case [ ] is base case, 0 is the base value
- Cons case recurses on component list bs
- What do multList and length have in common?

Forward Recursion

- In Structural Recursion, split input into components and (eventually) recurse
- Forward Recursion form of Structural Recursion
- In forward recursion, **first** call the function recursively on all recursive components, and then build final result from partial results
- Wait until whole structure has been traversed to start building answer

Forward Recursion: Examples

```
# let rec double_up list =
    match list
    with [ ] -> [ ]
      | (x :: xs) -> (x :: x :: double_up xs);;
val double_up : 'a list -> 'a list = <fun>

# let rec poor_rev list =
  match list
  with [] -> []
    | (x::xs) -> let r = poor_rev xs in r @ [x];;
val poor_rev : 'a list -> 'a list = <fun>
```

Forward Recursion: Examples

```
# let rec double_up list =
    match list
    with [ ] -> [ ]
      | (x :: xs) -> (x :: x :: double_up xs);;
val double_up : 'a list -> 'a list = <fun>
```

Base Case | Operator | Recursive Call

```
# let rec poor_rev list =
  match list
  with [] -> []
    | (x::xs) -> let r = poor_rev xs in r @ [x];;
val poor_rev : 'a list -> 'a list = <fun>
```

Base Case | Operator | Recursive Call

Recursing over lists

The Primitive Recursion Fairy

```
# let rec fold_right f list b =
  match list
  with [] -> b
  | (x :: xs) -> f x (fold_right f xs b);;
val fold_right : ('a -> 'b -> 'b) -> 'a list -> 'b -> 'b =
  <fun>
# fold_right
    (fun s -> fun () -> print_string s)
    ["hi"; "there"]
    ();;
therehi- : unit = ()
```


Folding Recursion : Length Example

```
# let rec length list = match list
  with [ ] -> 0   (* Nil case *)
  | a :: bs -> 1 + length bs;;  (* Cons case *)
val length : 'a list -> int = <fun>
# let length list =
fold_right (fun a -> fun r -> 1 + r) list 0;;
val length : 'a list -> int = <fun>
# length [5; 4; 3; 2];;
- : int = 4
```


Forward Recursion: Examples

```
# let rec double_up list =
    match list
    with [ ] -> [ ]
      | (x :: xs) -> (x :: x :: double_up xs);;
val double_up : 'a list -> 'a list = <fun>
```

Base Case | Operator | Recursive Call

```
# let double_up =
    fold_right (fun x -> fun r -> x :: x :: r) list [ ]
```

Operator | Recursive result | Base Case

```
# double_up  ["a";"b"];;
- : string list = ["a"; "a"; "b"; "b"]
```


- let rec multList_fr list =
  match list
    with [] -> 1
    | (x::xs) -> let r = (multList_fr xs) in
                 (x * r)

Folding Recursion

- multList folds to the right
- Same as:

```
# let multList list =
    List.fold_right
    (fun x -> fun p -> x * p)
    list 1;;
val multList : int list -> int = <fun>
# multList [2;4;6];;
- : int = 48
```

Terminology

- Available: A function call that can be executed by the current expression
- The fastest way to be unavailable is to be guarded by an abstraction (anonymous function, lambda lifted).
  - if (h x) then f x else (x + g x)
  - if (h x) then (fun x -> f x) else (g (x + x))

Not available

Terminology

- Tail Position: A subexpression s of expressions e, which is available and such that if evaluated, will be taken as the value of e (last thing done in this expression)
  - if (x>3) then x + 2 else x - 4
  - let x = 5 in x + 4
- Tail Call: A function call that occurs in tail position
  - if (h x) then f x else (x + g x)

Tail Recursion

- A recursive program is tail recursive if all recursive calls are tail calls
- Tail recursive programs may be optimized to be implemented as loops, thus removing the function call overhead for the recursive calls
- Tail recursion generally requires extra "accumulator" arguments to pass partial results
  - May require an auxiliary function

Tail Recursion - length

- How can we write length with tail recursion?

```
let length list =
   let rec length_aux list acc_length =
         match list
          with [ ] -> acc_length
            | (x::xs) ->
              length_aux xs (1 + acc_length)
    in length_aux list  0
```


Your turn: num_neg – tail recursive

```
# let num_neg list =
```


Your turn: num_neg – tail recursive

```
# let num_neg list =
 let rec num_neg_aux list curr_neg =




  in num_neg_aux  ?  ?
```

Your turn: num_neg – tail recursive

```
# let num_neg list =
 let rec num_neg_aux list curr_neg =
    match list with [] ->
      | (x :: xs) ->



  in num_neg_aux  ?  ?
```


Your turn: num_neg – tail recursive

```
# let num_neg list =
 let rec num_neg_aux list curr_neg =
    match list with [] -> curr_neg
      | (x :: xs) ->



  in num_neg_aux  ?  ?
```


Your turn: num_neg – tail recursive

```
# let num_neg list =
 let rec num_neg_aux list curr_neg =
    match list with [] -> curr_neg
      | (x :: xs) ->
        num_neg_aux xs  ?


  in num_neg_aux  ?  ?
```


Your turn: num_neg – tail recursive

```
# let num_neg list =
 let rec num_neg_aux list curr_neg =
    match list with [] -> curr_neg
      | (x :: xs) ->
        num_neg_aux xs
          (if x < 0 then 1 + curr_neg
           else curr_neg)
  in num_neg_aux  ?  ?
```


Your turn: num_neg – tail recursive

```
# let num_neg list =
 let rec num_neg_aux list curr_neg =
    match list with [] -> curr_neg
      | (x :: xs) ->
        num_neg_aux xs
          (if x < 0 then 1 + curr_neg
           else curr_neg)
  in num_neg_aux list  ?
```


Your turn: num_neg – tail recursive

```
# let num_neg list =
 let rec num_neg_aux list curr_neg =
    match list with [] -> curr_neg
      | (x :: xs) ->
        num_neg_aux xs
          (if x < 0 then 1 + curr_neg
           else curr_neg)
  in num_neg_aux list 0
```

```
let num_neg list =
List.fold_left
  (fun curr_neg -> (fun x ->
     (if x < 0 then 1 + curr_neg else curr_neg)
                          )
  )
0
list
```


Folding

```
# let rec fold_left f a list = match list
  with [] -> a | (x :: xs) -> fold_left f (f a x) xs;;
val fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a =
  <fun>
```

fold_left f a [x_1; x_2;…;x_n] = f(…(f (f a x_1) x_2)…)x_n

```
# let rec fold_right f list b = match list
  with [ ] -> b | (x :: xs) -> f x (fold_right f xs b);;
val fold_right : ('a -> 'b -> 'b) -> 'a list -> 'b -> 'b =
  <fun>
```

fold_right f [x_1; x_2;…;x_n] b = f x_1(f x_2 (…(f x_n b)…))

Folding

- Can replace recursion by fold_right in any forward primitive recursive definition
  - Primitive recursive means it only recurses on immediate subcomponents of recursive data structure
- Can replace recursion by fold_left in any tail primitive recursive definition

Mapping Recursion

```
# let rec map f list =
  match list
  with [] -> []
  | (h::t) -> (f h) :: (map f t);;
val map : ('a -> 'b) -> 'a list -> 'b list = <fun>
# map plus_two fib5;;
- : int list = [10; 7; 5; 4; 3; 3]
# map (fun x -> x - 1) fib6;;
: int list = [12; 7; 4; 2; 1; 0; 0]
```


Map is forward recursive

```
# let rec map f list =
  match list
  with [] -> []
  | (h::t) -> (f h) :: (map f t);;
val map : ('a -> 'b) -> 'a list -> 'b list = <fun>
# let map f list =
    List.fold_right (fun h -> fun r -> (f h) :: r)
                    list  [];;
val map : ('a -> 'b) -> 'a list -> 'b list = <fun>
```


Mapping Recursion

- Can use the higher-order recursive map function instead of direct recursion

```
# let doubleList list =
    List.map (fun x -> 2 * x) list;;
val doubleList : int list -> int list = <fun>
# doubleList [2;3;4];;
- : int list = [4; 6; 8]
```

Mapping Recursion

- Can use the higher-order recursive map function instead of direct recursion

```
# let doubleList list =
    List.map (fun x -> 2 * x) list;;
val doubleList : int list -> int list = <fun>
# doubleList [2;3;4];;
- : int list = [4; 6; 8]
```

- Same function, but no explicit recursion