Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Interpreter

- Takes abstract syntax trees as input
  - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
  - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next "state"
  - To get final value, put in a loop

Natural Semantics Example

- compute_exp (Var(v), m) = look_up v m
- compute_exp (Int(n), _) = Num (n)
- …
- compute_com(IfExp(b,c1,c2),m) =

```
    if compute_exp (b,m) = Bool(true)
    then compute_com (c1,m)
    else compute_com (c2,m)
```


Natural Semantics Example

- compute_com(While(b,c), m) =

```
  if compute_exp (b,m) = Bool(false)
  then m
  else compute_com
         (While(b,c), compute_com(c,m))
```

- May fail to terminate - exceed stack limits
- Returns no useful information then

Transition Semantics

- Form of operational semantics
- Describes how each program construct transforms machine state by *transitions*
- Rules look like
  (C, *m*) --> (C', *m'*) or (C,*m*) --> *m'*
- *C, C'* is code remaining to be executed
- m, m' represent the state/store/memory/ environment
  - Partial mapping from identifiers to values
  - Sometimes *m* (or *C*) not needed
- Indicates exactly one step of computation

Expressions and Values

- *C, C'* used for commands; *E, E' for* expressions; *U,V* for values
- Special class of expressions designated as *values*
  - Eg 2, 3 are values, but 2+3 is only an expression
- Memory only holds values
  - Other possibilities exist

Evaluation Semantics

- Transitions successfully stops when E/*C* is a value/memory
- Evaluation fails if no transition possible, but not at value/memory
- Value/memory is the final *meaning* of original expression/command (in the given state)
- Coarse semantics: final value / memory
- More fine grained: whole transition sequence

Simple Imperative Programming Language

- *I* ∈ *Identifiers*
- *N* ∈ *Numerals*
- *B* ::= true | false | *B* & *B* | *B* or *B* | not *B* | *E* < *E* | *E* = *E*
- *E::= N* | *I* | *E* + *E* | *E* * *E* | *E* - *E* | - *E*
- C::= skip | *C*;*C* | *I* ::= *E*
  | if *B* then *C* else *C* fi | while *B* do *C* od

Transitions for Expressions

- Numerals are values
- Boolean values = {true, false}
- Identifiers: (*I*,*m*) --> (*m*(*I*), *m*)

Boolean Operations:

- Operators: (short-circuit)

(false & *B*, *m*) --> (false,*m*)
(true & *B*, *m*) --> (*B*,*m*)

$$\frac{(B,\ m) \text{ --> } (B'',\ m)}{(B \,\&\, B',\ m) \text{ --> } (B'' \,\&\, B',\ m)}$$

(true or *B*, *m*) --> (true,*m*)
(false or *B*, *m*) --> (*B*,*m*)

$$\frac{(B,\ m) \text{ --> } (B'',\ m)}{(B \text{ or } B',\ m) \text{ --> } (B'' \text{ or } B',m)}$$

(not true, m) --> (false,*m*)
(not false, m) --> (true,*m*)

$$\frac{(B,\ m) \text{ --> } (B',\ m)}{(\text{not } B,\ m) \text{ --> } (\text{not } B',\ m)}$$

Relations

$$\frac{(E,\ m) \text{ --> } (E'',m)}{(E \sim E',\ m) \text{ --> } (E''\sim E',m)}$$

$$\frac{(E,\ m) \text{ --> } (E',m)}{(V \sim E,\ m) \text{ --> } (V\sim E',m)}$$

(*U* ~ *V*, *m*) --> (true,*m*) or (false,*m*)
depending on whether *U* ~ *V* holds or not

Arithmetic Expressions

$$\frac{(E,\ m) \text{ --> } (E'',m)}{(E \text{ op } E',\ m) \text{ --> } (E'' \text{ op } E',m)}$$

$$\frac{(E,\ m) \text{ --> } (E',m)}{(V \text{ op } E,\ m) \text{ --> } (V \text{ op } E',m)}$$

(*U op V*, *m*) --> (*N*,*m*) where *N* is the specified value for *U op V*

Commands - in English

- skip means done evaluating
- When evaluating an assignment, evaluate the expression first
- If the expression being assigned is already a value, update the memory with the new value for the identifier
- When evaluating a sequence, work on the first command in the sequence first
- If the first command evaluates to a new memory (ie completes), evaluate remainder with new memory

Commands

(skip, *m*) --> *m*

$$\frac{(E,m) \dashrightarrow (E',m)}{(I::=E,m) \dashrightarrow (I::=E',m)}$$

(*I*::=*V*,*m*) --> *m*[*I* <-- *V*]

$$\frac{(C,m) \dashrightarrow (C'',m')}{(C;C',\ m) \dashrightarrow (C'';C',m')} \qquad \frac{(C,m) \dashrightarrow m'}{(C;C',\ m) \dashrightarrow (C',m')}$$

If Then Else Command - in English

- If the boolean guard in an if_then_else is true, then evaluate the first branch
- If it is false, evaluate the second branch
- If the boolean guard is not a value, then start by evaluating it first.

If Then Else Command

(if true then *C* else *C'* fi, *m*) --> (*C*, *m*)

(if false then *C* else *C'* fi, *m*) --> (*C'*, *m*)

$$\frac{(B,m) \dashrightarrow (B',m)}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi},\ m) \dashrightarrow (\text{if } B' \text{ then } C \text{ else } C' \text{ fi},\ m)}$$

While Command

(while *B* do *C* od, *m*) -->
(if *B* then *C*; while *B* do *C* od else skip fi, m)

In English: Expand a While into a test of the boolean guard, with the true case being to do the body and then try the while loop again, and the false case being to stop.

Example Evaluation

- First step:

$$\frac{}{(\text{if } x > 5 \text{ then } y:= 2 + 3 \text{ else } y:=3 + 4 \text{ fi},\ \{x \to 7\}) \dashrightarrow ?}$$

Example Evaluation

- First step:

```
          (x > 5, {x -> 7}) --> ?
---------------------------------------------
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
                 {x -> 7})
                  --> ?
```


Example Evaluation

- First step:

```
      (x,{x -> 7}) --> (7, {x -> 7})
     ---------------------------------
          (x > 5, {x -> 7}) --> ?
---------------------------------------------
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
                 {x -> 7})
                  --> ?
```


Example Evaluation

- First step:

```
      (x,{x -> 7}) --> (7, {x -> 7})
     ---------------------------------
(x > 5, {x -> 7}) --> (7 > 5, {x -> 7})
---------------------------------------------
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
                 {x -> 7})
                  --> ?
```


Example Evaluation

- First step:

```
      (x,{x -> 7}) --> (7, {x -> 7})
     ---------------------------------
(x > 5, {x -> 7}) --> (7 > 5, {x -> 7})
---------------------------------------------
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
                 {x -> 7})
--> (if 7 > 5 then y:=2 + 3 else y:=3 + 4 fi,
                 {x -> 7})
```


Example Evaluation

- Second Step:

```
   (7 > 5, {x -> 7}) --> (true, {x -> 7})
---------------------------------------------
  (if 7 > 5 then y:=2 + 3 else y:=3 + 4 fi,
                 {x -> 7})
--> (if true then y:=2 + 3 else y:=3 + 4 fi,
                 {x -> 7})
```

- Third Step:

```
(if true then y:=2 + 3 else y:=3 + 4 fi, {x -> 7})
            -->(y:=2+3, {x->7})
```


Example Evaluation

- Fourth Step:

```
   (2+3, {x-> 7}) --> (5, {x -> 7})
--------------------------------------
(y:=2+3, {x->7}) --> (y:=5, {x->7})
```

- Fifth Step:

```
(y:=5, {x->7}) --> {y -> 5, x -> 7}
```

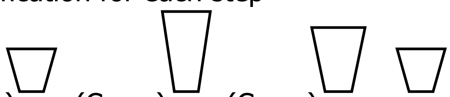
Example Evaluation

- Bottom Line:

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7})
--> (if 7 > 5 then y:=2 + 3 else y:=3 + 4 fi,
{x -> 7})
-->(if true then y:=2 + 3 else y:=3 + 4 fi,
{x -> 7})
-->(y:=2+3, {x->7})
--> (y:=5, {x->7}) --> {y -> 5, x -> 7}

Transition Semantics Evaluation

- A sequence of steps with trees of justification for each step



(C_1,m_1) --> (C_2,m_2) --> (C_3,m_3) --> … --> m

- Let -->* be the transitive closure of -->
- Ie, the smallest transitive relation containing -->

Adding Local Declarations

- Add to expressions:
- *E* ::= … | let *I* = *E* in *E'* | fun *I* -> *E* | *E E'*
- fun *I* -> *E* is a value
- Could handle local binding using state, but have assumption that evaluating expressions doesn't alter the environment
- We will use substitution here instead
- **Notation**: *E* [*E'* / *I*] means replace all free occurrence of *I* by *E'* in *E*

Call-by-value (Eager Evaluation)

(let *I* = *V* in *E*, *m*) --> (*E*[*V*/ *I*],*m*)

$$\frac{(E,\ m) \dashrightarrow (E'',m)}{(\text{let } I = E \text{ in } E',\ m) \dashrightarrow (\text{let } I = E'' \text{ in } E')}$$

((fun *I* -> *E*) *V*, *m*) --> (*E*[*V* / *I*],*m*)

$$\frac{(E',\ m) \dashrightarrow (E'',m)}{((\text{fun } I \text{ -> } E)\ E',\ m) \dashrightarrow ((\text{fun } I \text{ -> } E)\ E'',\ m)}$$

Call-by-name (Lazy Evaluation)

- (let *I* = *E* in *E'*, *m*) --> (*E'* [*E* / *I*],*m*)
- ((fun *I* -> *E'*) *E*, *m*) --> (*E'* [*E* / *I*],*m*)
- Question: Does it make a difference?
- It can depending on the language

Church-Rosser Property

- Church-Rosser Property: If E-->* E_1 and E-->* E_2, if there exists a value V such that E_1 -->* V, then E_2 -->* V
- Also called **confluence** or **diamond property**
- Example: E= 2 + 3 + 4

E_1 = 5 + 4 E_2= 2 + 7

V =9

Does It always Hold?

- No. Languages with side-effects tend not be Church-Rosser with the combination of call-by-name and call-by-value
- Alonzo Church and Barkley Rosser proved in 1936 the λ-calculus does have it
- Benefit of Church-Rosser: can check equality of terms by evaluating them (Given evaluation strategy might not terminate, though)

Transition Semantics for λ-Calculus

- Application (version 1)

$$(\lambda\ x\ .\ E)\ E' \dashrightarrow E[E'/x]$$

- Application (version 2)

$$(\lambda\ x\ .\ E)\ V \dashrightarrow E[V/x]$$

$$\frac{E' \dashrightarrow E''}{(\lambda\ x\ .\ E)\ E' \dashrightarrow (\lambda\ x\ .\ E)\ E''}$$