

Programming Languages and Compilers (CS 421)



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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha



Parser Code

- `<grammar>.ml` defines one parsing function per entry point
- Parsing function takes a lexing function (lexer buffer to token) and a lexer buffer as arguments
- Returns semantic attribute of corresponding entry point



Ocamlyacc Input

- File format:

```
%{
```

```
    <header>
```

```
%}
```

```
    <declarations>
```

```
%%
```

```
    <rules>
```

```
%%
```

```
    <trailer>
```



Ocamlyacc *<header>*

- Contains arbitrary Ocaml code
- Typically used to give types and functions needed for the semantic actions of rules and to give specialized error recovery
- May be omitted
- *<footer>* similar. Possibly used to call parser



Ocamlyacc <declarations>

- **%token** *symbol ... symbol*
- Declare given symbols as tokens
- **%token** <*type*> *symbol ... symbol*
- Declare given symbols as token constructors, taking an argument of type <*type*>
- **%start** *symbol ... symbol*
- Declare given symbols as entry points; functions of same names in <*grammar*>.ml



Ocamlyacc *<declarations>*

- **%type** *<type> symbol ... symbol*

Specify type of attributes for given symbols.

Mandatory for start symbols

- **%left** *symbol ... symbol*

- **%right** *symbol ... symbol*

- **%nonassoc** *symbol ... symbol*

Associate precedence and associativity to given symbols. Same line, same precedence; earlier line, lower precedence (broadest scope)



Ocamlyacc *<rules>*

- *nonterminal* :

symbol ... symbol { semantic_action }

| ...

| *symbol ... symbol { semantic_action }*

;

- Semantic actions are arbitrary Ocaml expressions
- Must be of same type as declared (or inferred) for *nonterminal*
- Access semantic attributes (values) of symbols by position: \$1 for first symbol, \$2 to second ...



Example - Base types

```
(* File: expr.ml *)
```

```
type expr =
```

```
  Term_as_Expr of term
```

```
  | Plus_Expr of (term * expr)
```

```
  | Minus_Expr of (term * expr)
```

```
and term =
```

```
  Factor_as_Term of factor
```

```
  | Mult_Term of (factor * term)
```

```
  | Div_Term of (factor * term)
```

```
and factor =
```

```
  Id_as_Factor of string
```

```
  | Parenthesized_Expr_as_Factor of expr
```




Example - Lexer (exprlex.mll)

```
{ (*open Exprparse*) }
let numeric = ['0' - '9']
let letter = ['a' - 'z' 'A' - 'Z']
rule token = parse
| "+" {Plus_token}
| "-" {Minus_token}
| "*" {Times_token}
| "/" {Divide_token}
| "(" {Left_parenthesis}
| ")" {Right_parenthesis}
| letter (letter|numeric|"_" )* as id {Id_token id}
| [' ' '\t' '\n'] {token lexbuf}
| eof {EOL}
```



Example - Parser (exprparse.mly)

```
%{ open Expr
```

```
%}
```

```
%token <string> Id_token
```

```
%token Left_parenthesis Right_parenthesis
```

```
%token Times_token Divide_token
```

```
%token Plus_token Minus_token
```

```
%token EOL
```

```
%start main
```

```
%type <expr> main
```

```
%%
```



Example - Parser (exprparse.mly)

expr:

term

{ Term_as_Expr \$1 }

| term Plus_token expr

{ Plus_Expr (\$1, \$3) }

| term Minus_token expr

{ Minus_Expr (\$1, \$3) }



Example - Parser (exprparse.mly)

term:

factor

{ Factor_as_Term \$1 }

| factor Times_token term

{ Mult_Term (\$1, \$3) }

| factor Divide_token term

{ Div_Term (\$1, \$3) }



Example - Parser (exprparse.mly)

factor:

Id_token

{ Id_as_Factor \$1 }

| Left_parenthesis expr Right_parenthesis

{ Parenthesized_Expr_as_Factor \$2 }

main:

| expr EOL

{ \$1 }



Example - Using Parser

```
# #use "expr.ml";;
```

```
...
```

```
# #use "exprparse.ml";;
```

```
...
```

```
# #use "exprlex.ml";;
```

```
...
```

```
# let test s =
```

```
  let lexbuf = Lexing.from_string (s^"\n") in  
    main token lexbuf;;
```



Example - Using Parser

```
# test "a + b";;
```

```
- : expr =
```

```
Plus_Expr
```

```
(Factor_as_Term (Id_as_Factor "a"),  
Term_as_Expr (Factor_as_Term  
  (Id_as_Factor "b")))
```



Disambiguating a Grammar

- Idea: Each non-terminal represents all strings having some property
- Identify these properties (often in terms of things that can't happen)
- Use these properties to inductively guarantee every string in language has a unique parse

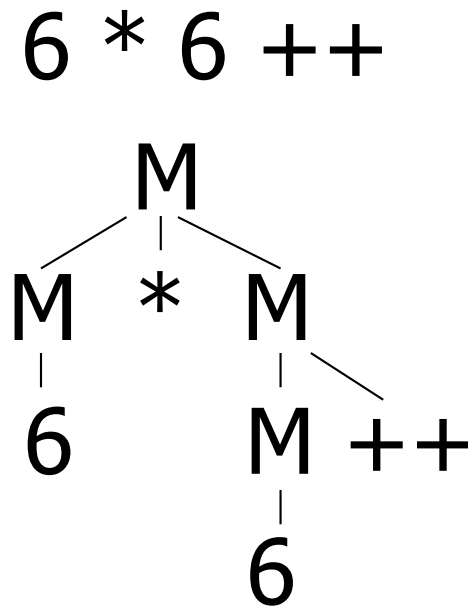
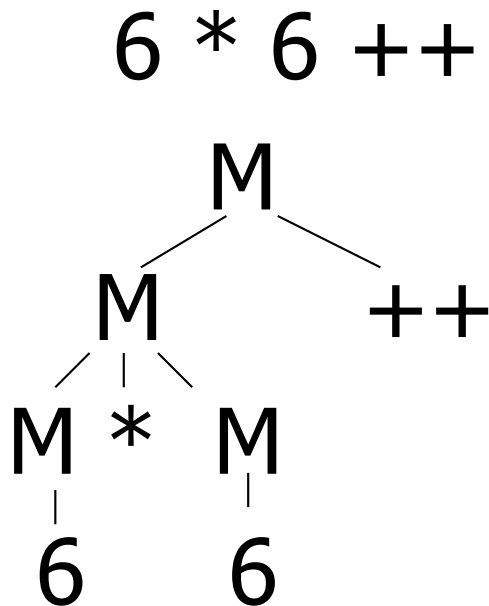


Steps to Grammar Disambiguation

- Identify the rules and a smallest use that display ambiguity
- Decide which parse to keep; why should others be thrown out?
- What syntactic restrictions on subexpressions are needed to throw out the bad (while keeping the good)?
- Add a new non-terminal and rules to describe this set of restricted subexpressions (called stratifying, or refactoring)
- **Characterize each non-terminal by a language invariant**
- Replace old rules to use new non-terminals
- Rinse and repeat

More Disambiguating Grammars

- $M ::= M * M \mid (M) \mid M ++ \mid 6$
- Ambiguous because of associativity of $*$
- because of conflict between $*$ and $++$:





$M ::= M * M \mid (M) \mid M ++ \mid 6$

- How to disambiguate?
- Choose associativity for $*$
- Choose precedence between $*$ and $++$
- Four possibilities
- Four different approaches
- Some easier than others
- Will do --- You choose


$$M ::= M * M \mid (M) \mid M ++ \mid 6$$

- Think about $6 * 6 ++ * 6 * 6 ++$
- Let's start with observations
- If $*$ binds less tightly than $++$, then no $*$ can be the immediate subtree to a $++$.
 - We would need a language for things that don't parse as $*$
- If $*$ binds more tightly than $++$, then ...
- The right subtree to $*$ can't be a $++$
- But the left can!
 - Need different languages of the left and right



$M ::= M * M \mid (M) \mid M ++ \mid 6$

- * higher prec than ++
 - $6 * 6 ++ \quad 6 ++ * 6$
- $M ::= M ++ \mid \text{StarExp} \mid (M) \mid 6$
- What is **StarExp**
- It is everything that parses as a * and can't parse as a ++
- But what is the associativity of *?
- Class chose left


$$M ::= M * M \mid (M) \mid M ++ \mid 6$$

- * higher prec than ++
 - $6 * 6 ++ \quad 6 ++ * 6$
- * Left assoc
- $M ::= M ++ \mid \text{StarExp} \mid (M) \mid 6$
- $\text{StarExp} ::= \text{PossStar} * \text{NoStarNoPlusPlus}$
- What is **PossStar**? It could it be a *****, but it also doesn't have to be.
- Can it be **++**? YES! It can be anything
- It is **M** !



$M ::= M * M \mid (M) \mid M ++ \mid 6$

- * higher prec than ++
 - $6 * 6 ++ \quad 6 ++ * 6$
- * Left assoc
- $M ::= M++ \mid \text{StarExp} \mid (M) \mid 6$
- $\text{StarExp} ::= M * \text{NoStarNoPlusPlus}$


$$M ::= M * M \mid (M) \mid M ++ \mid 6$$

- * higher prec than ++
 - $6 * 6 ++ \quad 6 ++ * 6$
- * Left assoc
- $M ::= M ++ \mid \text{StarExp} \mid (M) \mid 6$
- $\text{StarExp} ::= M * \text{NoStarNoPlusPlus}$
- But what is **NoStarNoPlusPlus**?
- Well, the other two original rules: $(M) \mid 6$


$$M ::= M * M \mid (M) \mid M ++ \mid 6$$

- * higher prec than ++
 - $6 * 6 ++ \quad 6 ++ * 6$
- * Left assoc
- $M ::= M ++ \mid \text{StarExp} \mid (M) \mid 6$
- $\text{StarExp} ::= M * \text{NoStarNoPlusPlus}$
- $\text{NoStarNoPlusPlus} ::= (M) \mid 6$
- But we have $(M) \mid 6$ twice, and it's the same language each time. Let's have one



$M ::= M * M \mid (M) \mid M ++ \mid 6$

- * higher prec than ++
 - $6 * 6 ++ \quad 6 ++ * 6$
- * Left assoc
- $M ::= M++ \mid \text{StarExp} \mid \text{NoStarNoPlusPlus}$
- $\text{StarExp} ::= M * \text{NoStarNoPlusPlus}$
- $\text{NoStarNoPlusPlus} ::= (M) \mid 6$



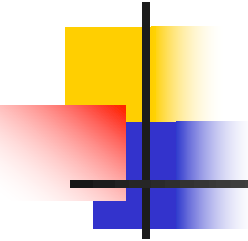
$M ::= M * M \mid (M) \mid M ++ \mid 6$

- * higher prec than ++
 - $6 * 6 ++ \quad 6 ++ * 6$
- * Left assoc
- $M ::= M ++ \mid \text{StarExp} \mid \text{NoStarNoPlusPlus}$
- $\text{StarExp} ::= M * \text{NoStarNoPlusPlus}$
- $\text{NoStarNoPlusPlus} ::= (M) \mid 6$



LR Parsing

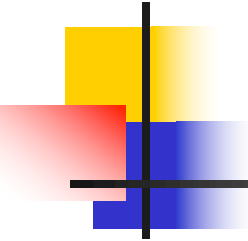
- Read tokens left to right (L)
- Create a rightmost derivation (R)
- How is this possible?
- Start at the bottom (left) and work your way up
- Last step has only one non-terminal to be replaced so is right-most
- Working backwards, replace mixed strings by non-terminals
- Always proceed so that there are no non-terminals to the right of the string to be replaced



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= \bullet (0 + 1) + 0$ shift



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

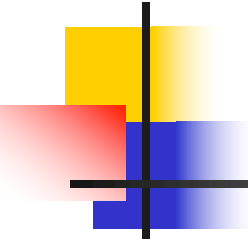
$\langle \text{Sum} \rangle \Rightarrow$

$$= (\bullet 0 + 1) + 0$$

$$= \bullet (0 + 1) + 0$$

shift

shift



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$$\Rightarrow (0 \bullet + 1) + 0$$

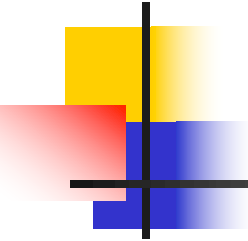
$$= (\bullet 0 + 1) + 0$$

$$= \bullet (0 + 1) + 0$$

reduce

shift

shift



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$$= (\langle \text{Sum} \rangle \bullet + 1) + 0$$

shift

$$\Rightarrow (0 \bullet + 1) + 0$$

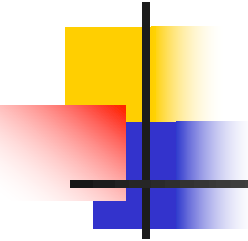
reduce

$$= (\bullet 0 + 1) + 0$$

shift

$$= \bullet (0 + 1) + 0$$

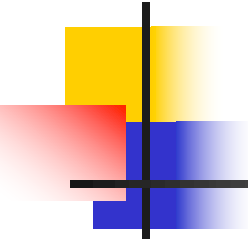
shift



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$=$	$(\langle \text{Sum} \rangle + \bullet 1) + 0$	shift
$=$	$(\langle \text{Sum} \rangle \bullet + 1) + 0$	shift
\Rightarrow	$(0 \bullet + 1) + 0$	reduce
$=$	$(\bullet 0 + 1) + 0$	shift
$=$	$\bullet (0 + 1) + 0$	shift



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$	reduce
$= (\langle \text{Sum} \rangle + \bullet 1) + 0$	shift
$= (\langle \text{Sum} \rangle \bullet + 1) + 0$	shift
$\Rightarrow (0 \bullet + 1) + 0$	reduce
$= (\bullet 0 + 1) + 0$	shift
$= \bullet (0 + 1) + 0$	shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= (\langle \text{Sum} \rangle \bullet) + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$ reduce
 $= (\langle \text{Sum} \rangle \bullet) + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= \langle \text{Sum} \rangle \bullet + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$ reduce
 $= (\langle \text{Sum} \rangle \bullet) + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
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 $= (\bullet 0 + 1) + 0$ shift
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Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= \langle \text{Sum} \rangle + \bullet 0$ shift
 $= \langle \text{Sum} \rangle \bullet + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$ reduce
 $= (\langle \text{Sum} \rangle \bullet) + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

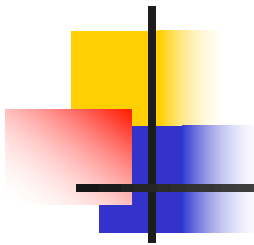
$\langle \text{Sum} \rangle \Rightarrow$
 $\Rightarrow \langle \text{Sum} \rangle + 0 \bullet$ reduce
 $= \langle \text{Sum} \rangle + \bullet 0$ shift
 $= \langle \text{Sum} \rangle \bullet + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$ reduce
 $= (\langle \text{Sum} \rangle \bullet) + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle$	$\Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$	●	reduce
	$\Rightarrow \langle \text{Sum} \rangle + 0$	●	reduce
	$= \langle \text{Sum} \rangle +$	● 0	shift
	$= \langle \text{Sum} \rangle$	● + 0	shift
	$\Rightarrow (\langle \text{Sum} \rangle)$	● + 0	reduce
	$= (\langle \text{Sum} \rangle$	●) + 0	shift
	$\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle$	●) + 0	reduce
	$\Rightarrow (\langle \text{Sum} \rangle + 1$	●) + 0	reduce
	$= (\langle \text{Sum} \rangle +$	● 1) + 0	shift
	$= (\langle \text{Sum} \rangle$	● + 1) + 0	shift
	$\Rightarrow (0$	● + 1) + 0	reduce
	$= ($	● 0 + 1) + 0	shift
	$=$	● (0 + 1) + 0	shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle$	●	\Rightarrow	$\langle \text{Sum} \rangle + \langle \text{Sum} \rangle$	●	reduce	
		\Rightarrow	$\langle \text{Sum} \rangle + 0$	●	reduce	
		=	$\langle \text{Sum} \rangle +$	●	0	shift
		=	$\langle \text{Sum} \rangle$	●	+ 0	shift
		\Rightarrow	$(\langle \text{Sum} \rangle)$	●	+ 0	reduce
		=	$(\langle \text{Sum} \rangle$	●) + 0	shift
		\Rightarrow	$(\langle \text{Sum} \rangle + \langle \text{Sum} \rangle$	●) + 0	reduce
		\Rightarrow	$(\langle \text{Sum} \rangle + 1$	●) + 0	reduce
		=	$(\langle \text{Sum} \rangle +$	●	1) + 0	shift
		=	$(\langle \text{Sum} \rangle$	●	+ 1) + 0	shift
		\Rightarrow	$(0$	●	+ 1) + 0	reduce
		=	$($	●	0 + 1) + 0	shift
		=	●	$(0 + 1) + 0$	shift	



Example

$$(0 + 1) + 0$$





Example

$$(0 + 1) + 0$$





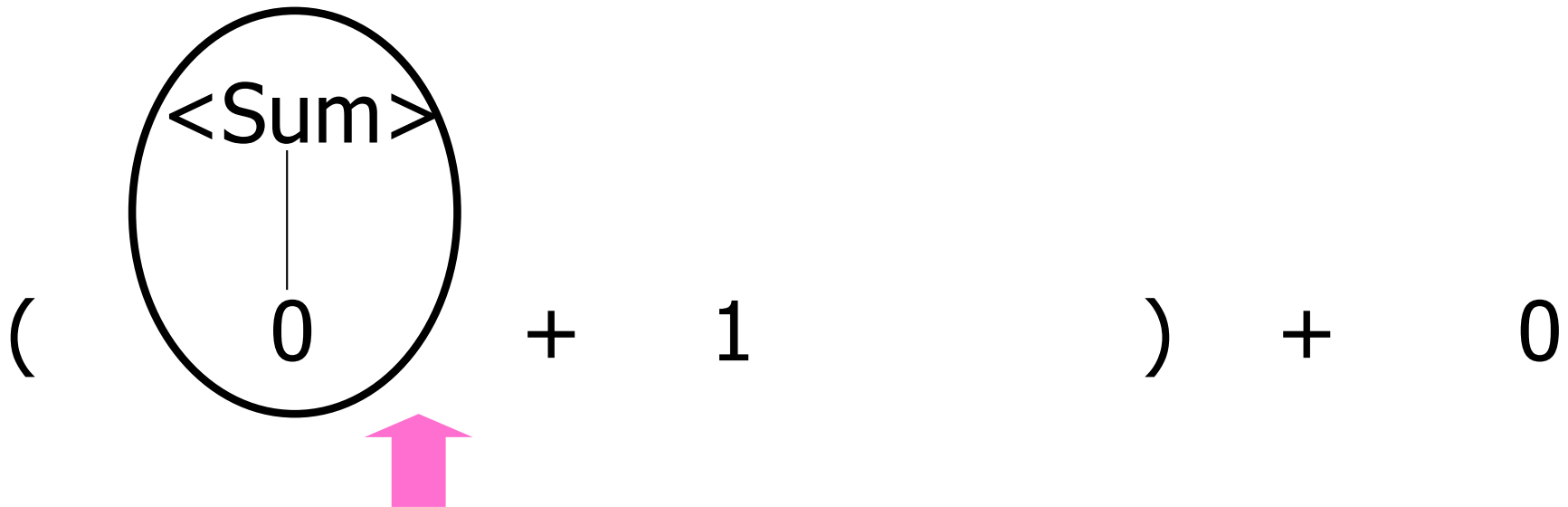
Example

$$(0 + 1) + 0$$



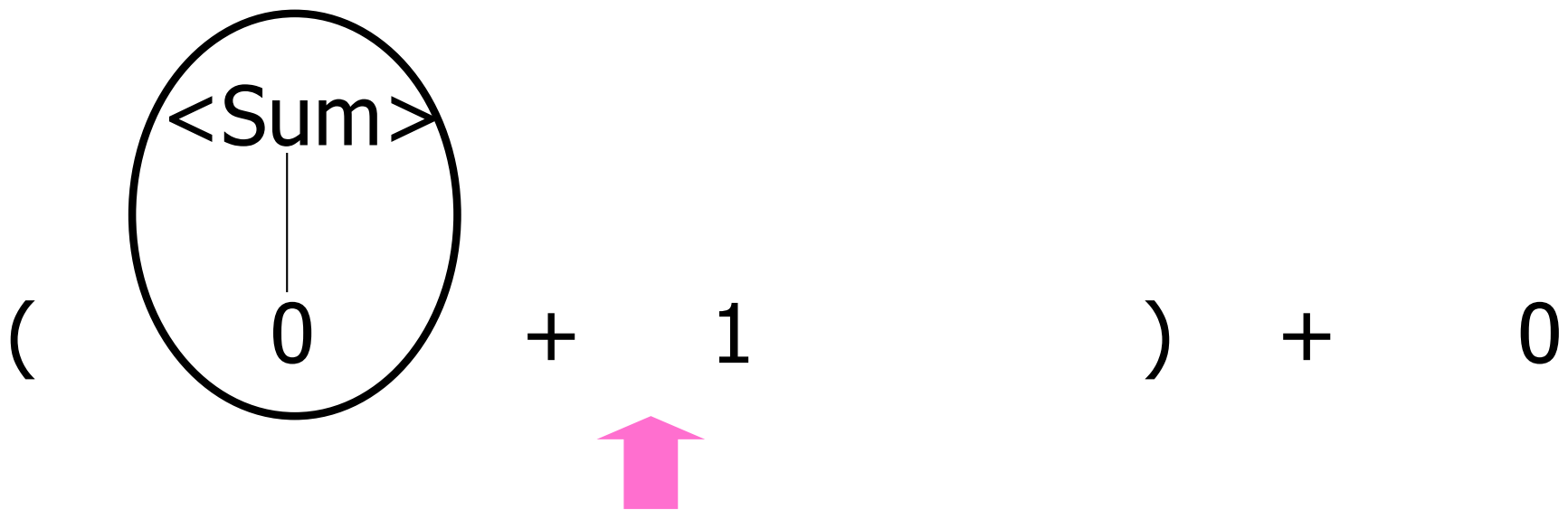


Example



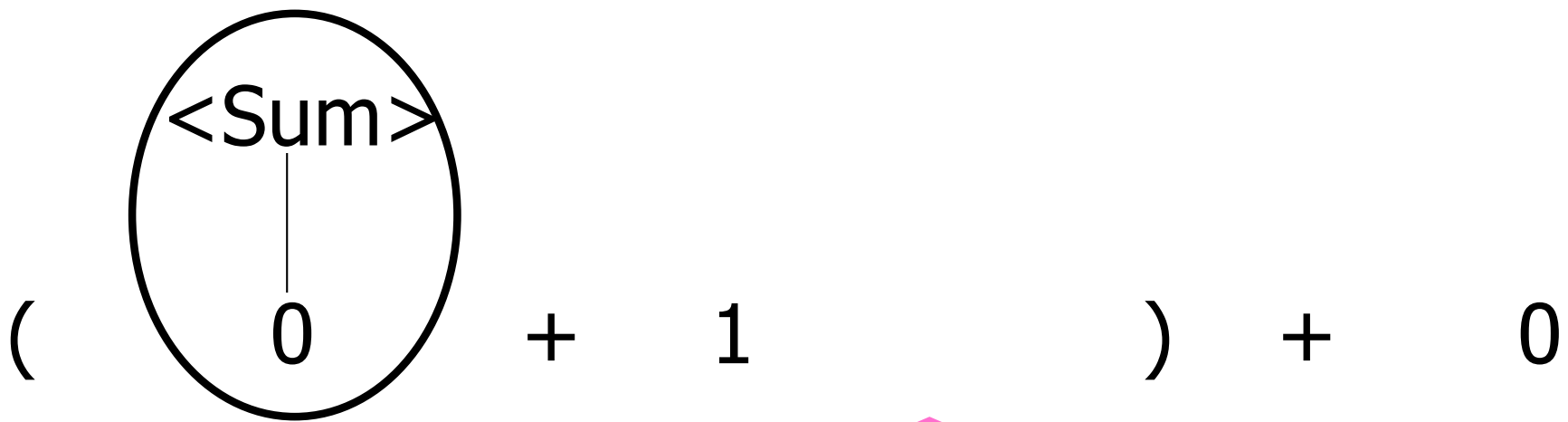


Example



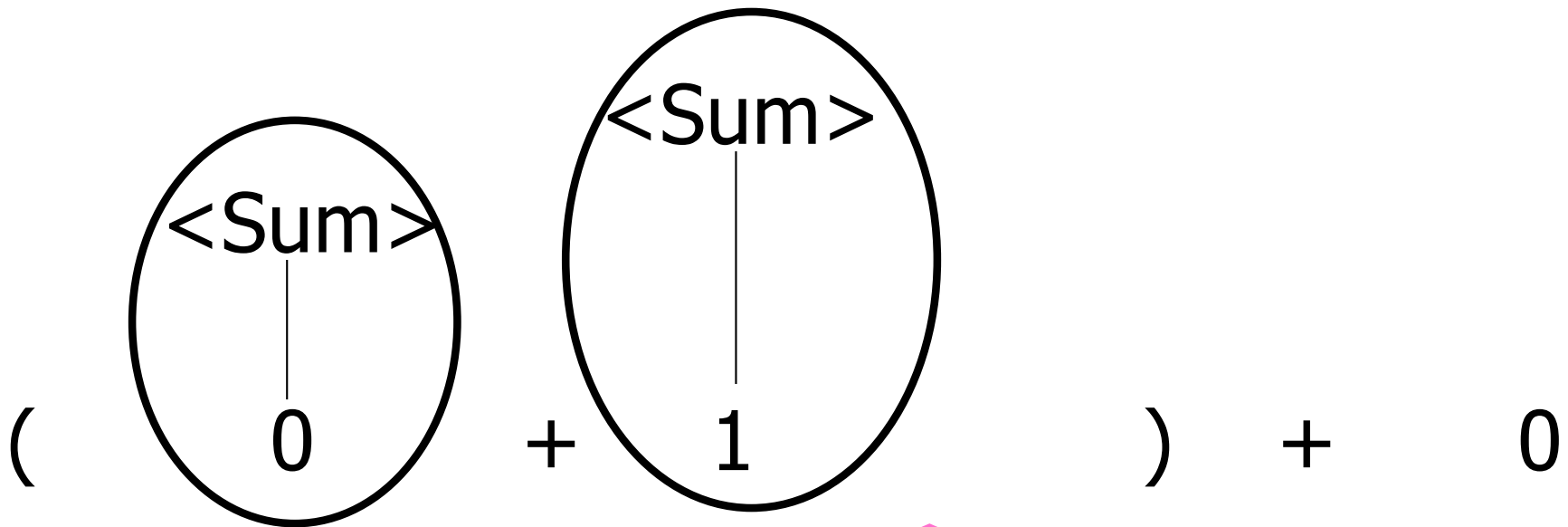


Example

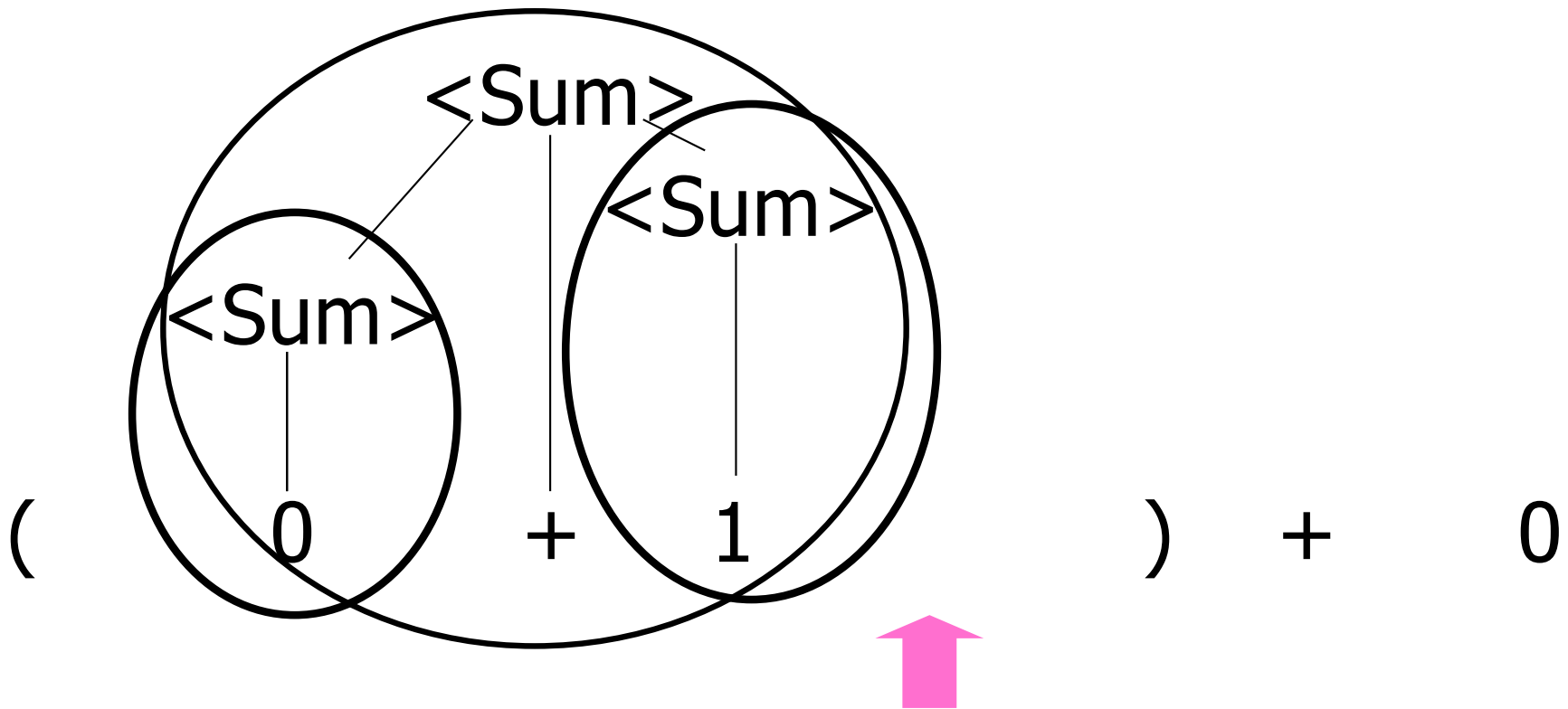




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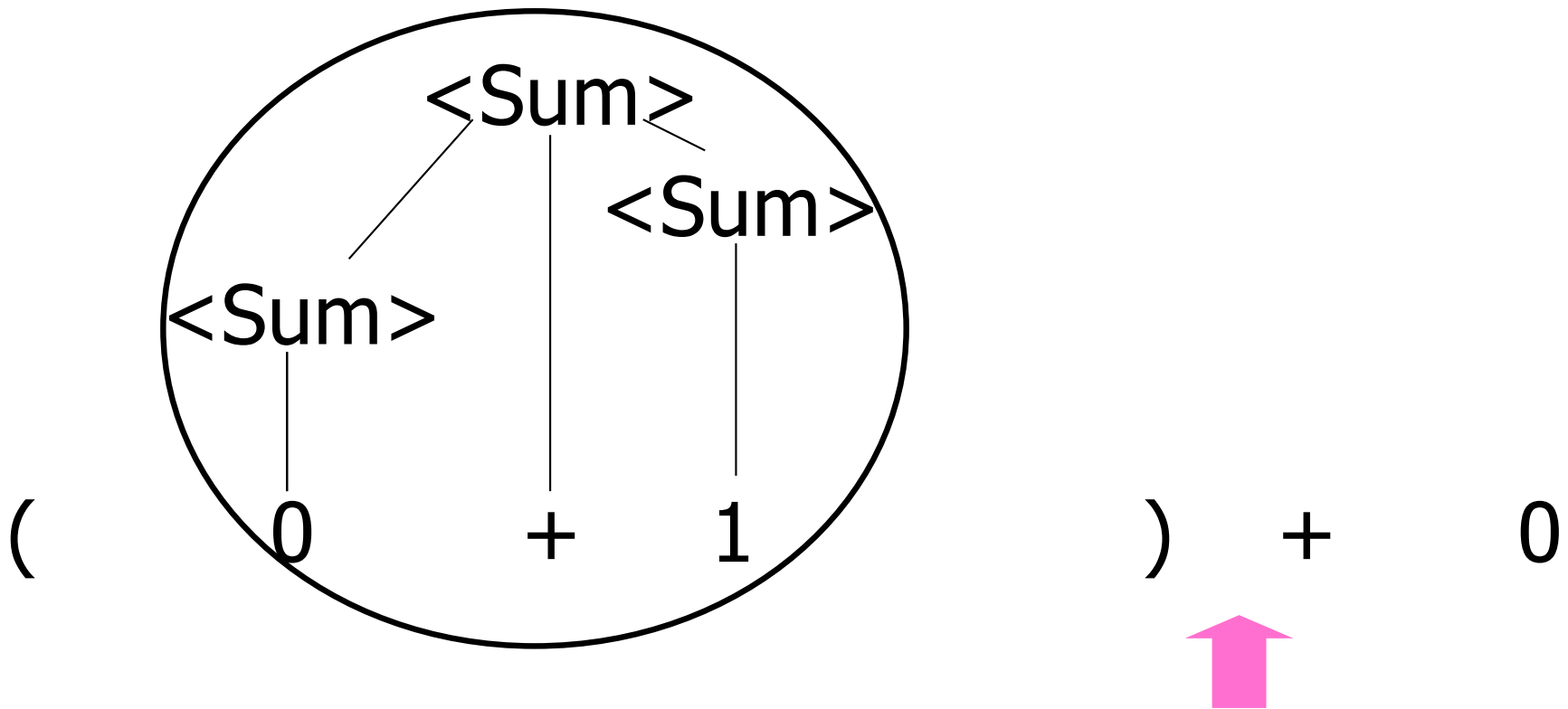


Example



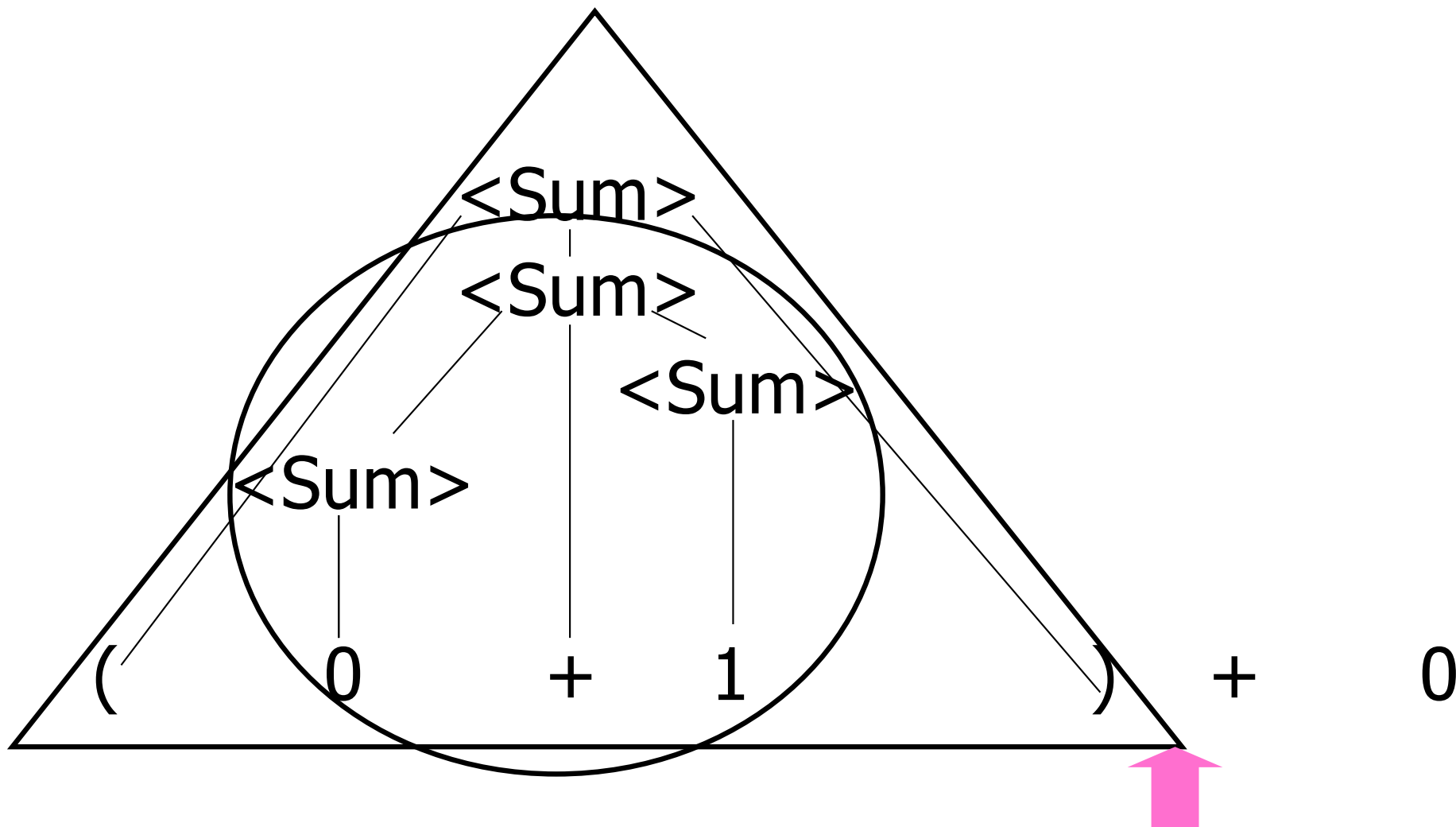


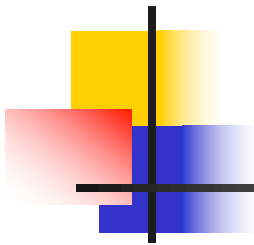
Example



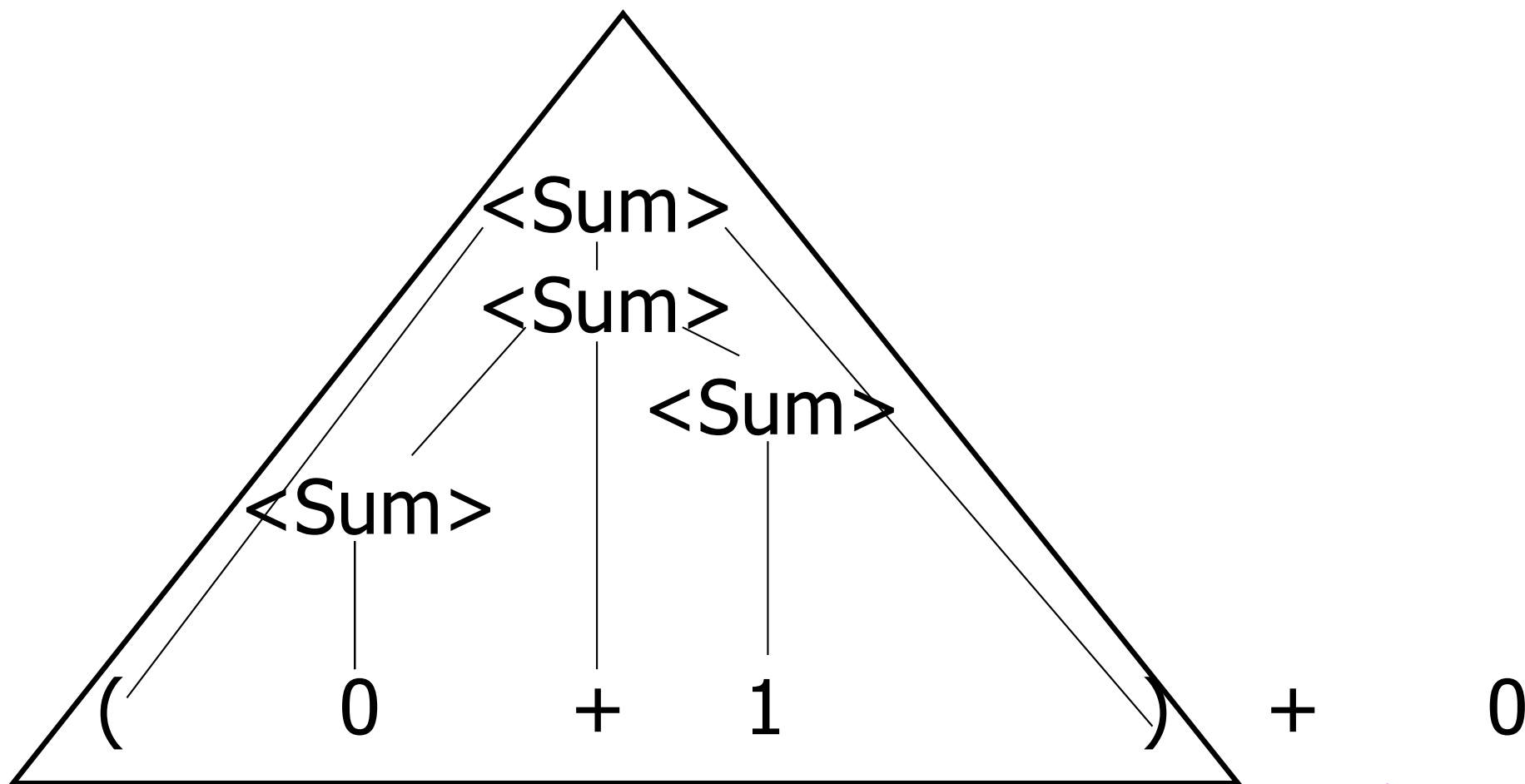


Example



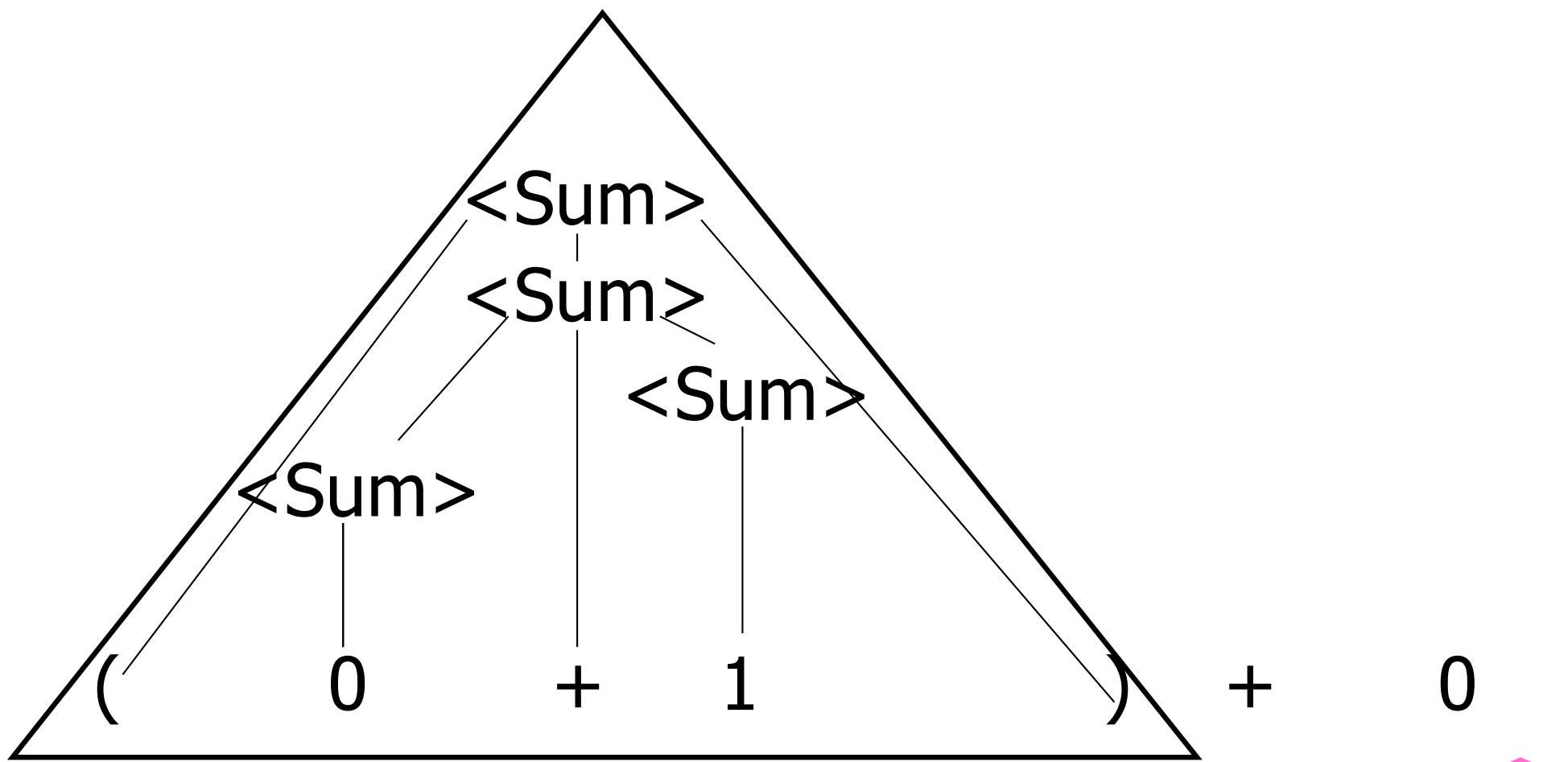


Example



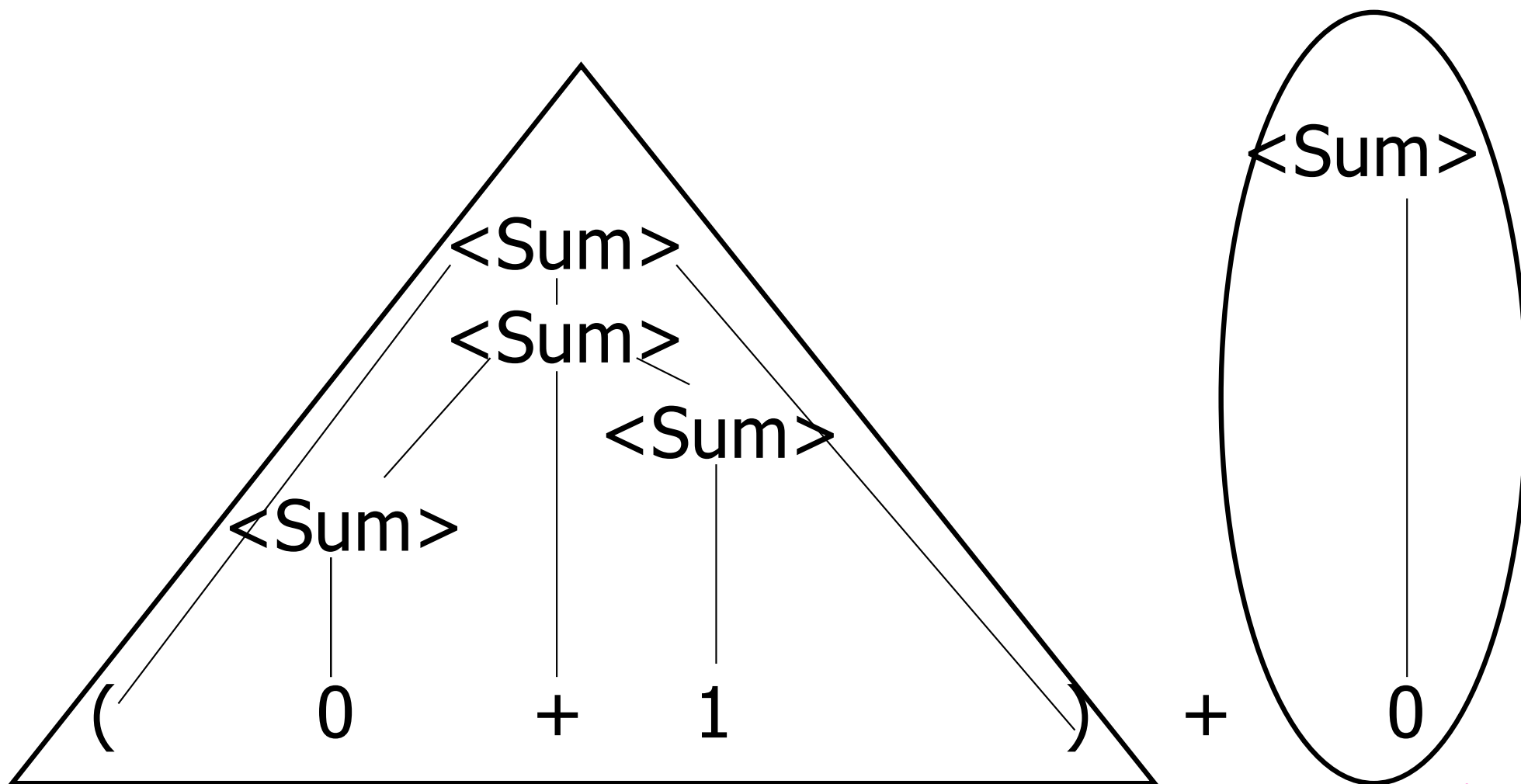


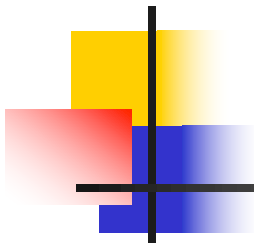
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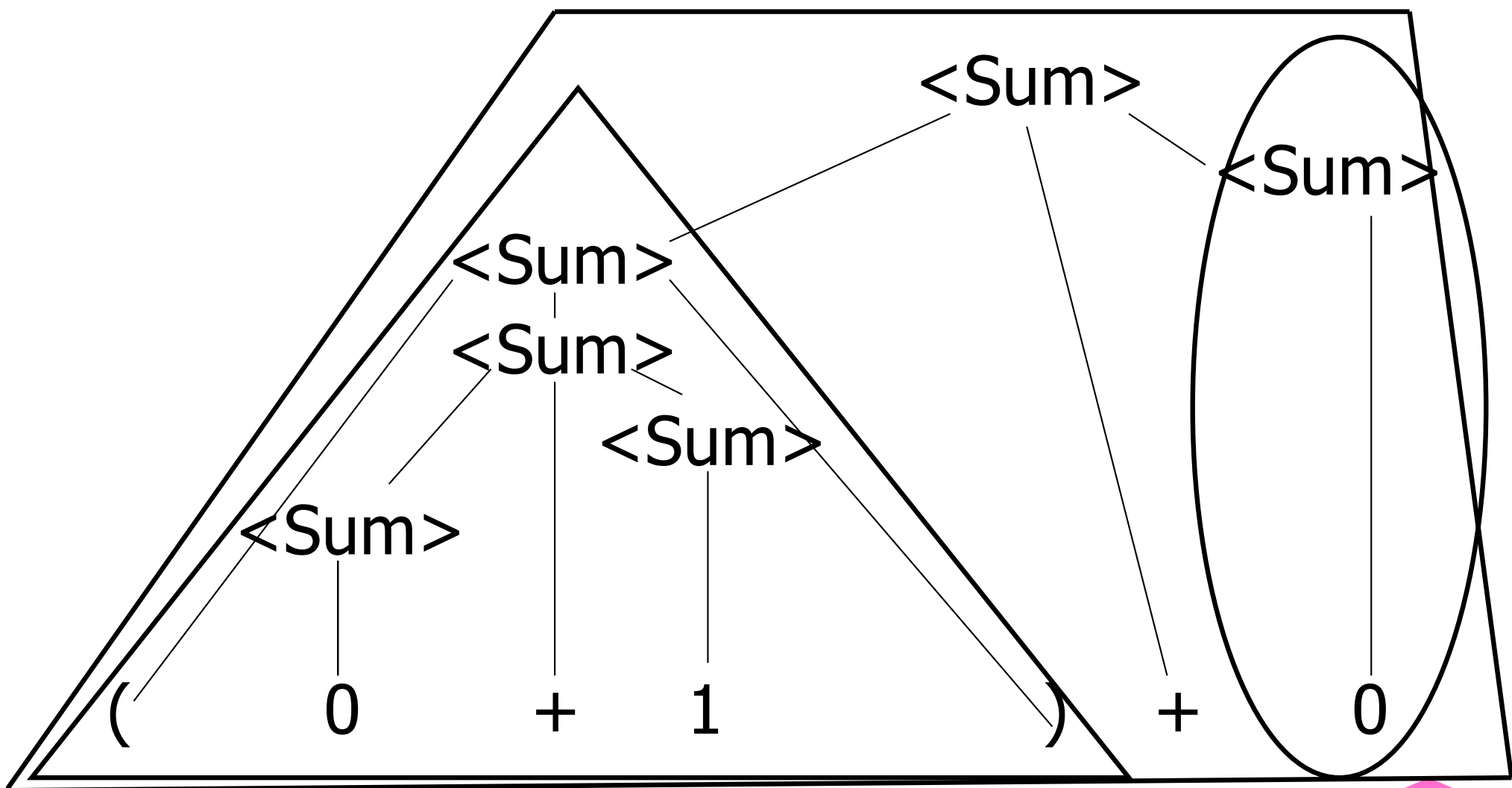


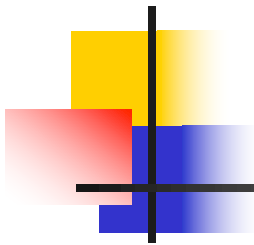
Example



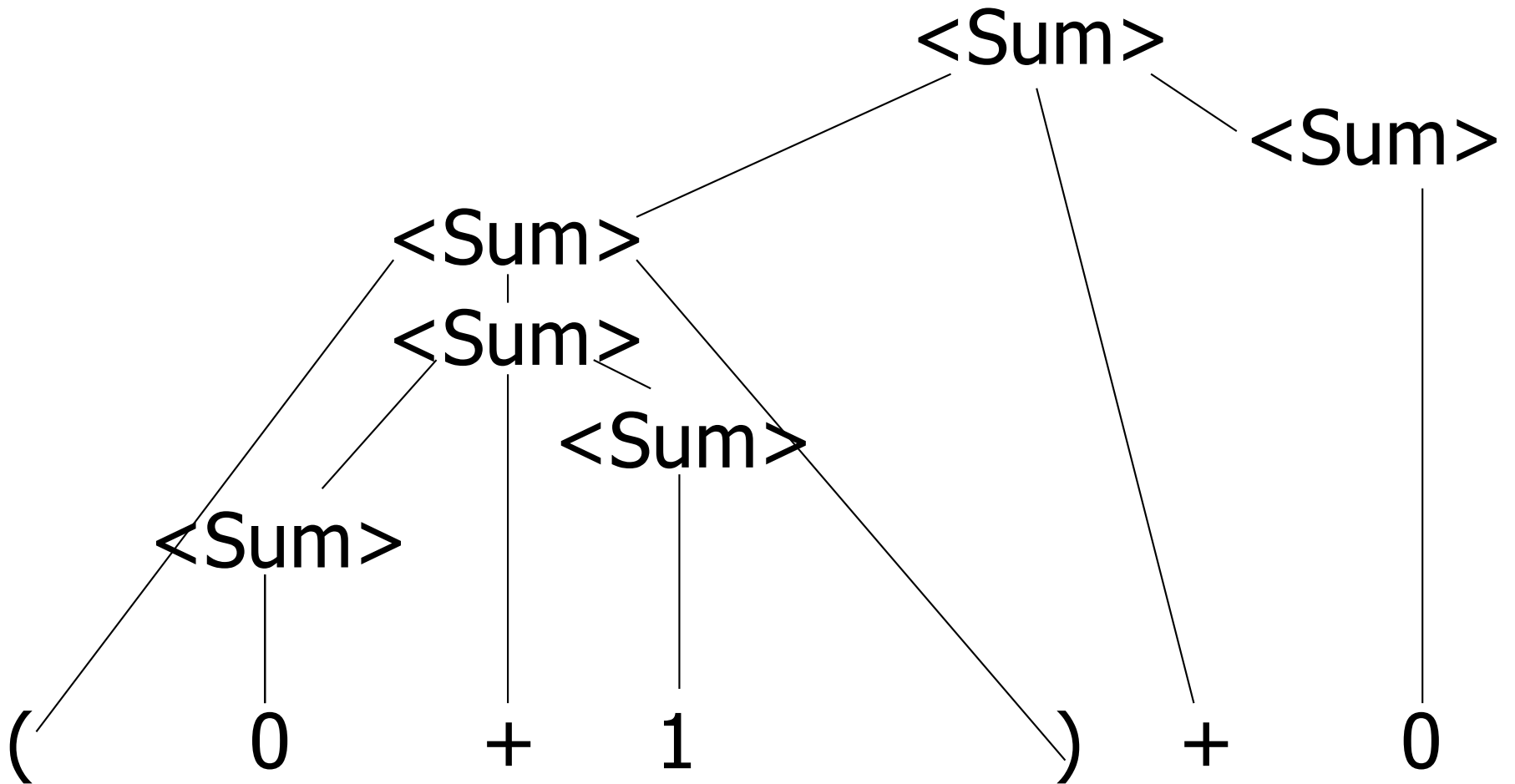


Example





Example





LR Parsing Tables

- Build a pair of tables, Action and Goto, from the grammar
 - This is the hardest part, we omit here
 - Rows labeled by states
 - For Action, columns labeled by terminals and “end-of-tokens” marker
 - (more generally strings of terminals of fixed length)
 - For Goto, columns labeled by non-terminals



Action and Goto Tables

- Given a state and the next input, Action table says either
 - **shift** and go to state n , or
 - **reduce** by production k (explained in a bit)
 - **accept** or **error**
- Given a state and a non-terminal, Goto table says
 - go to state m



LR(i) Parsing Algorithm

- Based on push-down automata
- Uses states and transitions (as recorded in Action and Goto tables)
- Uses a stack containing states, terminals and non-terminals



LR(i) Parsing Algorithm

0. Insure token stream ends in special “end-of-tokens” symbol
1. Start in state 1 with an empty stack
2. Push **state**(1) onto stack
- 3. Look at next i tokens from token stream ($toks$) (don't remove yet)
4. If top symbol on stack is **state**(n), look up action in Action table at ($n, toks$)



LR(i) Parsing Algorithm

5. If action = **shift** m ,

- a) Remove the top token from token stream and push it onto the stack
- b) Push **state**(m) onto stack
- c) Go to step 3



LR(i) Parsing Algorithm

6. If action = **reduce** k where production k is
 $E ::= u$
- a) Remove $2 * \text{length}(u)$ symbols from stack (u and all the interleaved states)
 - b) If new top symbol on stack is **state**(m), look up new state p in $\text{Goto}(m, E)$
 - c) Push E onto the stack, then push **state**(p) onto the stack
 - d) Go to step 3



LR(i) Parsing Algorithm

7. If action = **accept**

- Stop parsing, return success

8. If action = **error**,

- Stop parsing, return failure



Adding Synthesized Attributes

- Add to each **reduce** a rule for calculating the new synthesized attribute from the component attributes
- Add to each non-terminal pushed onto the stack, the attribute calculated for it
- When performing a **reduce**,
 - gather the recorded attributes from each non-terminal popped from stack
 - Compute new attribute for non-terminal pushed onto stack



Shift-Reduce Conflicts

- **Problem:** can't decide whether the action for a state and input character should be **shift** or **reduce**
- Caused by ambiguity in grammar
- Usually caused by lack of associativity or precedence information in grammar

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\bullet 0 + 1 + 0$ shift
 $\rightarrow 0 \bullet + 1 + 0$ reduce
 $\rightarrow \langle \text{Sum} \rangle \bullet + 1 + 0$ shift
 $\rightarrow \langle \text{Sum} \rangle + \bullet 1 + 0$ shift
 $\rightarrow \langle \text{Sum} \rangle + 1 \bullet + 0$ reduce
 $\rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet + 0$



Example - cont

- **Problem:** shift or reduce?
- You can shift-shift-reduce-reduce or reduce-shift-shift-reduce
- Shift first - right associative
- Reduce first- left associative



Reduce - Reduce Conflicts

- **Problem:** can't decide between two different rules to reduce by
- Again caused by ambiguity in grammar
- **Symptom:** RHS of one production suffix of another
- Requires examining grammar and rewriting it
- Harder to solve than shift-reduce errors



Example

■ $S ::= A \mid aB$ $A ::= abc$ $B ::= bc$

● abc shift

a ● bc shift

ab ● c shift

abc ●

■ Problem: reduce by $B ::= bc$ then by S
 $::= aB$, or by $A ::= abc$ then $S ::= A$?