Programming Languages and Compilers (CS 421)

Sasa Misailovic

4110 SC, UIUC

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Based on slides by Elsa Gunter, which are based in part on previous slides by Mattox Beckman and updated by Vikram Adve and Gul Agha

Moving On With Richer Expressions

- How do we extend the grammar to support nested additions, e.g., 1 * (0 + 1)

```
<exp>      ::= <factor>
             |  <factor> +  <exp>

<factor>   ::=  <bin>
             |   <bin>  *  <factor>

<bin>      ::=  0  | 1
```

Moving On With Richer Expressions

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```
<exp>      ::= <factor>
             |  <factor> +  <exp>

<factor>   ::=  <bin>
             |   <bin>  *  <factor>

<bin>      ::=  0  | 1 | ( <exp> )
```

Moving On With Richer Expressions

- How do we extend the grammar to support other operations, subtraction and division?

```
<exp>      ::= <factor>
             | <factor> + <exp> | <factor> - <exp>

<factor>   ::= <bin>
             | <bin> * <factor> | <bin> / <factor>

<bin>      ::= 0 | 1 | ( <exp> )
```

Disambiguating a Grammar

- <exp>::= 0|1| b<exp> | <exp>a
  | <exp>m<exp>
- Want ***a*** to have <u>higher precedence</u> than ***b***, which in turn has <u>higher precedence</u> than ***m***, and such that ***m*** <u>associates to the left</u>.
- Think of a,b,m as operators
  e.g., a is "++", b is "!" , m is "*"

Disambiguating a Grammar

- <exp>::= 0|1| b<exp> | <exp>a
  | <exp>m<exp>
- Want ***a*** to have <u>higher precedence</u> than ***b***, which in turn has <u>higher precedence</u> than ***m***, and such that ***m*** <u>associates to the left</u>.

- <exp> ::= <exp> m <not_m> | <not_m>
- <not_m> ::= b <not_m> | <not_b_m>
- <not_b_m> ::= <not_b_m>a | 0 | 1

Disambiguating a Grammar – Take 2

- <exp>::= 0|1| b<exp> | <exp>a
  | <exp>m<exp>
- Want ***b*** to have higher precedence than ***m***, which in turn has higher precedence than ***a***, and such that ***m*** associates to the right.

Disambiguating a Grammar – Take 2

- <exp>::= 0|1| b<exp> | <exp>a
  | <exp>m<exp>
- Want ***b*** to have higher precedence than ***m***, which in turn has higher precedence than ***a***, and such that ***m*** associates to the right.
- <exp> ::=
  <no_a_m> | <no_m> m <no_a>| <exp> a
- <no_a> ::= <no_a_m> | <no_a_m> m <no_a>
- <no_m> ::= <no_a_m> | <exp> a
- <no_a_m> ::= b <no_a_m> | 0 | 1

Disambiguating a Grammar – Take 3

- <exp>::= 0|1| b<exp> | <exp>a
  | <exp>m<exp>
- Want ***a*** has higher precedence than ***m***, which in turn has higher precedence than ***b***, and such that ***m*** associates to the right.
- For you…

Disambiguating Grammars – Dangling Else

- stmt ::= ...
  | **if** (expr) stmt
  | **if** (expr) stmt **else** stmt

- How can we parse
  if (e1) if (e2) s1 else s2 ?

Disambiguating Grammars – Dangling Else

- Try: let us try to differentiate if we have **if** inside the **then** branch or not….
- stmt = open_stmt | closed_stmt
- open_stmt ::= **if** (expr) stmt
  | **if** (expr) closed_stmt **else** open_stmt
- closed_stmt ::= non_if_statement
  | **if** (expr) closed_stmt **else** closed_stmt

- How can we parse **if** (e1) **if** (e2) s1 **else** s2 now ?

Programming Languages & Compilers

Three Main Topics of the Course

I

New Programming Paradigm

II

Language Translation

III

Language Semantics

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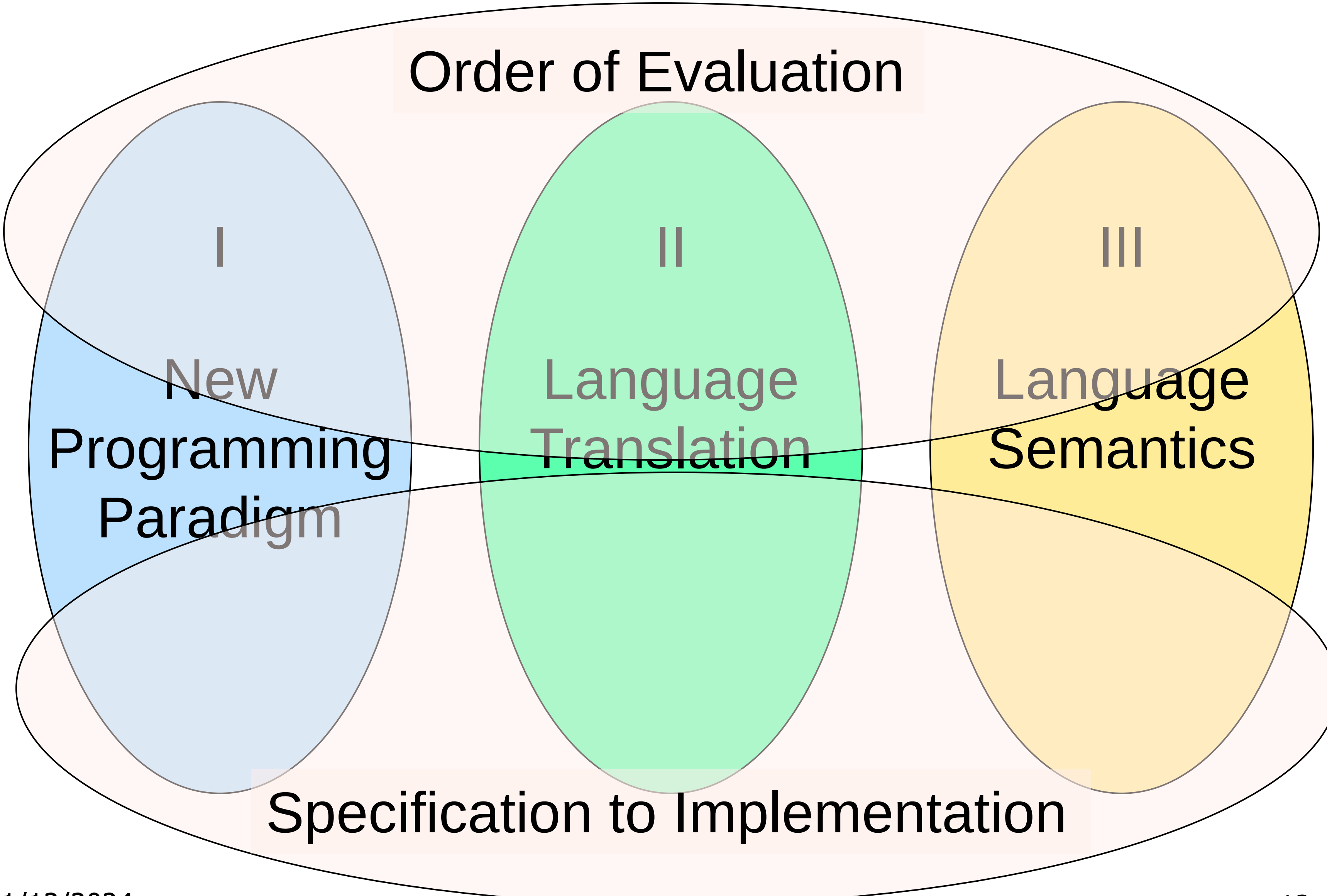
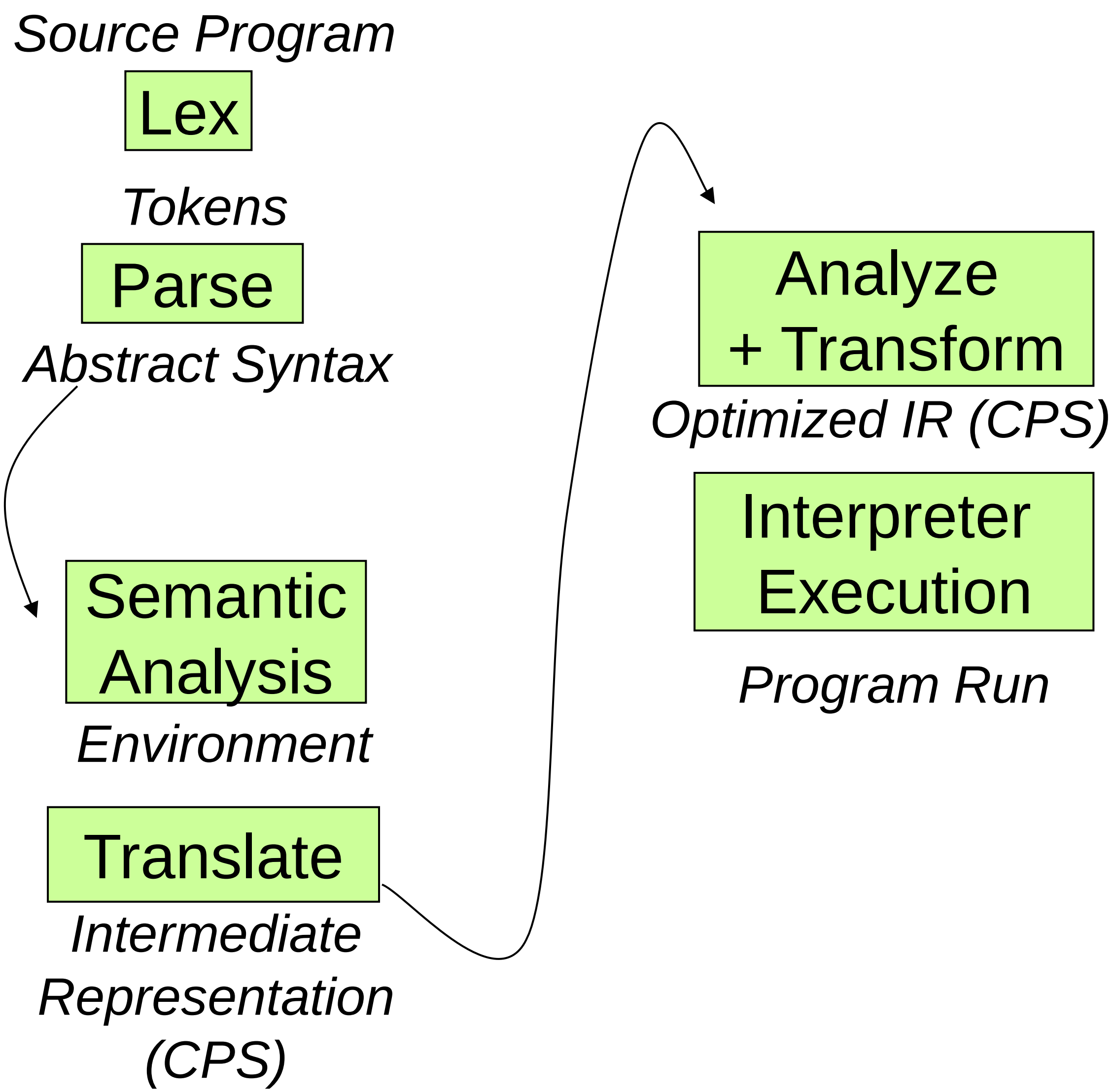


Major Phases of a PicoML Interpreter



Where do we stand?

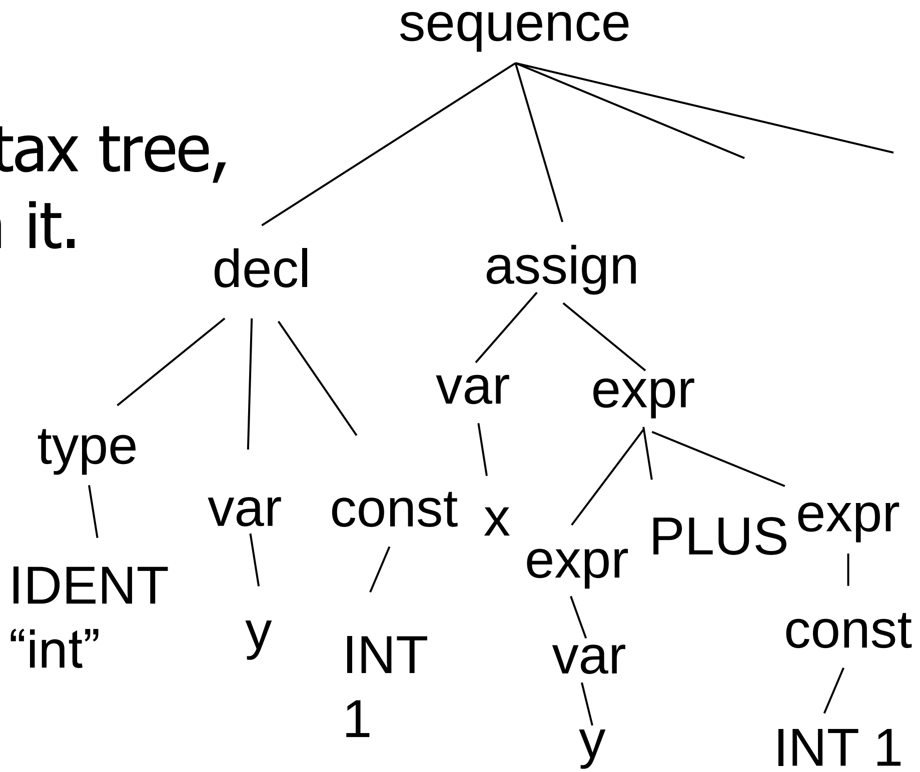
- Conceptually:

Input: “int y = 1; x = y + 1”

Lexer: [IDENT “int”, IDENT “main”, LPAREN, RPAREN, LCURLY, RCURLY]

Parser: turns this list into a syntax tree, so we can more easily work with it.

Typechecker: makes sure all variables are properly typed by traversing the syntax tree



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III : Language Semantics

Operational Semantics

Lambda Calculus

Axiomatic Semantics

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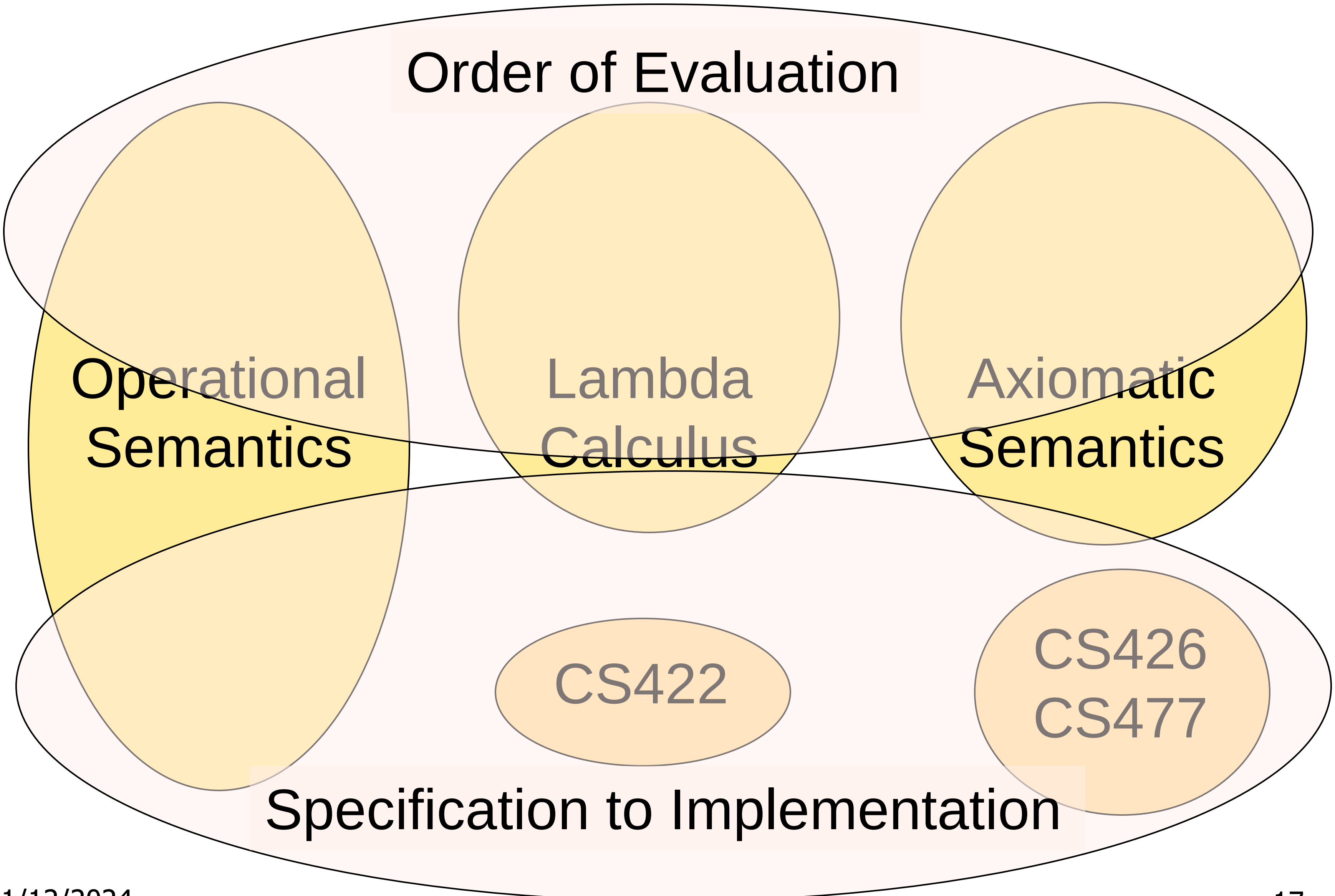


Semantics

- Expresses the meaning of syntax
- Static semantics
  - Meaning based only on the form of the expression without executing it
  - Usually restricted to type checking / type inference

Dynamic semantics

- Method of describing meaning of executing a program
- Several different types:
  - Operational Semantics
  - Axiomatic Semantics
  - Denotational Semantics

Dynamic Semantics

- Different languages better suited to different types of semantics
- Different types of semantics serve different purposes

Operational Semantics

- Start with a simple notion of machine
- Describe how to execute (implement) programs of language on virtual machine, by describing how to execute each program statement (ie, following the *structure* of the program)
- Meaning of program is how its execution changes the state of the machine
- Useful as basis for implementations

Axiomatic Semantics

- Also called Floyd-Hoare Logic
- Based on formal logic (first order predicate calculus)
- Axiomatic Semantics is a logical system built from *axioms* and *inference rules*
- Mainly suited to simple imperative programming languages

Axiomatic Semantics

- Used to formally prove a property (*post-condition*) of the *state* (the values of the program variables) after the execution of program, assuming another property (*pre-condition*) of the state before execution

- Written :

  {Precondition} Program {Postcondition}

- Source of idea of *loop invariant*

Denotational Semantics

- Construct a function $\mathcal{M}$ assigning a mathematical meaning to each program construct
- Lambda calculus (or its variants) often used as the range of the meaning function
- Meaning function is compositional: meaning of construct built from meaning of parts
- Useful for proving properties of programs

Natural Semantics

- Aka Structural Operational Semantics, aka "Big Step Semantics"
- Provide value for a program by rules and derivations, similar to type derivations
- Rule conclusions look like

$$(C, m) \Downarrow m'$$

or

$$(E, m) \Downarrow v$$

Simple Imperative Programming Language

- $I \in Identifiers$
- $N \in Numerals$
- B ::= true | false | B & B | B or B | not B | $E < E$ | $E = E$
- $E ::= N \mid I \mid E + E \mid E * E \mid E - E \mid - E \mid (E)$
- C::= skip | C;C | I := E | if B then C else C fi | while B do C od

Natural Semantics of Atomic Expressions

- Identifiers: $(I,m) \Downarrow m(I)$
- Numerals are values: $(N,m) \Downarrow N$

- Booleans: $(\mathrm{true},m) \Downarrow \mathrm{true}$

  $(\mathrm{false}\ ,m) \Downarrow \mathrm{false}$

Booleans Warmup: Negation

$$\frac{(B,\ m) \Downarrow \text{true}}{(\text{not } B,\ m) \Downarrow \text{false}} \qquad \frac{(B,\ m) \Downarrow \text{false}}{(\text{not } B,\ m) \Downarrow \text{true}}$$

Booleans: Conjunction(&) and Disjunction(or)

$$\frac{(B, m) \Downarrow \text{false}}{(B \,\&\, B', m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{true} \quad (B', m) \Downarrow b}{(B \,\&\, B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(B \text{ or } B', m) \Downarrow \text{true}} \qquad \frac{(B, m) \Downarrow \text{false} \quad (B', m) \Downarrow b}{(B \text{ or } B', m) \Downarrow b}$$

Relations Warmup

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad b = (U = V)}{(E = E', m) \Downarrow b}$$

- By $U \sim V = b$, we mean does (the meaning of) the relation $\sim$ hold on the meaning of U and V
- May be specified by a mathematical expression/equation or rules matching U and V

Relations: General Case (~ is any relational operator)

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad (U \sim V) = b}{(E \sim E', m) \Downarrow b}$$

- By $U \sim V = b$, we mean does (the meaning of) the relation ~ hold on the meaning of U and V
- May be specified by a mathematical expression/equation or rules matching U and V

Arithmetic Expressions

$$\frac{(E, m) \Downarrow U \qquad (E', m) \Downarrow V \quad U \; op \; V = N}{(E \; op \; E', m) \Downarrow N}$$

where N is the specified value for $U \; op \; V$

Commands

Skip: $(\text{skip}, m) \Downarrow m$

Assignment:
$$\frac{(E,m) \Downarrow V}{(I{:=}E,m) \Downarrow m\,[I \text{ <-- } V\,]} \quad (=\{\mathrm{I} \text{ -> } \mathrm{V}\}+\mathrm{m})$$

Sequencing:
$$\frac{(C,m) \Downarrow m' \qquad (C',m') \Downarrow m''}{(C;C', m) \Downarrow m''}$$

If Then Else Command

$$\frac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

$$\frac{(B,m) \Downarrow \text{false} \quad (C',m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

While Command

$$\frac{(B,m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m}$$

$$\frac{(B,m)\Downarrow\text{true} \quad (C,m)\Downarrow m' \quad (\text{while } B \text{ do } C \text{ od}, m')\Downarrow m''}{(\text{while } B \text{ do C od}, m) \Downarrow m''}$$

Example: If Then Else Rule

$$\frac{}{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi}, \{x \text{ -> } 7\}) \Downarrow \, ?}$$

Example: If Then Else Rule

$$\frac{\overline{(x > 5, \{x \text{ -> } 7\}) \Downarrow ?}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \{x \text{ -> } 7\}) \Downarrow\ ?}$$

Example: Arith Relation

$$\dfrac{\dfrac{\begin{array}{c} ? > ? = ? \\ (x,\{x\text{->}7\})\Downarrow ? \quad (5,\{x\text{->}7\})\Downarrow ? \end{array}}{(x > 5, \{x \text{ -> } 7\})\Downarrow ?}}{\begin{array}{c}(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \\ \{x \text{ -> } 7\}) \Downarrow ?\end{array}}$$

Example: Identifier(s)

$$\cfrac{\cfrac{7 > 5 = \text{true} \quad \cfrac{(x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow ?}}{}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \{x \text{ -> } 7\}) \Downarrow ?}$$

Example: Arith Relation

$$\cfrac{\cfrac{7 > 5 = \text{true} \quad (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow \text{true}}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\ \{x \text{ -> } 7\}) \Downarrow\ ?}$$

Example: If Then Else Rule

$$\frac{\dfrac{7 > 5 = \text{true} \qquad (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow \text{true}} \qquad \dfrac{}{(y{:=}\ 2 + 3,\ \{x\text{-> }7\}) \Downarrow\ ?}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\ \{x \text{ -> } 7\}) \Downarrow\ ?}$$

Example: Assignment

$$\frac{\dfrac{7 > 5 = \text{true} \qquad (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5, \{x \text{ -> } 7\})\Downarrow \text{true}} \qquad \dfrac{\dfrac{}{(2+3, \{x\text{->}7\})\Downarrow ?}}{(y:= 2 + 3, \{x\text{-> } 7\}) \Downarrow ?}}{(\text{if } x > 5 \text{ then } y:= 2 + 3 \text{ else } y:=3 + 4 \text{ fi}, \{x \text{ -> } 7\}) \Downarrow ?}$$

Example: Arith Op

$$
\dfrac{
\dfrac{7 > 5 = \text{true} \qquad (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow \text{true}}
\qquad
\dfrac{\dfrac{? + ? = ? \qquad (2,\{x\text{->}7\})\Downarrow ? \quad (3,\{x\text{->}7\})\Downarrow ?}{(2+3,\ \{x\text{->}7\})\Downarrow ?}}{(y{:=}\ 2 + 3,\ \{x\text{->}\ 7\}) \Downarrow ?}
}{
(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\ \{x \text{ -> } 7\}) \Downarrow ?
}
$$

Example: Numerals

$$
\dfrac{
\dfrac{7 > 5 = \text{true} \qquad (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow \text{true}}
\qquad
\dfrac{\dfrac{2 + 3 = 5 \qquad (2,\{x\text{->}7\})\Downarrow 2 \quad (3,\{x\text{->}7\})\ \Downarrow 3}{(2+3,\ \{x\text{->}7\})\Downarrow ?}}{(y{:=}\ 2 + 3,\ \{x\text{-> } 7\})\ \Downarrow ?}
}{
(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\ \{x \text{ -> } 7\})\ \Downarrow ?
}
$$

Example: Arith Op

$$
\dfrac{
\dfrac{
\dfrac{(x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow \text{true}}\ \ 7 > 5 = \text{true}
}{}
\qquad
\dfrac{
\dfrac{
\dfrac{(2,\{x\text{->}7\})\Downarrow 2 \quad (3,\{x\text{->}7\})\Downarrow 3}{(2+3,\ \{x\text{->}7\})\Downarrow 5}\ \ 2 + 3 = 5
}{(y{:=}\ 2 + 3,\ \{x\text{-> } 7\})}
}{\Downarrow ?}
}{
\begin{array}{c}(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\\ \{x \text{ -> } 7\}) \Downarrow\ ?\end{array}
}
$$

Example: Assignment

$$\frac{\dfrac{\begin{array}{c}7 > 5 = \text{true}\\ (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5\end{array}}{(x > 5, \{x \text{->} 7\})\Downarrow \text{true}} \qquad \dfrac{\dfrac{\begin{array}{c}2 + 3 = 5\\ (2,\{x\text{->}7\})\Downarrow 2 \quad (3,\{x\text{->}7\})\Downarrow 3\end{array}}{(2+3, \{x\text{->}7\})\Downarrow 5}}{\begin{array}{c}(y{:=}\ 2 + 3, \{x\text{->}\ 7\}\\ \Downarrow \{x\text{->}7, y\text{->}5\}\end{array}}}{\begin{array}{c}(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\\ \{x \text{->}\ 7\})\Downarrow ?\end{array}}$$

Example: If Then Else Rule

$$
\dfrac{
\dfrac{\begin{array}{c}7 > 5 = \text{true}\\ \text{(x,\{x->7\})}\Downarrow\text{7} \quad \text{(5,\{x->7\})}\Downarrow\text{5}\end{array}}{\text{(x > 5, \{x -> 7\})}\Downarrow\text{true}}
\qquad
\dfrac{\dfrac{\begin{array}{c}2 + 3 = 5\\ \text{(2,\{x->7\})}\Downarrow\text{2} \quad \text{(3,\{x->7\})}\Downarrow\text{3}\end{array}}{\text{(2+3, \{x->7\})}\Downarrow\text{5}}}{\begin{array}{c}\text{(y:= 2 + 3, \{x-> 7\}}\\ \Downarrow \text{\{x->7, y->5\}}\end{array}}
}{
\begin{array}{c}\text{(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,}\\ \text{\{x -> 7\})} \Downarrow \text{\{x->7, y->5\}}\end{array}
}
$$

Comment

- Simple Imperative Programming Language introduces variables *implicitly* through assignment
- The let-in command introduces scoped variables *explictly*
- Clash of constructs apparent in awkward semantics

Interpretation Versus Compilation

- A **compiler** from language L1 to language L2 is a program that takes an L1 program and for each piece of code in L1 generates a piece of code in L2 of same meaning
- An **interpreter** of L1 in L2 is an L2 program that executes the meaning of a given L1 program
- Compiler would examine the body of a loop once; an interpreter would examine it every time the loop was executed

Interpreter

- An *Interpreter* represents the operational semantics of a language L1 (source language) in the language of implementation L2 (target language)
- Built incrementally
  - Start with literals
  - Variables
  - Primitive operations
  - Evaluation of expressions
  - Evaluation of commands/declarations

Interpreter

- Takes abstract syntax trees as input
  - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
  - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next "state"
  - To get final value, put in a loop

Natural Semantics Interpreter Implementation

- Identifiers: $(k,m) \Downarrow m(k)$
- Numerals are values: $(N,m) \Downarrow N$
- Conditionals: $\dfrac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$ $\dfrac{(B,m) \Downarrow \text{false} \quad (C',m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$

```
compute_exp (Var(v), m) = look_up v m
compute_exp (Int(n), _) = Num (n)
…
compute_com (IfExp(b,c1,c2), m) =
        if compute_exp (b,m) = Bool(true)
        then compute_com (c1,m)
        else compute_com (c2,m)
```

Natural Semantics Interpreter Implementation

- Loop: $\dfrac{(B, m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m}$ $\dfrac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m' \quad (\text{while } B \text{ do } C \text{ od}, m') \Downarrow m''}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m''}$

```
compute_com (While(b,c), m) =
      if compute_exp (b,m) = Bool(false)
      then m
      else compute_com
              (While(b,c), compute_com(c,m))
```

- May fail to terminate - exceed stack limits
  - Returns no useful information then

Transition Semantics

- Form of operational semantics
- Describes how each program construct transforms machine state by *transitions*
- Rules look like

  (C, *m*) --> (C', *m'*) or (C,*m*) --> *m'*
- *C, C'* is code remaining to be executed
- m, m' represent the state/store/memory/environment
  - Partial mapping from identifiers to values
  - Sometimes *m* (or *C*) not needed
- Indicates exactly one step of computation

Expressions and Values

- *C, C'* used for commands; *E, E' for* expressions; *U,V* for values
- Special class of expressions designated as *values*
  - Eg 2, 3 are values, but 2+3 is only an expression
- Memory only holds values
  - Other possibilities exist

Evaluation Semantics

- Transitions successfully stops when E/*C* is a value/memory
- Evaluation fails if no transition possible, but not at value/memory
- Value/memory is the final *meaning* of original expression/command (in the given state)
- Coarse semantics: final value / memory
- More fine grained: whole transition sequence

Simple Imperative Programming Language

- $I \in$ *Identifiers*
- $N \in$ *Numerals*
- B ::= true | false | B & B | B or B | not B | $E < E$ | $E = E$
- $E ::= N \mid I \mid E + E \mid E * E \mid E - E \mid - E$
- C::= skip | $C;C$ | I ::= E
  | if B then C else C fi | while B do C od

Transitions for Expressions

- Numerals are values

- Boolean values = {true, false}

- Identifiers: (I,m) --> $(m(I), m)$

Boolean Operations:

- Operators: (short-circuit)

(false & *B*, *m*) --> (false,*m*)

(true & *B*, *m*) --> (*B*,*m*)

$$\frac{(B, m) \dashrightarrow (B'', m)}{(B \,\&\, B', m) \dashrightarrow (B'' \,\&\, B', m)}$$

(true or *B*, *m*) --> (true,*m*)

(false or *B*, *m*) --> (*B*,*m*)

$$\frac{(B, m) \dashrightarrow (B'', m)}{(B \text{ or } B', m) \dashrightarrow (B'' \text{ or } B', m)}$$

(not true, m) --> (false,*m*)

(not false, m) --> (true,*m*)

$$\frac{(B, m) \dashrightarrow (B', m)}{(\text{not } B, m) \dashrightarrow (\text{not } B', m)}$$

Relations

$$\frac{(E, m) \dashrightarrow (E'', m)}{(E \sim E', m) \dashrightarrow (E'' \sim E', m)}$$

$$\frac{(E, m) \dashrightarrow (E', m)}{(V \sim E, m) \dashrightarrow (V \sim E', m)}$$

($U \sim V$, m) --> (true,m) or (false,m) depending on whether $U \sim V$ holds or not

Arithmetic Expressions

$$\frac{(E, m) \dashrightarrow (E'', m)}{(E \; op \; E', m) \dashrightarrow (E'' \; op \; E', m)}$$

$$\frac{(E, m) \dashrightarrow (E', m)}{(V \; op \; E, m) \dashrightarrow (V \; op \; E', m)}$$

$(U \; op \; V, m) \dashrightarrow (N, m)$ where N is the specified value for $U \; op \; V$

Commands - in English

- skip means done evaluating
- When evaluating an assignment, evaluate the expression first
- If the expression being assigned is already a value, update the memory with the new value for the identifier
- When evaluating a sequence, work on the first command in the sequence first
- If the first command evaluates to a new memory (ie completes), evaluate remainder with new memory

Commands

$$(\text{skip}, m) \dashrightarrow m$$

$$\frac{(E,m) \dashrightarrow (E',m)}{(I::=E,m) \dashrightarrow (I::=E',m)}$$

$$(I::=V,m) \dashrightarrow m[I \leftarrow V]$$

$$\frac{(C,m) \dashrightarrow (C'',m')}{(C;C', m) \dashrightarrow (C'';C',m')} \qquad \frac{(C,m) \dashrightarrow m'}{(C;C', m) \dashrightarrow (C',m')}$$

If Then Else Command - in English

- If the boolean guard in an if_then_else is true, then evaluate the first branch
- If it is false, evaluate the second branch
- If the boolean guard is not a value, then start by evaluating it first.

If Then Else Command

(if true then C else C' fi, m) --> (C, m)

(if false then C else C' fi, m) --> (C', m)

$$\frac{(B,m) \dashrightarrow (B',m)}{\begin{array}{c}(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \\ \dashrightarrow (\text{if } B' \text{ then } C \text{ else } C' \text{ fi}, m)\end{array}}$$

What should while transition to?

(while B do C od, m) → ?

Wrong! BAD

$$\frac{(B, m) \rightarrow (B', m)}{(\text{while } B \text{ do } C \text{ od}, m) \rightarrow (\text{while } B' \text{ do } C \text{ od}, m)}$$

While Command

(while *B* do *C* od, *m*) -->

(if *B* then *C*; while *B* do *C* od else skip fi, m)

In English: Expand a While into a test of the boolean guard, with the true case being to do the body and then try the while loop again, and the false case being to stop.

Example Evaluation

- First step:

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7})
--> ?

Example Evaluation

- First step:

$$\frac{(x > 5, \{x \rightarrow 7\}) \dashrightarrow ?}{\begin{array}{c}(\text{if } x > 5 \text{ then } y{:}= 2 + 3 \text{ else } y{:}=3 + 4 \text{ fi},\\ \{x \rightarrow 7\})\end{array}}$$

--> ?

Example Evaluation

- First step:

$$\frac{\dfrac{(x,\{x \to 7\}) \dashrightarrow (7, \{x \to 7\})}{(x > 5, \{x \to 7\}) \dashrightarrow ?}}{\text{(if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi, } \{x \to 7\}) \dashrightarrow ?}$$

Example Evaluation

- First step:

$$\dfrac{\dfrac{(x,\{x \rightarrow 7\}) \dashrightarrow (7, \{x \rightarrow 7\})}{(x > 5, \{x \rightarrow 7\}) \dashrightarrow (7 > 5, \{x \rightarrow 7\})}}{\begin{array}{c}(\text{if } x > 5 \text{ then } y{:}= 2 + 3 \text{ else } y{:}=3 + 4 \text{ fi},\\ \{x \rightarrow 7\})\\ \dashrightarrow\ ?\end{array}}$$

Example Evaluation

- First step:

$$\dfrac{\dfrac{(x,\{x \rightarrow 7\}) \longrightarrow (7, \{x \rightarrow 7\})}{(x > 5, \{x \rightarrow 7\}) \longrightarrow (7 > 5, \{x \rightarrow 7\})}}{\begin{array}{c}(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \\ \{x \rightarrow 7\}) \\ \longrightarrow (\text{if } 7 > 5 \text{ then } y{:=}2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \\ \{x \rightarrow 7\})\end{array}}$$

Example Evaluation

- Second Step:

$$\frac{(7 > 5, \{x \to 7\}) \longrightarrow (\text{true}, \{x \to 7\})}{\begin{array}{c}(\text{if } 7 > 5 \text{ then } y{:=}2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\\ \{x \to 7\})\\ \longrightarrow (\text{if true then } y{:=}2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\\ \{x \to 7\})\end{array}}$$

- Third Step:

$$(\text{if true then } y{:=}2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \{x \to 7\})$$

$$\longrightarrow (y{:=}2{+}3, \{x \to 7\})$$

Example Evaluation

- Fourth Step:

$$\frac{(2+3,\ \{x\text{-}>\ 7\})\ \text{-->}\ (5,\ \{x\ \text{-}>\ 7\})}{(y{:=}2+3,\ \{x\text{-}>7\})\ \text{-->}\ (y{:=}5,\ \{x\text{-}>7\})}$$

- Fifth Step:

(y:=5, {x->7}) --> {y -> 5, x -> 7}

Example Evaluation

- Bottom Line:

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi, {x -> 7})

--> (if 7 > 5 then y:=2 + 3 else y:=3 + 4 fi, {x -> 7})

-->(if true then y:=2 + 3 else y:=3 + 4 fi, {x -> 7})

-->(y:=2+3, {x->7})

--> (y:=5, {x->7}) --> {y -> 5, x -> 7}

Transition Semantics Evaluation

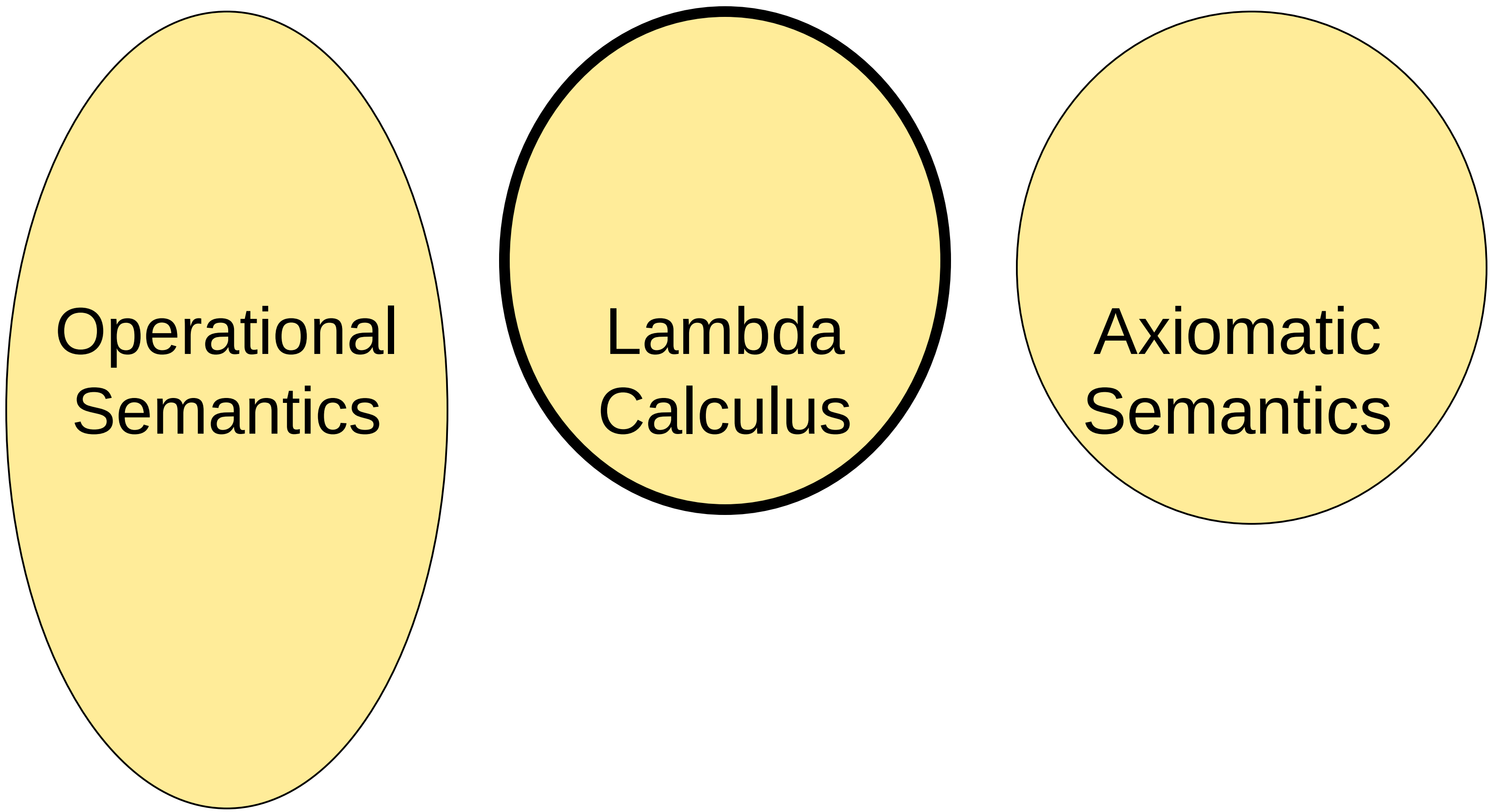
- A sequence of steps with trees of justification for each step

$$(C_1,m_1) \dashrightarrow (C_2,m_2) \dashrightarrow (C_3,m_3) \dashrightarrow \ldots \dashrightarrow m$$

- Let -->* be the transitive closure of -->
- Ie, the smallest transitive relation containing -->

Programming Languages & Compilers

III : Language Semantics



Lambda Calculus - Motivation

- Aim is to capture the essence of functions, function applications, and evaluation
- $\lambda-$calculus is a theory of computation
- "The Lambda Calculus: Its Syntax and Semantics". H. P. Barendregt. North Holland, 1984

Lambda Calculus - Motivation

- All *sequential programs* may be viewed as functions from input (initial state and input values) to output (resulting state and output values).
- λ-calculus is a mathematical formalism of functions and functional computations
- Two flavors: typed and untyped

Untyped λ-Calculus

- Only three kinds of expressions:
  - Variables: x, y, z, w, …
  - Abstraction: λ x. e
    (Function creation, think fun x -> e)
  - Application: $e_1\ e_2$
  - Parenthesized expression: (e)

Untyped λ-Calculus Grammar

- Formal BNF Grammar:
  - <expression> ::= <variable>
    | <abstraction>
    | <application>
    | (<expression>)
  - <abstraction>
    ::= λ<variable>.<expression>
  - <application>
    ::= <expression> <expression>

Untyped λ-Calculus Terminology

- Occurrence: a location of a subterm in a term
- Variable binding: λ x. e is a binding of x in e
- Bound occurrence: all occurrences of x in λ x. e
- Free occurrence: one that is not bound
- Scope of binding: in λ x. e, all occurrences in e not in a subterm of the form λ x. e’ (same x)
- Free variables: all variables having free occurrences in a term

Example

- Label occurrences and scope:

$$(\lambda\ x.\ y\ \lambda\ y.\ y\ (\lambda\ x.\ x\ y)\ x)\ x$$

1 2 3 4 5 6 7 8 9

Example

- Label occurrences and scope:
- $$(\lambda\ \underset{1}{x}.\ \overset{\text{free}}{\underset{2}{y}}\ \lambda\ \underset{3}{y}.\ \underset{4}{y}\ (\lambda\ \underset{5}{x}.\ \underset{6}{x}\ \underset{7}{y})\ \underset{8}{x})\ \overset{\text{free}}{\underset{9}{x}}$$

Untyped λ-Calculus

- How do you compute with the λ-calculus?
- Roughly speaking, by substitution:

- $(\lambda\ x.\ e_1)\ e_2 \Rightarrow^* e_1\,[e_2\,/\,x]$

- * Modulo all kinds of subtleties to avoid free variable capture

Transition Semantics for λ-Calculus

$$\frac{E \dashrightarrow E''}{E\ E' \dashrightarrow E''\ E'}$$

- Application (version 1 - Lazy Evaluation)

$$(\lambda\ x\ .\ E)\ E' \dashrightarrow E[E'/x]$$

- Application (version 2 - Eager Evaluation)

$$\frac{E' \dashrightarrow E''}{(\lambda\ x\ .\ E)\ E' \dashrightarrow (\lambda\ x\ .\ E)\ E''}$$

$$\frac{}{(\lambda\ x\ .\ E)\ V \dashrightarrow E[V/x]}$$

V - variable or abstraction (value)

How Powerful is the Untyped λ-Calculus?

- The untyped λ-calculus is Turing Complete
  - Can express any sequential computation
- Problems:
  - How to express basic data: booleans, integers, etc?
  - How to express recursion?
  - Constants, if_then_else, etc, are conveniences; can be added as syntactic sugar

Typed vs Untyped λ-Calculus

- The *pure* λ-calculus has no notion of type: (f f) is a legal expression
- Types restrict which applications are valid
- Types are not syntactic sugar! They disallow some terms
- Simply typed λ-calculus is less powerful than the untyped λ-Calculus: NOT Turing Complete (no recursion)

α Conversion

1. α-conversion:
   2. λ x. exp --α--> λ y. (exp [y/x])
3. Provided that
   1. y is not free in exp
   2. No free occurrence of x in exp becomes bound in exp when replaced by y

λ x. x (λ y. x y) -×-> λ y. y(λ y.y y)

α Conversion Non-Examples

1. Error: y is not free in term second

$$\lambda\, x.\ x\ y\ \cancel{\text{--}\alpha\text{-->}}\ \lambda\, y.\ y\ y$$

2. Error: free occurrence of x becomes bound in wrong way when replaced by y

$$\lambda\, x.\ \underbrace{\lambda\, y.\ x\ y}_{\text{exp}}\ \cancel{\text{--}\alpha\text{-->}}\ \lambda\, y.\ \underbrace{\lambda\, y.\ y\ y}_{\text{exp}[y/x]}$$

But $\lambda\, x.\ (\lambda\, y.\ y)\ x\ \text{--}\alpha\text{-->}\ \lambda\, y.\ (\lambda\, y.\ y)\ y$

And $\lambda\, y.\ (\lambda\, y.\ y)\ y\ \text{--}\alpha\text{-->}\ \lambda\, x.\ (\lambda\, y.\ y)\ x$

Congruence

- Let ~ be a relation on lambda terms. ~ is a congruence if
- it is an equivalence relation
- If $e_1 \sim e_2$ then
  - $(e\ e_1) \sim (e\ e_2)$ and $(e_1 e) \sim (e_2\ e)$
  - $\lambda\, x.\ e_1 \sim \lambda\, x.\ e_2$

α Equivalence

- α equivalence is the smallest congruence containing α conversion

- One usually treats α-equivalent terms as equal - i.e. use α equivalence classes of terms

Example

Show: λ x. (λ y. y x) x ~α~ λ y. (λ x. x y) y

- λ x. (λ y. y x) x --α--> λ z. (λ y. y z) z so
  λ x. (λ y. y x) x ~α~ λ z. (λ y. y z) z
- (λ y. y z) --α--> (λ x. x z) so

  (λ y. y z) ~α~ (λ x. x z) so

  (λ y. y z) z ~α~ (λ x. x z) z so

  λ z. (λ y. y z) z ~α~ λ z. (λ x. x z) z
- λ z. (λ x. x z) z --α--> λ y. (λ x. x y) y so

  λ z. (λ x. x z) z ~α~ λ y. (λ x. x y) y
- λ x. (λ y. y x) x ~α~ λ y. (λ x. x y) y