

Programming Languages and Compilers (CS 421)

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<https://courses.engr.illinois.edu/cs421/fa2024/CS421C>

Based on slides by Elsa Gunter, which are based in part on previous slides by Mattox Beckman and updated by Vikram Adve and Gul Agha

- Last time we were doing context-free grammars
- Some grammars can have ambiguity (multiple parse trees)
- We will continue with disambiguation on Thursday.
- Today, let's see how we can parse "good" grammars

How do you know you have ambiguity?

- The Ocaml parser generator (ocamlyacc) will report ambiguity in the grammar as “conflicts”:
- ***Shift/reduce:*** Usually caused by lack of associativity or precedence information in grammar
- ***Reduce/reduce:*** can't decide between two different rules to reduce by; Not always clear what the problem is, but often right-hand side of one production is the suffix of another
- We will explain what these conflicts mean next time!

Where do we stand?

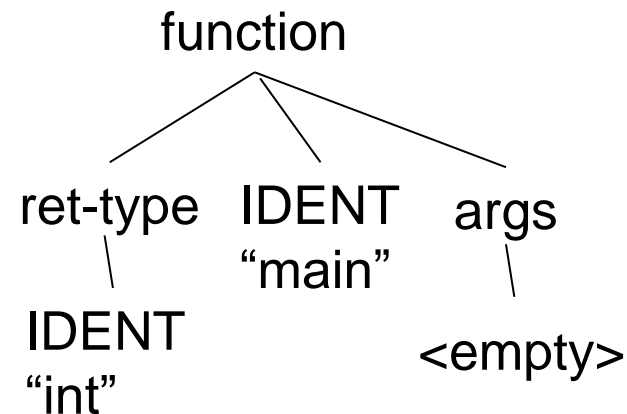
- Conceptually:

Input: "int main () { }"

Lexer*: ["int", "main", "(", ")", "{", "}"]

Lexer: [IDENT "int", IDENT "main", LPAREN, RPAREN, LCURLY, RCURLY]

Parser: turns this list into a tree, so we can more easily work with it.



Parser Code

- Ocaml yacc is a parser generator for Ocaml
 - Similar generators exist for other languages
 - Search under: Yacc, Bison, Menhir...
 - Another family: Antlr
- Input: high level specification (<grammar>.mly file)
- Output: tokens (<grammar>.mli) and generated parser (<grammar>.ml)
 - <grammar>.ml defines a parsing function per entry point
 - Parsing function takes a lexing function (lexer buffer to token) and a lexer buffer as arguments
 - Returns semantic attribute of corresponding entry point

Ocamlyacc Input

- *<grammar>*.mly File format:

%o{

<header>

%o}

<declarations>

%o%

<rules>

%o%

<trailer>

Ocamlyacc <*header*>

- Contains arbitrary Ocaml code
- Typically used to give types and functions needed for the semantic actions of rules and to give specialized error recovery
- May be omitted
- <*trailer*> similar. Possibly used to call parser

Ocamlyacc Input

- *<grammar>.mly* File format:

%{

<header>

%}

<declarations>

%%

<rules>

%%

<trailer>

Ocamlyacc *<declarations>*

- **%token** *symbol ... symbol*
Declare given symbols as tokens
- **%token** *<type> symbol ... symbol*
- Declare given symbols as token constructors, taking an argument of type *<type>*
- **%start** *symbol ... symbol*
Declare given symbols as entry points; functions of same names in *<grammar>.ml*

Ocamlyacc <declarations>

- **%type** <type> symbol ... symbol

Specify type of attributes for given symbols.
Mandatory for start symbols

- **%left** symbol ... symbol
- **%right** symbol ... symbol
- **%nonassoc** symbol ... symbol

Associate precedence and associativity to given symbols. Same line, same precedence; earlier line, lower precedence (broadest scope)

Ocamlyacc Input

- *<grammar>.mly* File format:

%{

<header>

%}

<declarations>

%%

<rules>

%%

<trailer>

Ocamlyacc *<rules>*

- *nonterminal* :

symbol ... symbol { semantic_action }

| ...

| *symbol ... symbol { semantic_action }*

;

- Semantic actions are arbitrary Ocaml expressions
- Must be of same type as declared (or inferred) for *nonterminal*
- Access semantic attributes (values) of symbols by position: \$1 for first symbol, \$2 to second ...

Example - Grammar

A slight variation of what we've seen earlier:

$\text{Expr} ::= \text{Term} \mid \text{Term} + \text{Expr} \mid \text{Term} - \text{Expr}$

$\text{Term} ::= \text{Factor} \mid \text{Factor} * \text{Term} \mid \text{Factor} / \text{Term}$

$\text{Factor} ::= \text{Id} \mid (\text{Expr})$

Example - Base types

```
Expr ::= Term | Term + Expr | Term - Expr
Term  ::= Factor | Factor * Term | Factor / Term
Factor ::= Id | ( Expr )
```

```
(* File: expr.ml *)
type expr =
  | Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
and term =
  | Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
and factor =
  | Id_as_Factor of string
  | Parenthesized_Expr_as_Factor of expr
```

Parser's role

- Parser's role: Converts text from the grammar to a datastructure defined by this base type:

```
Expr ::= Term | Term + Expr | Term - Expr
Term ::= Factor | Factor * Term | Factor / Term
Factor ::= Id | ( Expr )
```

- Example: "X + Y"

Example - Base types

```
(* File: expr.ml *)
type expr =
  Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
and term =
  Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
```

Example - Lexer

```
Expr ::= Term | Term + Expr | Term - Expr
Term  ::= Factor | Factor * Term | Factor / Term
Factor ::= Id | ( Expr )
```

```
{ open Exprparse }
```

```
let numeric = ['0' - '9']
```

```
let letter = ['a' - 'z' 'A' - 'Z']
```

```
rule token = parse
```

```
| "+" {Plus_token}
```

```
| "-" {Minus_token}
```

```
| "*" {Times_token}
```

```
| "/" {Divide_token}
```

```
| "(" {Left_parenthesis}
```

```
| ")" {Right_parenthesis}
```

```
| letter (letter|numeric|"_" )* as id {Id_token id}
```

```
| [' ' '\t' '\n'] {token lexbuf}
```

```
| eof {EOL}
```

← The lexer uses the tokens defined in the parser file (see next slide)

Example - Parser (exprparse.mly)

```
%{  
    open Expr  
%}  
%token <string> Id_token  
%token Left_parenthesis Right_parenthesis  
%token Times_token Divide_token  
%token Plus_token Minus_token  
%token EOL  
  
%start main  
%type <expr> main  
%%
```

Example - Parser (exprparse.mly)

```
Expr ::= Term | Term + Expr | Term - Expr
Term  ::= Factor | Factor * Term | Factor / Term
Factor ::= Id | ( Expr )
```

expr:

```
term
  { Term_as_Expr $1 }
| term Plus_token expr
  { Plus_Expr ($1, $3) }
| term Minus_token expr
  { Minus_Expr ($1, $3) }
```

Example - Base types

```
(* File: expr.ml *)
type expr =
  Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
```

Example - Parser (exprparse.mly)

```
Expr ::= Term | Term + Expr | Term - Expr
Term  ::= Factor | Factor * Term | Factor / Term
Factor ::= Id | ( Expr )
```

expr:

```
term①
  { Term_as_Expr $1 }
```

```
| term① Plus_token expr③
  { Plus_Expr ($1, $3) }
```

```
| term① Minus_token expr③
  { Minus_Expr ($1, $3) }
```

Example - Base types

```
(* File: expr.ml *)
type expr =
  Term_as_Expr of term
| Plus_Expr of (term * expr)
| Minus_Expr of (term * expr)
```

Example - Parser (exprparse.mly)

```
Expr ::= Term | Term + Expr | Term - Expr
Term  ::= Factor | Factor * Term | Factor / Term
Factor ::= Id | ( Expr )
```

term:

factor

{ Factor_as_Term \$1 }

| factor Times_token term

{ Mult_Term (\$1, \$3) }

| factor Divide_token term

{ Div_Term (\$1, \$3) }

Example - Base types

```
(* File: expr.ml *)
type expr =
  Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
and term =
  Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
```

Example - Parser (exprparse.mly)

```
Expr ::= Term | Term + Expr | Term - Expr
Term  ::= Factor | Factor * Term | Factor / Term
Factor ::= Id | ( Expr )
```

factor:

```
  Id_token
    { Id_as_Factor $1 }
| Left_parenthesis expr Right_parenthesis
  { Parenthesized_Expr_as_Factor $2 }
```

main:

```
| expr EOL
  { $1 }
```

Recall, we previously defined:

```
%start main
%type <expr> main
```

Example - Base types

```
(* File: expr.ml *)
type expr =
  Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
and term =
  Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
and factor =
  Id_as_Factor of string
  | Parenthesized_Expr_as_Factor of expr
```

- Call:

- `$ ocaml yacc options exprparse.mly`

- Get:

- Tokens: `exprparse.mli` (can be used in lexer)

- Parser: `exprparse.ml`
(included in the rest of code)

Example - Using Parser

```
# #use "expr.ml";;  
...  
# #use "exprparse.ml";;  
...  
# #use "exprlex.ml";;  
...  
# let test s =  
    let lexbuf = Lexing.from_string (s ^ "\n") in  
    main token lexbuf;;
```

Example - Using Parser

```
# test "a + b";;
```

```
- : expr =
```

```
Plus_Expr
```

```
(Factor_as_Term (Id_as_Factor "a"),
```

```
Term_as_Expr
```

```
(Factor_as_Term (Id_as_Factor "b"))
```

```
)
```

Example - Base types

```
(* File: expr.ml *)
```

```
type expr =
```

```
  Term_as_Expr of term
```

```
  | Plus_Expr of (term * expr)
```

```
  | Minus_Expr of (term * expr)
```

```
and term =
```

```
  Factor_as_Term of factor
```

```
  | Mult_Term of (factor * term)
```

```
  | Div_Term of (factor * term)
```

```
and factor =
```

```
  Id_as_Factor of string
```

```
  | Parenthesized_Expr_as_Factor of expr
```


LR Parsing

General plan:

- Read tokens left to right (L)
- Create a rightmost derivation (R)

How is this possible?

- Start at the bottom (left) and work your way up
- Last step has only one non-terminal to be replaced so is right-most
- Working backwards, replace mixed strings by non-terminals
- Always proceed so that there are no non-terminals to the right of the string to be replaced

Example: $\langle \text{Sum} \rangle ::= 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

(0 + 1) + 0



Example: $\langle \text{Sum} \rangle ::= 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

(0 + 1) + 0

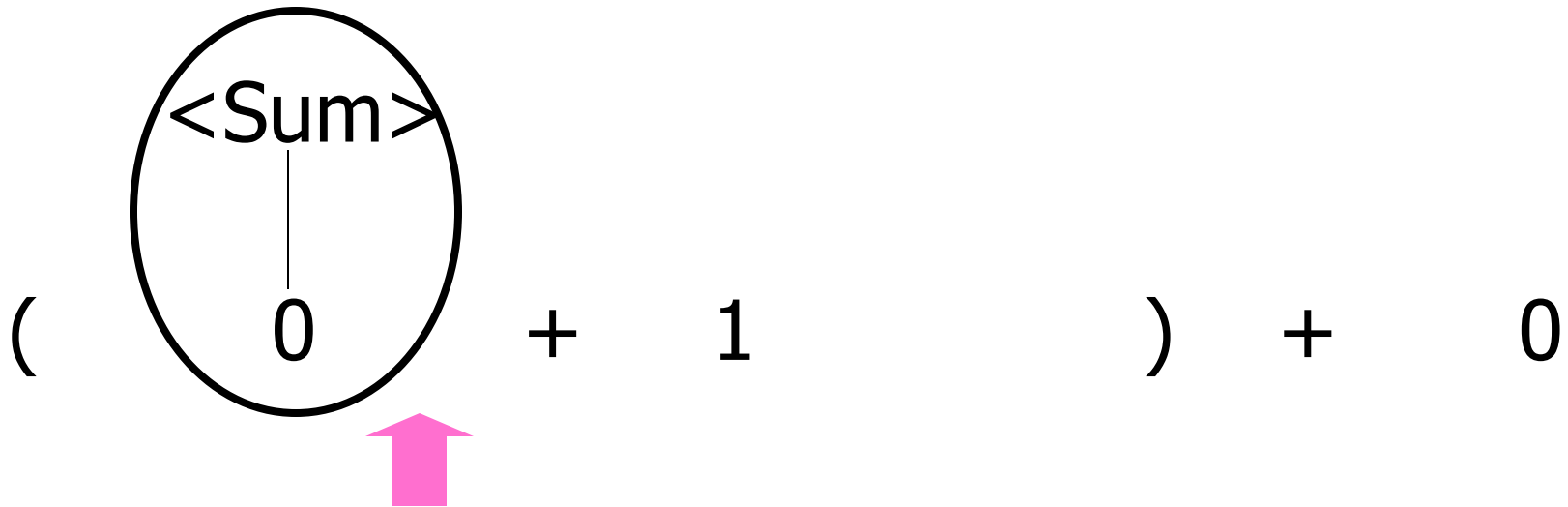


Example: $\langle \text{Sum} \rangle ::= 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

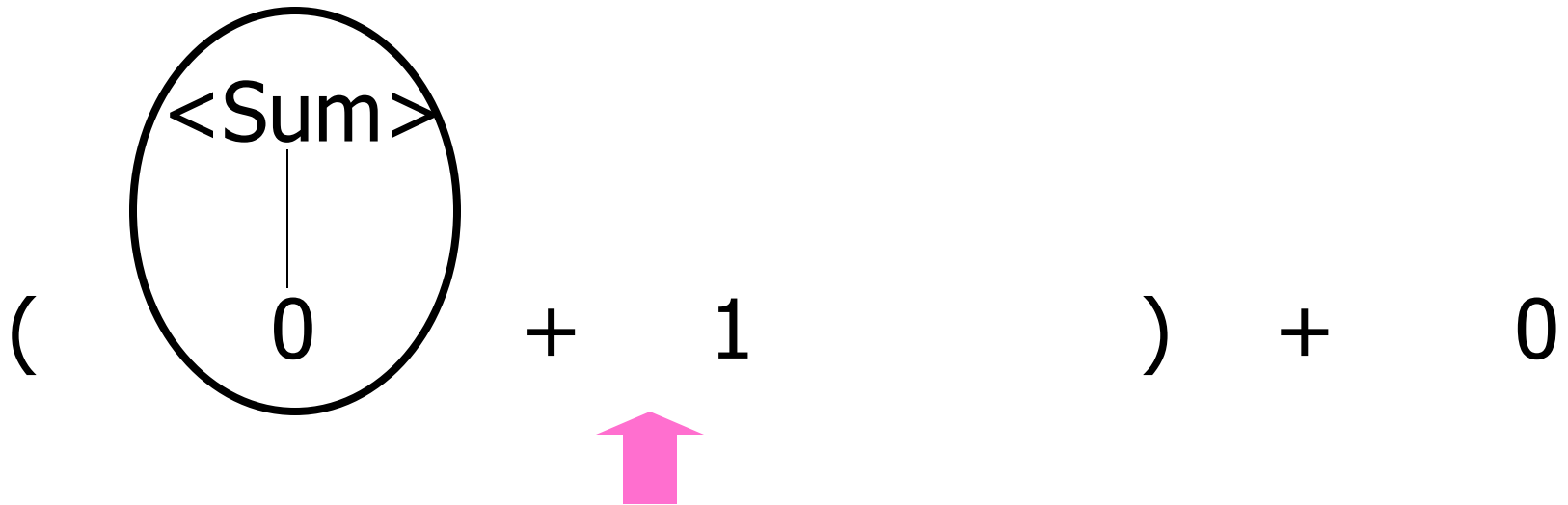
(0 + 1) + 0



Example: $\langle \text{Sum} \rangle ::= 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$



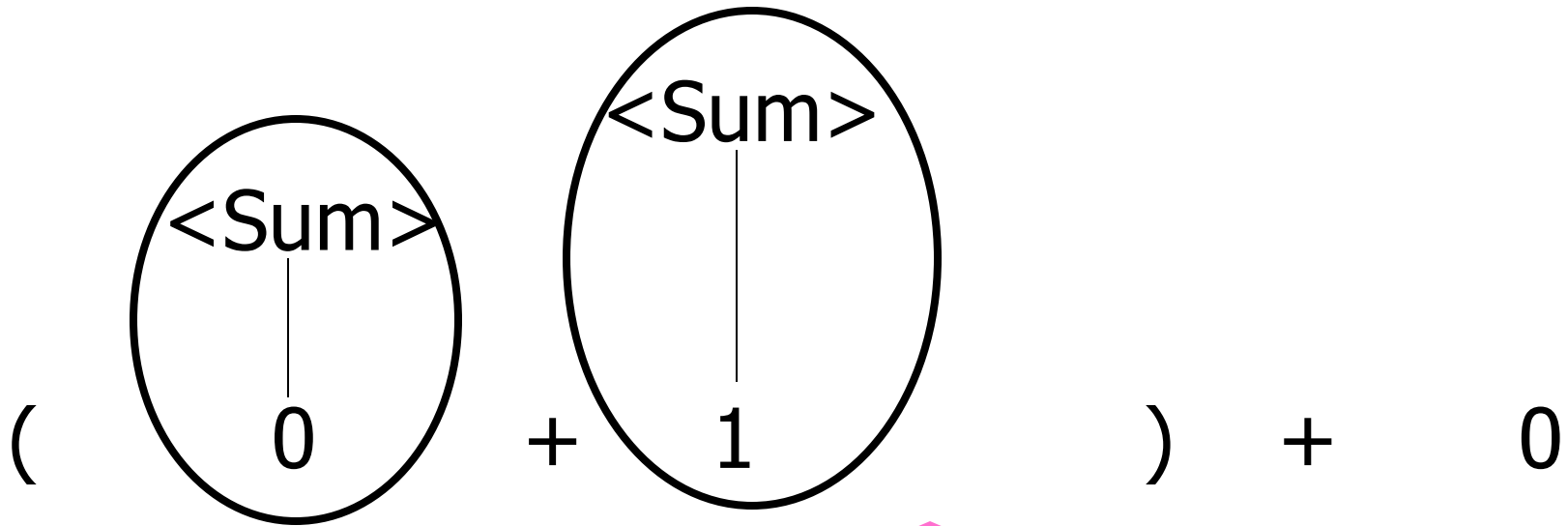
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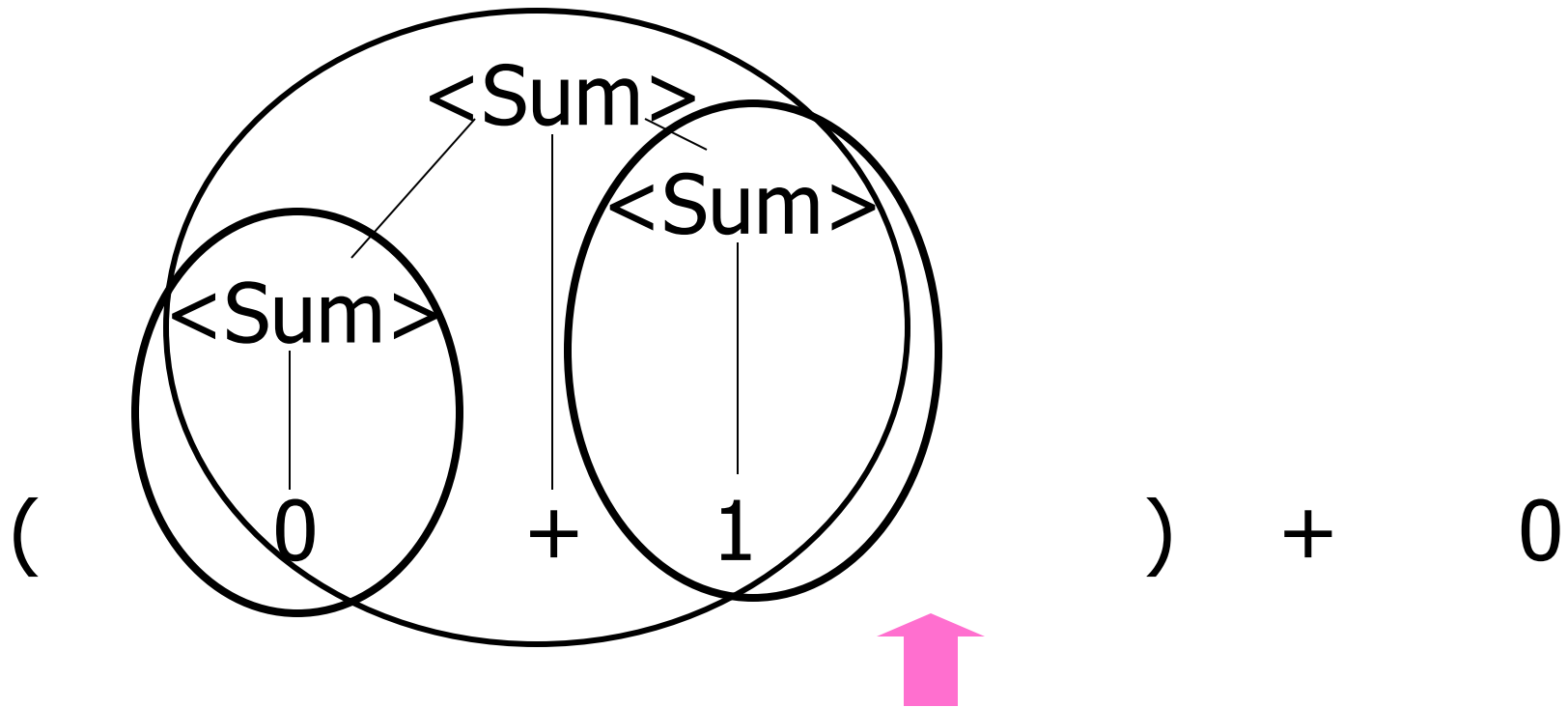
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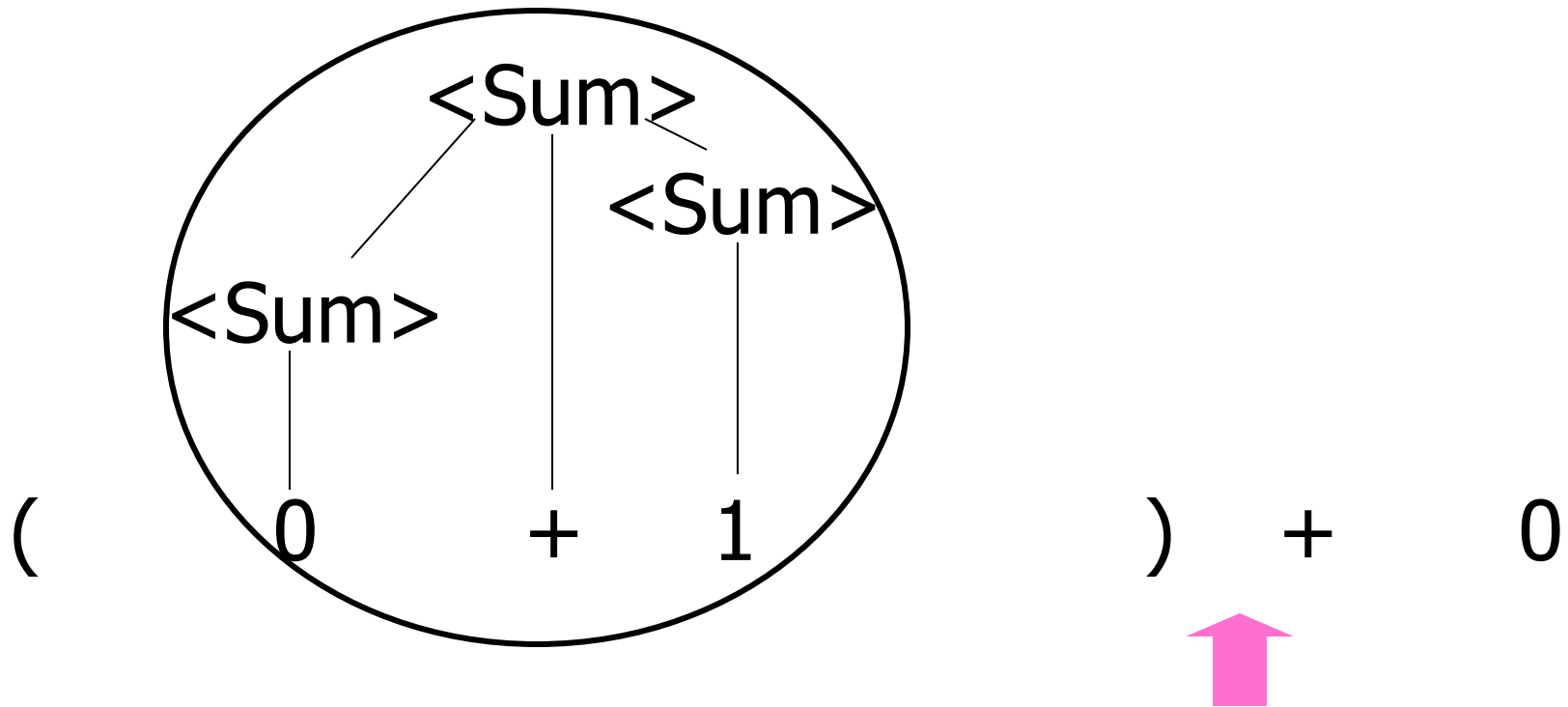
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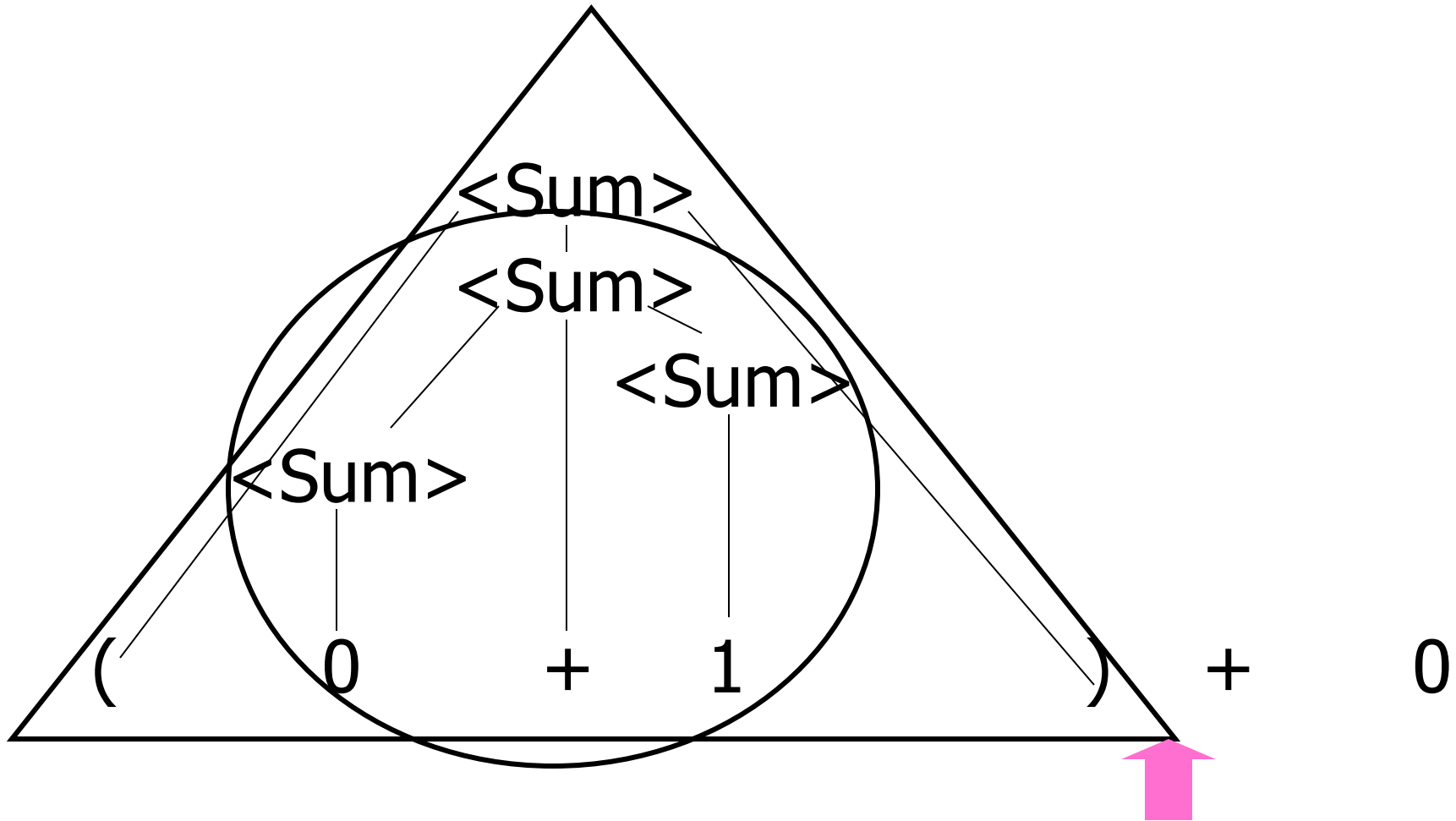
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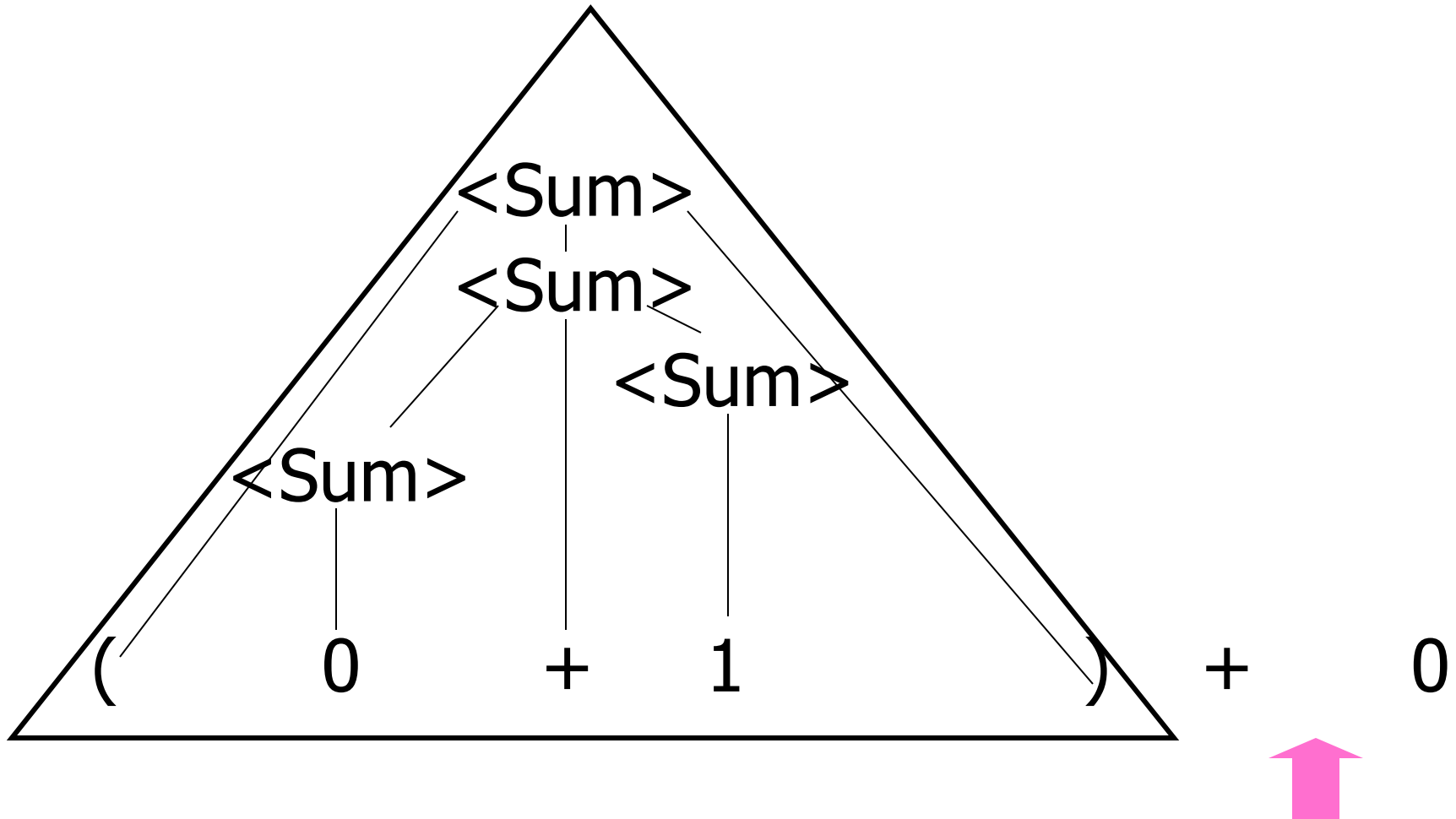
Example: $\langle \text{Sum} \rangle ::= 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$



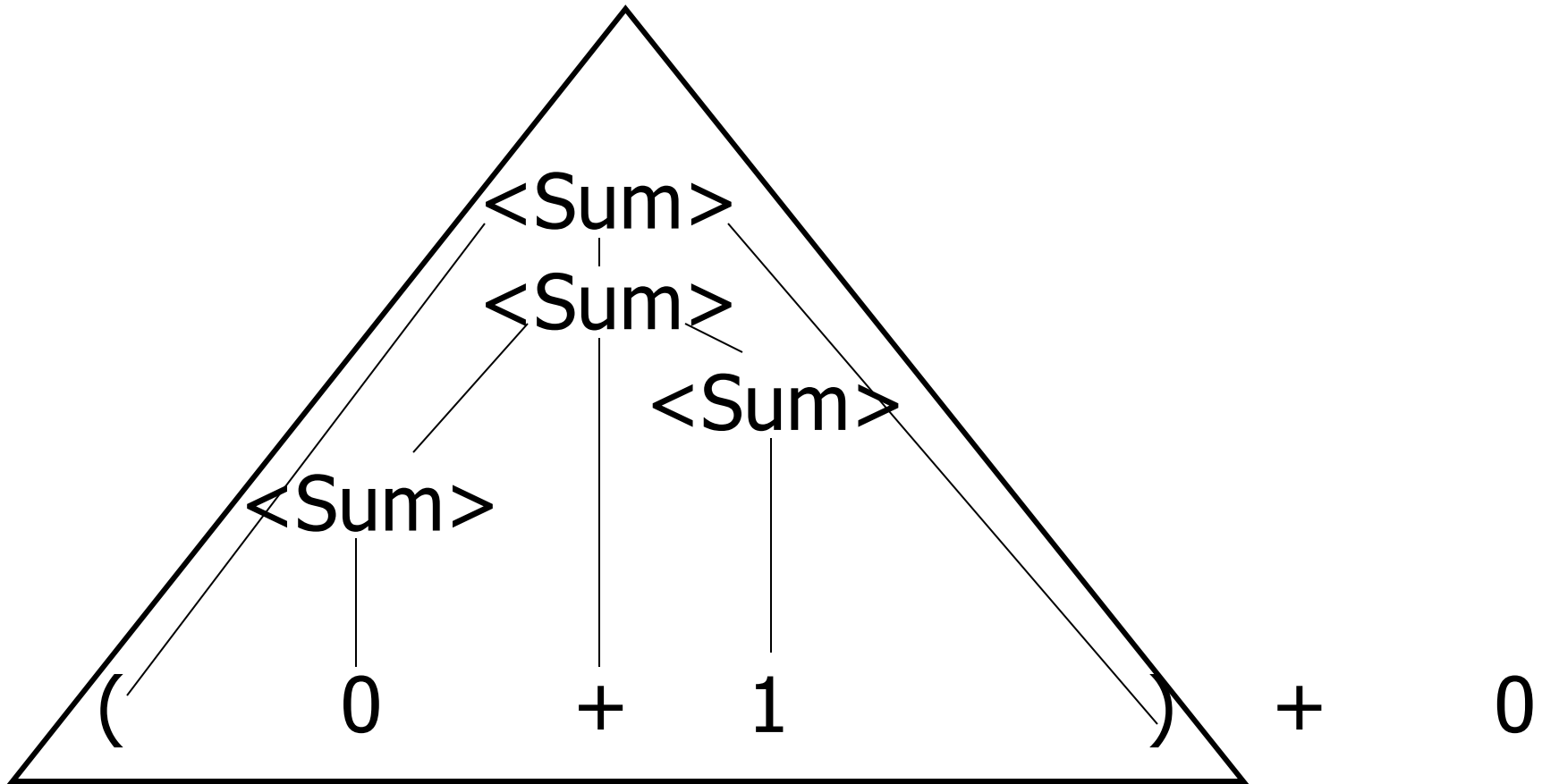
Example: $\langle \text{Sum} \rangle ::= 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$



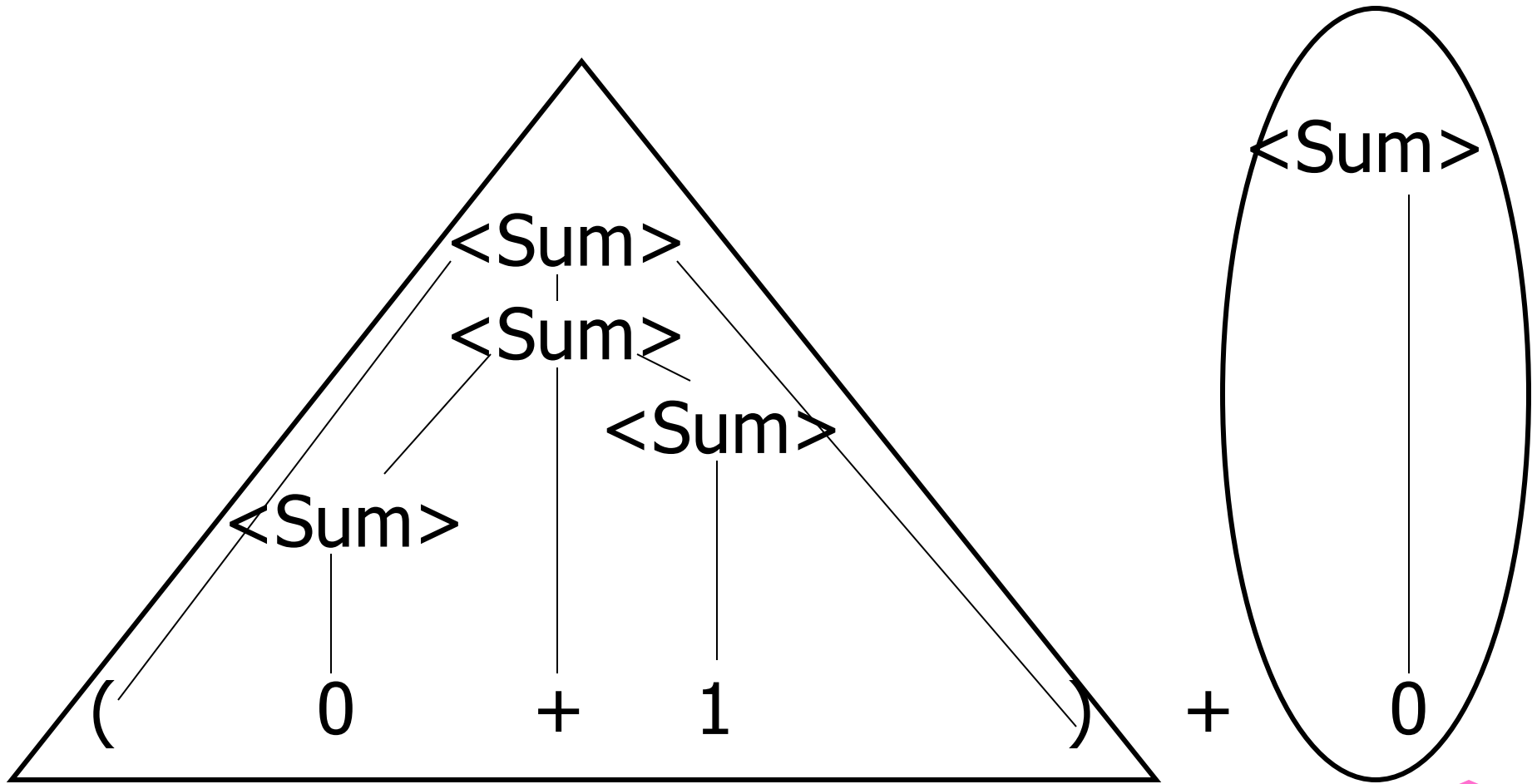
Example: $\langle \text{Sum} \rangle ::= 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
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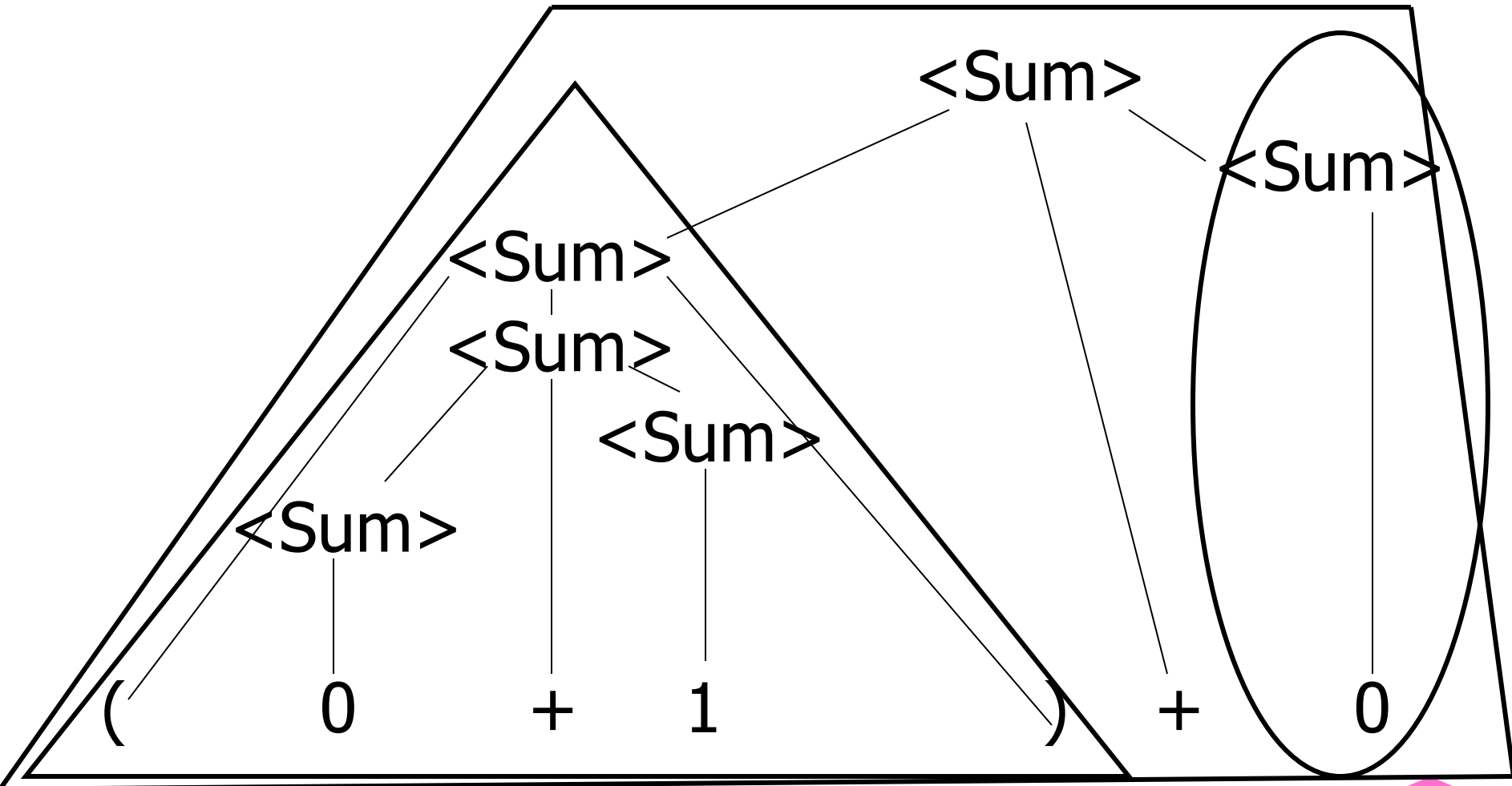
Example: $\langle \text{Sum} \rangle ::= 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$



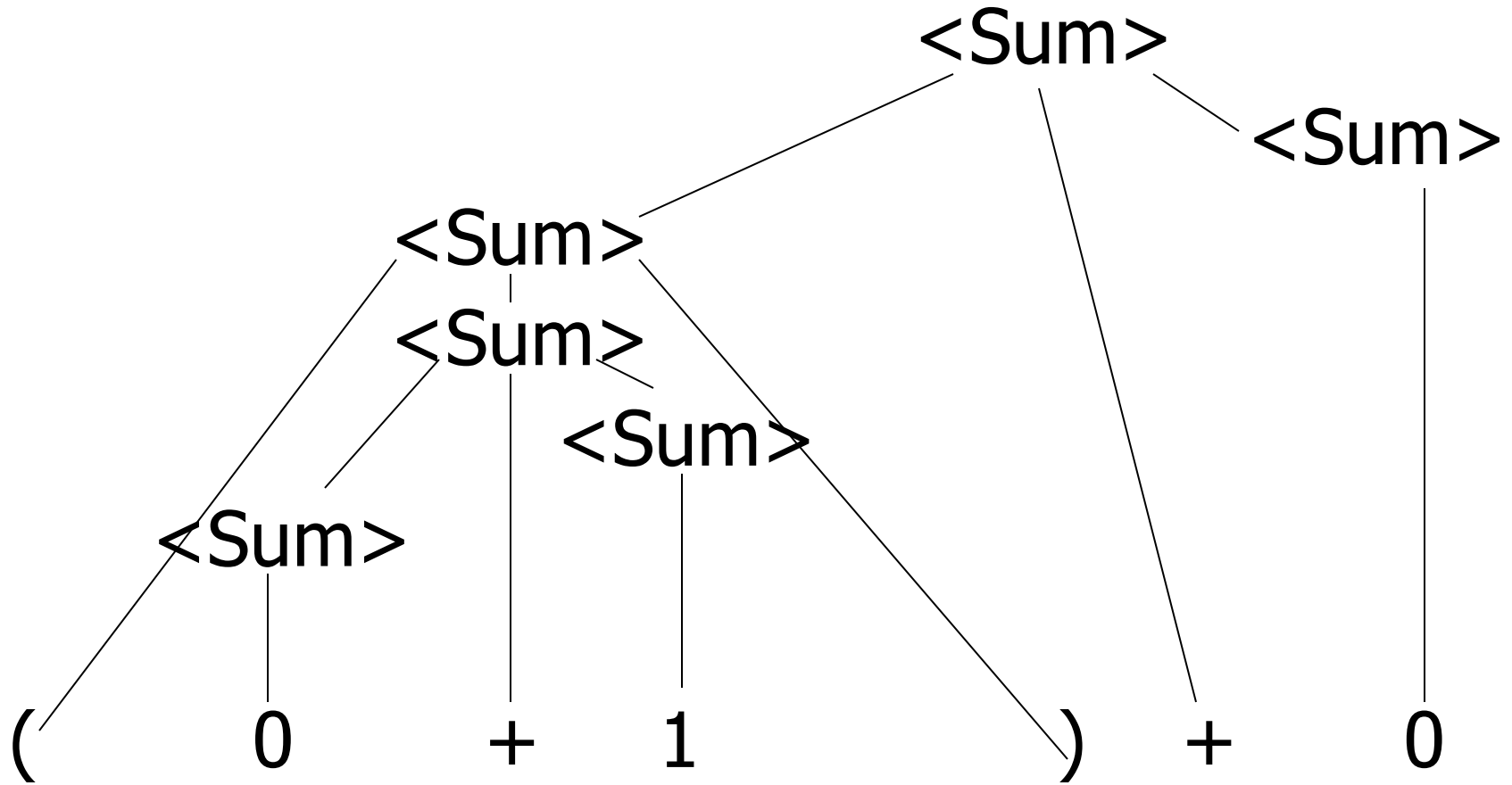
Example: $\langle \text{Sum} \rangle ::= 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$



Example: $\langle \text{Sum} \rangle ::= 0 \mid 1 \mid (\langle \text{Sum} \rangle) \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$



Example: $\langle \text{Sum} \rangle ::= 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$



LR Parsing Tables

- Build a pair of tables, Action and Goto, from the grammar
 - This is the hardest part, we omit here
 - Rows labeled by states
 - For Action, columns labeled by terminals and “end-of-tokens” marker
 - (more generally strings of terminals of fixed length)
 - For Goto, columns labeled by non-terminals

Action and Goto Tables

- Given a state and the next input, Action table says either
 - **shift** and go to state n , or
 - **reduce** by production k (explained in a bit)
 - **accept** or **error**
- Given a state and a non-terminal, Goto table says
 - go to state m

LR(i) Parsing Algorithm

- Based on push-down automata
- Uses states and transitions (as recorded in Action and Goto tables)
- Uses a stack containing states, terminals and non-terminals

LR(i) Parsing Algorithm

0. Ensure token stream ends in special “end-of-tokens” symbol
1. Start in state 1 with an empty stack
2. Push **state**(1) onto stack
3. Look at next i tokens from token stream (*toks*) (don't remove yet)
4. If top symbol on stack is **state**(n), look up action in Action table at $(n, toks)$

LR(i) Parsing Algorithm

5. If action = **shift** m ,

- a) Remove the top token from token stream and push it onto the stack
- b) Push **state**(m) onto stack
- c) Go to step 3

LR(i) Parsing Algorithm

6. If action = **reduce** k where production k is

$E ::= u$

- a) Remove $2 * \text{length}(u)$ symbols from stack (u and all the interleaved states)
- b) If new top symbol on stack is **state**(m), look up new state p in $\text{Goto}(m, E)$
- c) Push E onto the stack, then push **state**(p) onto the stack
- d) Go to step 3

LR(i) Parsing Algorithm

7. If action = **accept**

- Stop parsing, return success

8. If action = **error**,

- Stop parsing, return failure

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

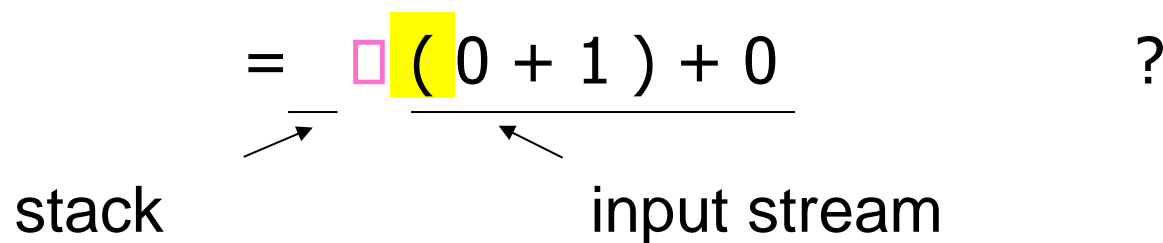
$= \square (0 + 1) + 0$ shift

Recall: LR(i) Parsing Algorithm

0. Ensure token stream ends in special “end-of-tokens” symbol
1. Start in state 1 with an empty stack
2. Push **state**(1) onto stack
- 3. Look at next i tokens from token stream ($toks$) (don't remove yet)
4. If top symbol on stack is **state**(n), look up action in Action table at $(n, toks)$

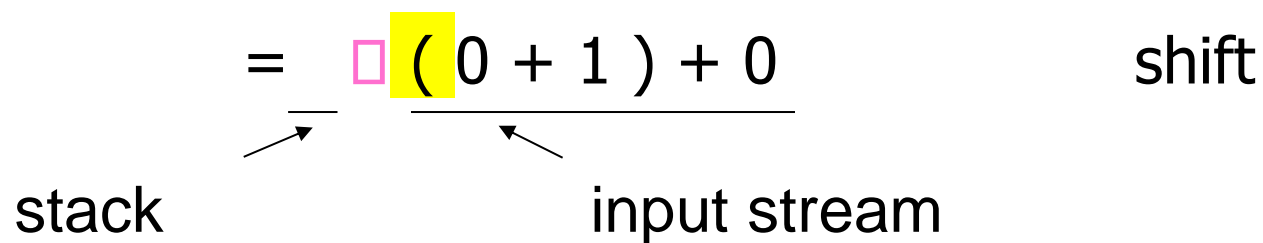
Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
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Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$



Recall: LR(i) Parsing Algorithm

5. If action = **shift** m ,

- a) Remove the top token from token stream and push it onto the stack
- b) Push **state**(m) onto stack
- c) Go to step 3

From: □ (0 + 1) + 0

To: (□ 0 + 1) + 0

stack

input stream

We don't visualize putting state(m) here...

Recall: LR(i) Parsing Algorithm

5. If action = **shift** m ,

- a) Remove the top token from token stream and push it onto the stack
- b) Push **state**(m) onto stack
- c) Go to step 3

If we wanted to visualize that state(m) on the stack it would look like this...

From: $\square (0 + 1) + 0$

To: $(\text{<state(m)>} \square 0 + 1) + 0$

stack

input stream

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$$\begin{aligned}
 &= (\square 0 + 1) + 0 && ? \\
 &= \square (0 + 1) + 0 && \text{shift}
 \end{aligned}$$

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

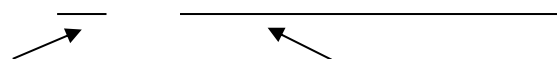
$\langle \text{Sum} \rangle \Rightarrow$

$$= (\square 0 + 1) + 0$$

$$= \square (0 + 1) + 0$$

shift

shift



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$$\Rightarrow (0 \square + 1) + 0$$

$$= (\square 0 + 1) + 0$$

$$= \square (0 + 1) + 0$$

?

shift

shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (0 \square + 1) + 0$ reduce
 $= (\square 0 + 1) + 0$ shift
 $= \square (0 + 1) + 0$ shift

Recall: LR(i) Parsing Algorithm

6. If action = **reduce** k where production k is
 $E ::= u$
- a) Remove $2 * \text{length}(u)$ symbols from stack (u and all the interleaved states)
 - b) If new top symbol on stack is **state**(m), look up new state p in $\text{Goto}(m, E)$
 - c) Push E onto the stack, then push **state**(p) onto the stack
 - d) Go to step 3

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= (\langle \text{Sum} \rangle \square + 1) + 0$ shift
 $\Rightarrow (0 \square + 1) + 0$ reduce
 $= (\square 0 + 1) + 0$ shift
 $= \square (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= (\langle \text{Sum} \rangle + \square 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \square + 1) + 0$ shift
 $\Rightarrow (0 \square + 1) + 0$ reduce
 $= (\square 0 + 1) + 0$ shift
 $= \square (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (\langle \text{Sum} \rangle + 1 \square) + 0$?
$= (\langle \text{Sum} \rangle + \square 1) + 0$	shift
$= (\langle \text{Sum} \rangle \square + 1) + 0$	shift
$\Rightarrow (0 \square + 1) + 0$	reduce
$= (\square 0 + 1) + 0$	shift
$= \square (0 + 1) + 0$	shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (\langle \text{Sum} \rangle + 1 \square) + 0$	reduce
$= (\langle \text{Sum} \rangle + \square 1) + 0$	shift
$= (\langle \text{Sum} \rangle \square + 1) + 0$	shift
$\Rightarrow (0 \square + 1) + 0$	reduce
$= (\square 0 + 1) + 0$	shift
$= \square (0 + 1) + 0$	shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square) + 0 \quad ?$
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \square) + 0 \quad \text{reduce}$
 $= (\langle \text{Sum} \rangle + \square 1) + 0 \quad \text{shift}$
 $= (\langle \text{Sum} \rangle \square + 1) + 0 \quad \text{shift}$
 $\Rightarrow (0 \square + 1) + 0 \quad \text{reduce}$
 $= (\square 0 + 1) + 0 \quad \text{shift}$
 $= \square (0 + 1) + 0 \quad \text{shift}$

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \square) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \square 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \square + 1) + 0$ shift
 $\Rightarrow (0 \square + 1) + 0$ reduce
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$\langle \text{Sum} \rangle \Rightarrow$

$= (\langle \text{Sum} \rangle \square) + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square) + 0$ reduce
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$\langle \text{Sum} \rangle \Rightarrow$

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 $= \langle \text{Sum} \rangle \square + 0$ shift
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 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle$	\Rightarrow		
	\Rightarrow	$\langle \text{Sum} \rangle + 0 \square$	reduce
	$=$	$\langle \text{Sum} \rangle + \square 0$	shift
	$=$	$\langle \text{Sum} \rangle \square + 0$	shift
	\Rightarrow	$(\langle \text{Sum} \rangle) \square + 0$	reduce
	$=$	$(\langle \text{Sum} \rangle \square) + 0$	shift
	\Rightarrow	$(\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square) + 0$	reduce
	\Rightarrow	$(\langle \text{Sum} \rangle + 1 \square) + 0$	reduce
	$=$	$(\langle \text{Sum} \rangle + \square 1) + 0$	shift
	$=$	$(\langle \text{Sum} \rangle \square + 1) + 0$	shift
	\Rightarrow	$(0 \square + 1) + 0$	reduce
	$=$	$(\square 0 + 1) + 0$	shift
	$=$	$\square (0 + 1) + 0$	shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle$	$\Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square$	reduce
	$\Rightarrow \langle \text{Sum} \rangle + 0 \square$	reduce
	$= \langle \text{Sum} \rangle + \square 0$	shift
	$= \langle \text{Sum} \rangle \square + 0$	shift
	$\Rightarrow (\langle \text{Sum} \rangle) \square + 0$	reduce
	$= (\langle \text{Sum} \rangle \square) + 0$	shift
	$\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square) + 0$	reduce
	$\Rightarrow (\langle \text{Sum} \rangle + 1 \square) + 0$	reduce
	$= (\langle \text{Sum} \rangle + \square 1) + 0$	shift
	$= (\langle \text{Sum} \rangle \square + 1) + 0$	shift
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Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
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$\langle \text{Sum} \rangle \square$	$\Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square$	reduce
	$\Rightarrow \langle \text{Sum} \rangle + 0 \square$	reduce
	$= \langle \text{Sum} \rangle + \square 0$	shift
	$= \langle \text{Sum} \rangle \square + 0$	shift
	$\Rightarrow (\langle \text{Sum} \rangle) \square + 0$	reduce
	$= (\langle \text{Sum} \rangle \square) + 0$	shift
	$\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square) + 0$	reduce
	$\Rightarrow (\langle \text{Sum} \rangle + 1 \square) + 0$	reduce
	$= (\langle \text{Sum} \rangle + \square 1) + 0$	shift
	$= (\langle \text{Sum} \rangle \square + 1) + 0$	shift
	$\Rightarrow (0 \square + 1) + 0$	reduce
	$= (\square 0 + 1) + 0$	shift
	$= \square (0 + 1) + 0$	shift

Recall: LR(i) Parsing Algorithm

7. If action = **accept**

- Stop parsing, return success

8. If action = **error**,

- Stop parsing, return failure

Adding Synthesized Attributes

- Add to each **reduce** a rule for calculating the new synthesized attribute from the component attributes
- Add to each non-terminal pushed onto the stack, the attribute calculated for it
- When performing a **reduce**,
 - gather the recorded attributes from each non-terminal popped from stack
 - Compute new attribute for non-terminal pushed onto stack

- Back to Ambiguous Grammars...

How do you know you have ambiguity?

- The Ocaml parser generator (ocamlyacc) will report ambiguity in the grammar as “conflicts”:
- ***Shift/reduce:*** Usually caused by lack of associativity or precedence information in grammar
- ***Reduce/reduce:*** can't decide between two different rules to reduce by; Not always clear what the problem is, but often right-hand side of one production is the suffix of another
- We will explain what these conflicts mean next time!

Shift-Reduce Conflicts

- **Problem:** can't decide whether the action for a state and input character should be **shift** or **reduce**
- Caused by ambiguity in grammar
- Usually caused by lack of associativity or precedence information in grammar

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

-> $\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square + 0 \quad ??$
-> $\langle \text{Sum} \rangle + 1 \square + 0 \quad \text{reduce}$
-> $\langle \text{Sum} \rangle + \square 1 + 0 \quad \text{shift}$
-> $\langle \text{Sum} \rangle \square + 1 + 0 \quad \text{shift}$
-> $0 \square + 1 + 0 \quad \text{reduce}$
 $\square 0 + 1 + 0 \quad \text{shift}$

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

- **Problem:** shift or reduce?

-> $\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square + 0 \quad ??$

- You can shift-shift-reduce-reduce or reduce-shift-shift-reduce

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

■ **Problem:** shift or reduce?

-> $\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square + 0 \quad ??$

■ You can **shift-shift-reduce-reduce** or
reduce-shift-shift-reduce

-> $\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square \quad \text{reduce}$

-> $\langle \text{Sum} \rangle + \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square \quad \text{reduce}$

-> $\langle \text{Sum} \rangle + \langle \text{Sum} \rangle + \square 0 \quad \text{shift}$

-> $\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square + 0 \quad \text{shift}$

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

- **Problem:** shift or reduce?

-> $\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square + 0 \quad ??$

- You can shift-shift-reduce-reduce or
reduce-shift-shift-reduce

-> $\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square$ reduce

-> $\langle \text{Sum} \rangle + \square 0$ shift

-> $\langle \text{Sum} \rangle \square + 0$ shift

-> $\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square + 0$ reduce

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

- **Problem:** shift or reduce?

-> $\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \square + 0 \quad ??$

- You can shift-shift-reduce-reduce or reduce-shift-shift-reduce
- Shift first - right associative
- Reduce first- left associative

Reduce - Reduce Conflicts

- **Problem:** can't decide between two different rules to reduce by
- Again, caused by ambiguity in grammar

- **Symptom:** RHS of one production suffix of another
- Requires examining grammar and rewriting it
- Harder to solve than shift-reduce errors

Example

■ $S ::= A \mid aB$ $A ::= abc$ $B ::= bc$

□ abc shift

■ Problem: reduce by $B ::= bc$ then by $S ::= aB$, or by $A ::= abc$ then $S ::= A$?

- Back to Grammar Disambiguation...

Disambiguating a Grammar

- Idea: Each non-terminal represents all strings having some property
- Identify these properties (often in terms of things that can't happen)
- Use these properties to inductively guarantee every string in language has a unique parse

Steps to Grammar Disambiguation

- Identify the rules and a smallest use that display ambiguity
- Decide which parse to keep; why should others be thrown out?
- What syntactic restrictions on subexpressions are needed to throw out the bad (while keeping the good)?
- Add a new non-terminal and rules to describe this set of restricted subexpressions (called stratifying, or refactoring)
- **Characterize each non-terminal by a language invariant**
- Replace old rules to use new non-terminals
- Rinse and repeat

How do we disambiguate in this case?

- Our old friend:

$$\begin{aligned} \langle \text{exp} \rangle & ::= \langle \text{factor} \rangle \\ & \quad | \langle \text{factor} \rangle + \langle \text{factor} \rangle \end{aligned}$$
$$\begin{aligned} \langle \text{factor} \rangle & ::= \langle \text{bin} \rangle \\ & \quad | \langle \text{bin} \rangle * \langle \text{exp} \rangle \end{aligned}$$
$$\langle \text{bin} \rangle ::= 0 \mid 1$$

- How do we make multiplication have higher precedence than addition?

Moving On With Richer Expressions

- How do we extend the grammar to support nested additions, e.g., $1 * (0 + 1)$

$\langle \text{exp} \rangle ::= \langle \text{factor} \rangle$
 $\quad \quad \quad | \langle \text{factor} \rangle + \langle \text{exp} \rangle$

$\langle \text{factor} \rangle ::= \langle \text{bin} \rangle$
 $\quad \quad \quad | \langle \text{bin} \rangle * \langle \text{factor} \rangle$

$\langle \text{bin} \rangle ::= 0 \mid 1$

Moving On With Richer Expressions

- How do we extend the grammar to support nested additions, e.g., $1 * (0 + 1)$

$\langle \text{exp} \rangle ::= \langle \text{factor} \rangle$
 $\quad \quad \quad | \langle \text{factor} \rangle + \langle \text{exp} \rangle$

$\langle \text{factor} \rangle ::= \langle \text{bin} \rangle$
 $\quad \quad \quad | \langle \text{bin} \rangle * \langle \text{factor} \rangle$

$\langle \text{bin} \rangle ::= 0 \mid 1 \mid (\langle \text{exp} \rangle)$

Moving On With Richer Expressions

- How do we extend the grammar to support other operations, subtraction and division?

$\langle \text{exp} \rangle ::= \langle \text{factor} \rangle$
 $\quad \quad \quad | \langle \text{factor} \rangle + \langle \text{exp} \rangle | \langle \text{factor} \rangle - \langle \text{exp} \rangle$

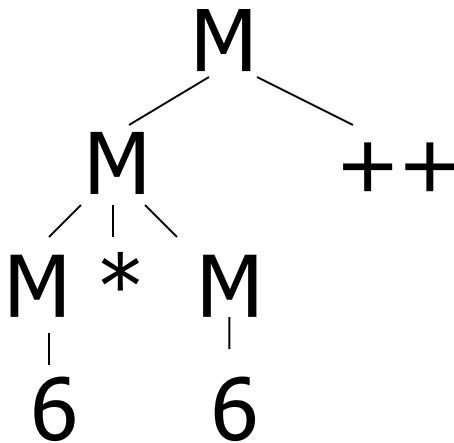
$\langle \text{factor} \rangle ::= \langle \text{bin} \rangle$
 $\quad \quad \quad | \langle \text{bin} \rangle * \langle \text{factor} \rangle | \langle \text{bin} \rangle / \langle \text{factor} \rangle$

$\langle \text{bin} \rangle ::= 0 | 1 | (\langle \text{exp} \rangle)$

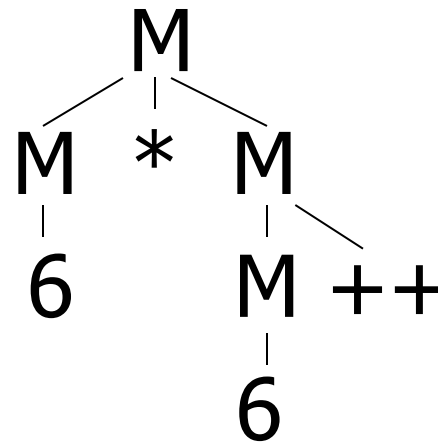
More Disambiguating Grammars

- $M ::= M * M \mid (M) \mid M ++ \mid 6$
- Ambiguous because of associativity of $*$
- because of conflict between $*$ and $++$:

■ $6 * 6 ++$



$6 * 6 ++$



$M ::= M * M \mid (M) \mid M ++ \mid 6$

- How to disambiguate?
- Choose associativity for $*$
- Choose precedence between $*$ and $++$
- Four possibilities
- Four different approaches
- Some easier than others

$M ::= M * M \mid (M) \mid M ++ \mid 6$

- Think about $6 * 6 ++ * 6 * 6 ++$
- Let's start with observations
- If $*$ binds less tightly than $++$, then no $*$ can be the immediate subtree to a $++$.
 - We would need a language for things that don't parse as $*$
- If $*$ binds more tightly than $++$, then ...
- The right subtree to $*$ can't be a $++$
- But the left can!
 - Need different languages of the left and right

$M ::= M * M \mid (M) \mid M ++ \mid 6$

- * higher prec than ++

- $6 * 6 ++ \quad 6 ++ * 6$

- $M ::= M ++ \mid \text{StarExp} \mid (M) \mid 6$

- What is **StarExp**

- It is everything that parses as a * and can't parse as a ++

- But what is the associativity of *?

- Class chose left

$M ::= M * M \mid (M) \mid M ++ \mid 6$

- * higher prec than ++
 - $6 * 6 ++ \quad 6 ++ * 6$
- * Left assoc
- $M ::= M ++ \mid \text{StarExp} \mid (M) \mid 6$
- $\text{StarExp} ::= \text{PossStar} * \text{NoStarNoPlusPlus}$
- What is **PossStar**? It could it be a *, but it also doesn't have to be.
- Can it be ++? YES! It can be anything
- It is **M** !

$M ::= M * M \mid (M) \mid M ++ \mid 6$

- * higher prec than ++
 - $6 * 6 ++ \quad 6 ++ * 6$
- * Left assoc
- $M ::= M ++ \mid \text{StarExp} \mid (M) \mid 6$
- $\text{StarExp} ::= M * \text{NoStarNoPlusPlus}$

$M ::= M * M \mid (M) \mid M ++ \mid 6$

- * higher prec than ++
 - $6 * 6 ++ \quad 6 ++ * 6$
- * Left assoc
- $M ::= M ++ \mid \text{StarExp} \mid (M) \mid 6$
- $\text{StarExp} ::= M * \text{NoStarNoPlusPlus}$
- But what is NoStarNoPlusPlus ?
- Well, the other two original rules: $(M) \mid 6$

$M ::= M * M \mid (M) \mid M ++ \mid 6$

- * higher prec than ++
 - $6 * 6 ++ \quad 6 ++ * 6$
- * Left assoc
- $M ::= M ++ \mid \text{StarExp} \mid (M) \mid 6$
- $\text{StarExp} ::= M * \text{NoStarNoPlusPlus}$
- $\text{NoStarNoPlusPlus} ::= (M) \mid 6$
- But we have $(M) \mid 6$ twice, and it's the same language each time. Let's have one

$M ::= M * M \mid (M) \mid M ++ \mid 6$

- * higher prec than ++
 - $6 * 6 ++ \quad 6 ++ * 6$
- * Left assoc
- $M ::= M ++ \mid \text{StarExp} \mid \text{NoStarNoPlusPlus}$
- $\text{StarExp} ::= M * \text{NoStarNoPlusPlus}$
- $\text{NoStarNoPlusPlus} ::= (M) \mid 6$

$M ::= M * M \mid (M) \mid M ++ \mid 6$

- * higher prec than ++
 - $6 * 6 ++ \quad 6 ++ * 6$
- * Left assoc
- $M ::= M ++ \mid \text{StarExp} \mid \text{NoStarNoPlusPlus}$
- $\text{StarExp} ::= M * \text{NoStarNoPlusPlus}$
- $\text{NoStarNoPlusPlus} ::= (M) \mid 6$

Disambiguating a Grammar

- $\langle \text{exp} \rangle ::= 0 | 1 | b \langle \text{exp} \rangle | \langle \text{exp} \rangle a$
| $\langle \text{exp} \rangle m \langle \text{exp} \rangle$
- Want ***a*** to have higher precedence than ***b***, which in turn has higher precedence than ***m***, and such that ***m*** associates to the left.
- Think of *a*, *b*, *m* as operators
e.g., *a* is “++”, *b* is “!”, *m* is “*”

Disambiguating a Grammar

- $\langle \text{exp} \rangle ::= 0 | 1 | b \langle \text{exp} \rangle | \langle \text{exp} \rangle a$
| $\langle \text{exp} \rangle m \langle \text{exp} \rangle$
- Want ***a*** to have higher precedence than ***b***, which in turn has higher precedence than ***m***, and such that ***m*** associates to the left.
- $\langle \text{exp} \rangle ::= \langle \text{exp} \rangle m \langle \text{not}_m \rangle | \langle \text{not}_m \rangle$
- $\langle \text{not}_m \rangle ::= b \langle \text{not}_m \rangle | \langle \text{not}_b_m \rangle$
- $\langle \text{not}_b_m \rangle ::= \langle \text{not}_b_m \rangle a | 0 | 1$

Disambiguating a Grammar – Take 2

- $\langle \text{exp} \rangle ::= 0 | 1 | b \langle \text{exp} \rangle | \langle \text{exp} \rangle a$
| $\langle \text{exp} \rangle m \langle \text{exp} \rangle$
- Want ***b*** to have higher precedence than ***m***, which in turn has higher precedence than ***a***, and such that ***m*** associates to the right.

Disambiguating a Grammar – Take 2

- $\langle \text{exp} \rangle ::= 0 | 1 | b \langle \text{exp} \rangle | \langle \text{exp} \rangle a$
| $\langle \text{exp} \rangle m \langle \text{exp} \rangle$
- Want ***b*** to have higher precedence than ***m***, which in turn has higher precedence than ***a***, and such that ***m*** associates to the right.
- $\langle \text{exp} \rangle ::=$
 $\langle \text{no_a_m} \rangle | \langle \text{no_m} \rangle m \langle \text{no_a} \rangle | \langle \text{exp} \rangle a$
- $\langle \text{no_a} \rangle ::= \langle \text{no_a_m} \rangle | \langle \text{no_a_m} \rangle m \langle \text{no_a} \rangle$
- $\langle \text{no_m} \rangle ::= \langle \text{no_a_m} \rangle | \langle \text{exp} \rangle a$
- $\langle \text{no_a_m} \rangle ::= b \langle \text{no_a_m} \rangle | 0 | 1$

Disambiguating a Grammar – Take 3

- $\langle \text{exp} \rangle ::= 0 | 1 | b \langle \text{exp} \rangle | \langle \text{exp} \rangle a$
| $\langle \text{exp} \rangle m \langle \text{exp} \rangle$
- Want ***a*** has higher precedence than ***m***, which in turn has higher precedence than ***b***, and such that ***m*** associates to the right.
- For you...

Disambiguating Grammars – Dangling Else

- $\text{stmt} ::= \dots$
 - | **if** (expr) stmt
 - | **if** (expr) stmt **else** stmt

- How can we parse
if (e1) if (e2) s1 else s2 ?

Disambiguating Grammars – Dangling Else

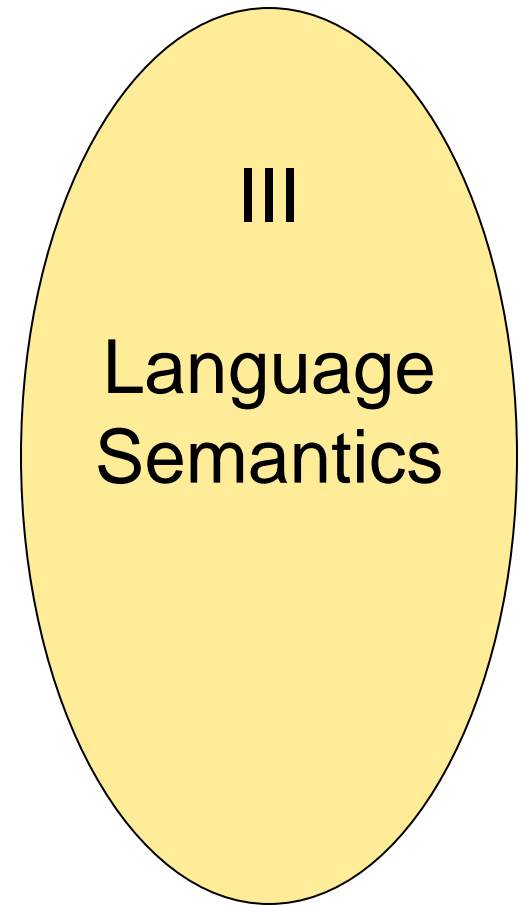
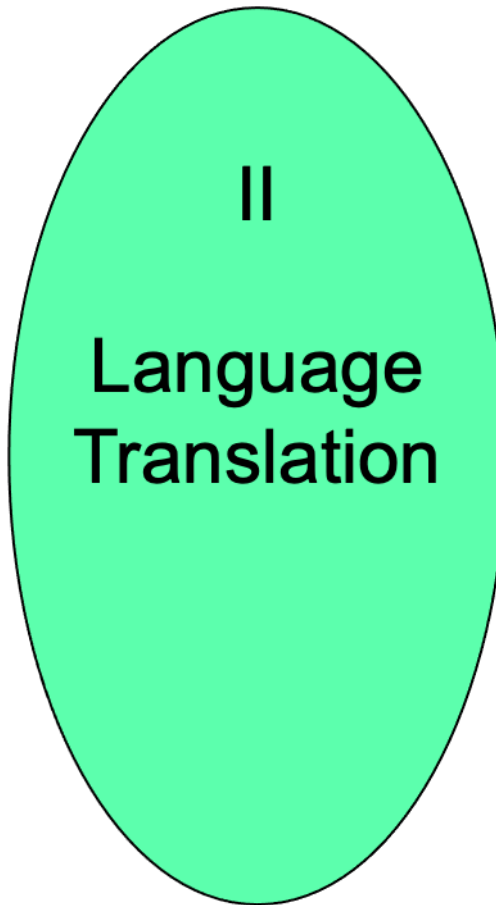
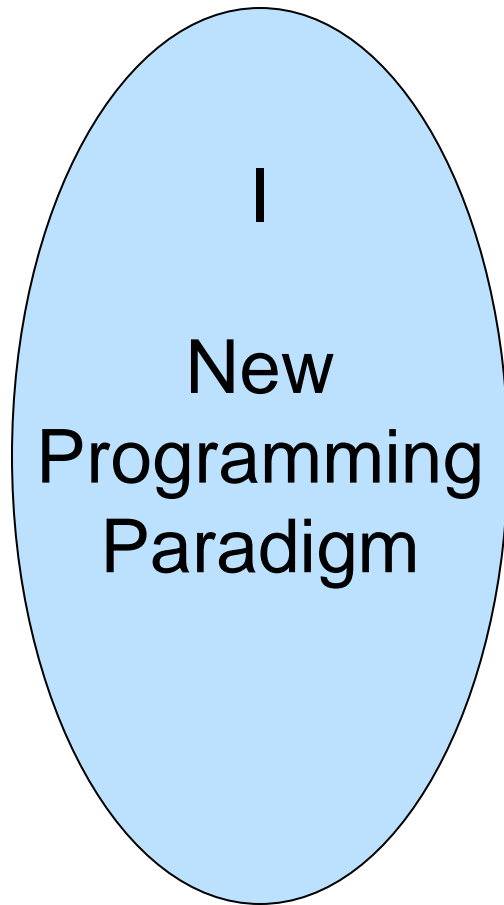
- Try: let us try to differentiate if we have **if** inside the **then** branch or not....
- $\text{stmt} = \text{open_stmt} \mid \text{closed_stmt}$
- $\text{open_stmt} ::= \mathbf{if} (\text{expr}) \text{stmt}$
 - | $\mathbf{if} (\text{expr}) \text{closed_stmt} \mathbf{else} \text{open_stmt}$
- $\text{closed_stmt} ::= \text{non_if_statement}$
 - | $\mathbf{if} (\text{expr}) \text{closed_stmt} \mathbf{else} \text{closed_stmt}$
- How can we parse **if (e1) if (e2) s1 else s2** now ?

Disambiguating Grammars – Overlapping

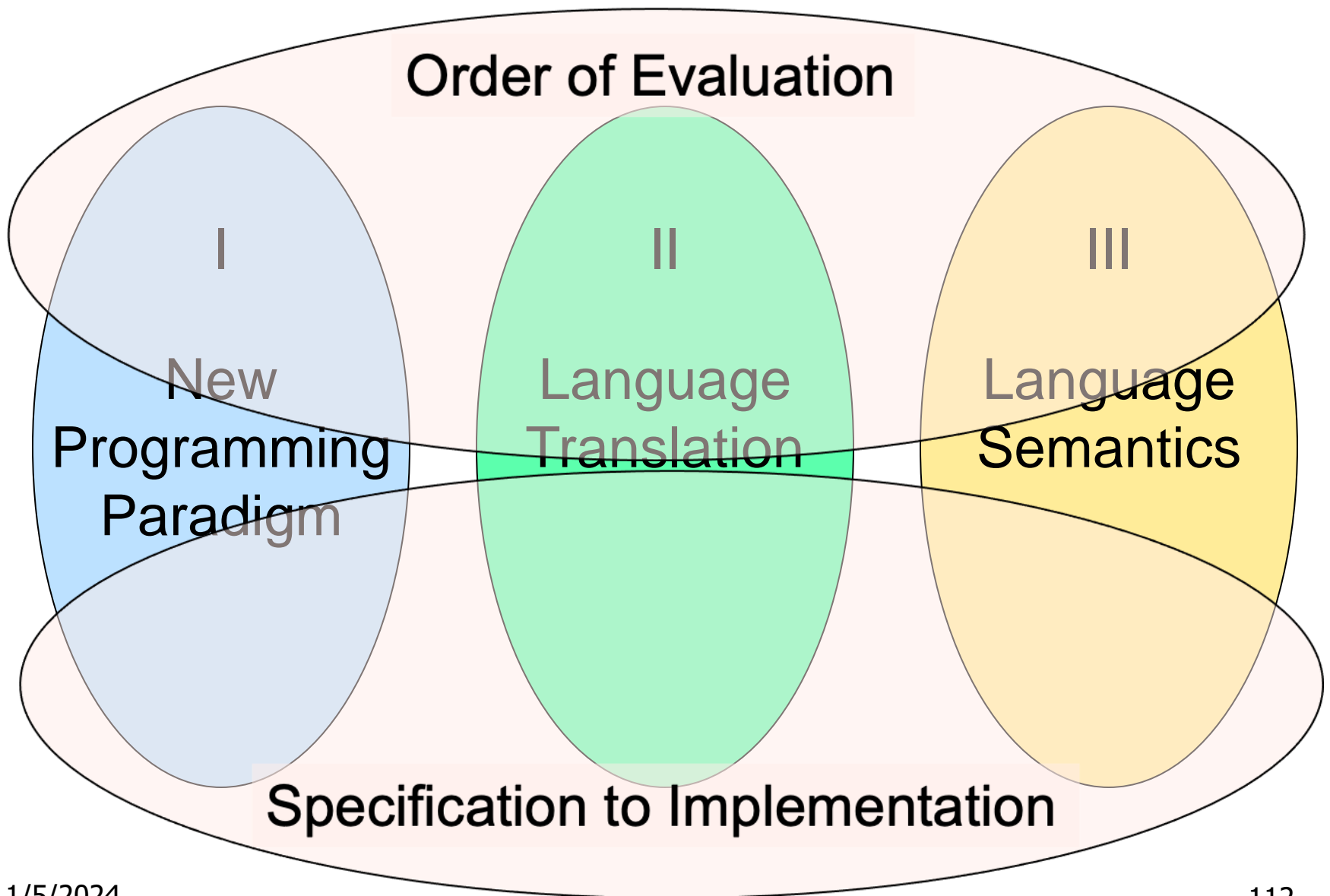
- $seq = \varepsilon \mid may_word \mid word\ seq$
- $may_word = \varepsilon \mid \text{"word"}$
- How do you parse "word"? And ε ?
- How do you fix it?

Programming Languages & Compilers

Three Main Topics of the Course



Programming Languages & Compilers



Programming Languages & Compilers

III : Language Semantics

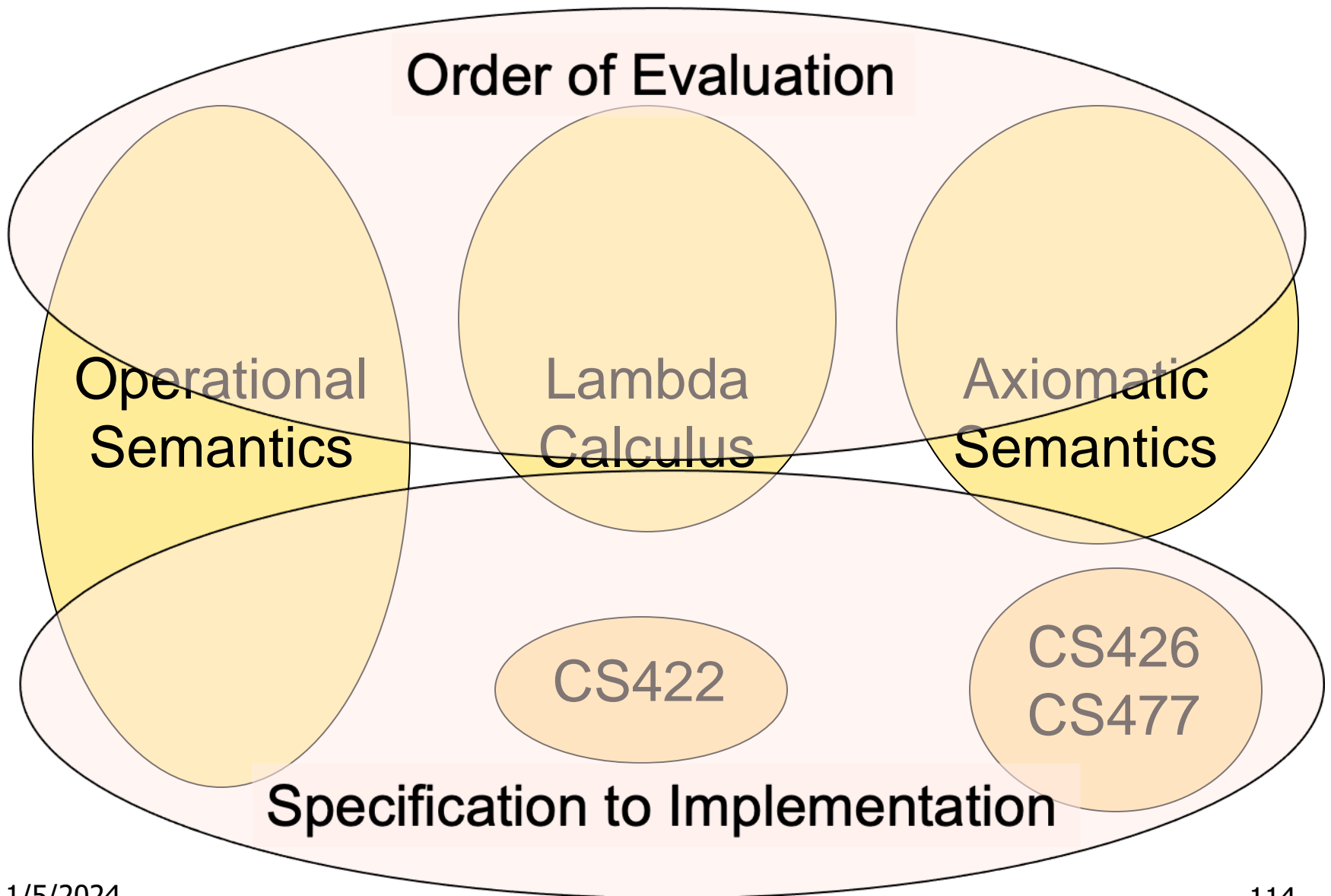


Operational
Semantics

Lambda
Calculus

Axiomatic
Semantics

Programming Languages & Compilers



Semantics

- Expresses the meaning of syntax
- Static semantics
 - Meaning based only on the form of the expression without executing it
 - Usually restricted to type checking / type inference

Dynamic semantics

- Method of describing meaning of executing a program
- Several different types:
 - Operational Semantics
 - Axiomatic Semantics
 - Denotational Semantics

Dynamic Semantics

- Different languages better suited to different types of semantics
- Different types of semantics serve different purposes