Programming Languages and Compilers (CS 421)

Elsa L Gunter
2112 SC, UIUC

https://courses.engr.illinois.edu/cs421/fa2017/CS421D

Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Lists

- List can take one of two forms:
  - Empty list, written [ ]
  - Non-empty list, written x :: xs
    - x is head element, xs is tail list, :: called "cons"
  - Syntactic sugar: [x] == x :: [ ]
  - [x1; x2; …; xn] == x1 :: x2 :: … :: xn :: [ ]

Lists

```
# let fib5 = [8;5;3;2;1;1];;
val fib5 : int list = [8; 5; 3; 2; 1; 1]
# let fib6 = 13 :: fib5;;
val fib6 : int list = [13; 8; 5; 3; 2; 1; 1]
# (8::5::3::2::1::1::[ ]) = fib5;;
- : bool = true
# fib5 @ fib6;;
- : int list = [8; 5; 3; 2; 1; 1; 13; 8; 5; 3; 2; 1;
  1]
```


Lists are Homogeneous

```
# let bad_list = [1; 3.2; 7];;
Characters 19-22:
  let bad_list = [1; 3.2; 7];;
                      ^^^
This expression has type float but is here
  used with type int
```


Question

- Which one of these lists is invalid?

1. [2; 3; 4; 6]
2. [2,3; 4,5; 6,7]
3. [(2.3,4); (3.2,5); (6,7.2)]
4. [["hi"; "there"]; ["wahcha"]; [ ]; ["doin"]]

Answer

- Which one of these lists is invalid?

1. [2; 3; 4; 6]
2. [2,3; 4,5; 6,7]
3. [(2.3,4); (3.2,5); (6,7.2)]
4. [["hi"; "there"]; ["wahcha"]; [ ]; ["doin"]]

- 3 is invalid because of last pair

Functions Over Lists

```
# let rec double_up list =
    match list
    with [ ] -> [ ]  (* pattern before ->,
                        expression after *)
       | (x :: xs) -> (x :: x :: double_up xs);;
val double_up : 'a list -> 'a list = <fun>
# let fib5_2 = double_up fib5;;
val fib5_2 : int list = [8; 8; 5; 5; 3; 3; 2; 2; 1;
  1; 1; 1]
```


Functions Over Lists

```
# let silly = double_up ["hi"; "there"];;
val silly : string list = ["hi"; "hi"; "there"; "there"]
# let rec poor_rev list =
  match list
  with [] -> []
     | (x::xs) -> poor_rev xs @ [x];;
val poor_rev : 'a list -> 'a list = <fun>
# poor_rev silly;;
- : string list = ["there"; "there"; "hi"; "hi"]
```


Structural Recursion

- Functions on recursive datatypes (eg lists) tend to be recursive
- Recursion over recursive datatypes generally by structural recursion
  - Recursive calls made to components of structure of the same recursive type
  - Base cases of recursive types stop the recursion of the function

Question: Length of list

- Problem: write code for the length of the list
  - How to start?

```
let length l =
```


Question: Length of list

- Problem: write code for the length of the list
  - How to start?

```
let rec length l =
   match l with
```


Question: Length of list

- Problem: write code for the length of the list
  - What patterns should we match against?

```
let rec length l =
   match l with
```

Question: Length of list

- Problem: write code for the length of the list
  - What patterns should we match against?

```
let rec length l =
   match l with [] ->
    | (a :: bs) ->
```


Question: Length of list

- Problem: write code for the length of the list
  - What result do we give when l is empty?

```
let rec length l =
   match l with [] -> 0
    | (a :: bs) ->
```


Question: Length of list

- Problem: write code for the length of the list
  - What result do we give when l is not empty?

```
let rec length l =
   match l with [] -> 0
    | (a :: bs) ->
```


Question: Length of list

- Problem: write code for the length of the list
  - What result do we give when l is not empty?

```
let rec length l =
   match l with [] -> 0
    | (a :: bs) -> 1 + length bs
```


Structural Recursion : List Example

```
# let rec length list = match list
  with [ ] -> 0   (* Nil case *)
  | x :: xs -> 1 + length xs;;  (* Cons case *)
val length : 'a list -> int = <fun>
# length [5; 4; 3; 2];;
- : int = 4
```

- Nil case [ ] is base case
- Cons case recurses on component list xs

Same Length

- How can we efficiently answer if two lists have the same length?

Same Length

- How can we efficiently answer if two lists have the same length?

```
let rec same_length list1 list2 =
   match list1 with [] ->
     (match list2 with [] -> true
       | (y::ys) -> false)
   | (x::xs) ->
     (match list2 with [] -> false
       | (y::ys) -> same_length xs ys)
```


Higher-Order Functions Over Lists

```
# let rec map f list =
  match list
  with [] -> []
  | (h::t) -> (f h) :: (map f t);;
val map : ('a -> 'b) -> 'a list -> 'b list = <fun>
# map plus_two fib5;;
- : int list = [10; 7; 5; 4; 3; 3]
# map (fun x -> x - 1) fib6;;
: int list = [12; 7; 4; 2; 1; 0; 0]
```


Iterating over lists

```
# let rec fold_left f a list =
  match list
  with [] -> a
  | (x :: xs) -> fold_left f (f a x) xs;;
val fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a =
   <fun>
# fold_left
   (fun () -> print_string)
   ()
   ["hi"; "there"];;
hithere- : unit = ()
```


Recursing over lists

The Primitive Recursion Fairy

```
# let rec fold_right f list b =
  match list
  with [] -> b
  | (x :: xs) -> f x (fold_right f xs b);;
val fold_right : ('a -> 'b -> 'b) -> 'a list -> 'b -> 'b =
   <fun>
# fold_right
    (fun s -> fun () -> print_string s)
    ["hi"; "there"]
    ();;
therehi- : unit = ()
```


Forward Recursion

- In Structural Recursion, split input into components and (eventually) recurse
- Forward Recursion form of Structural Recursion
- In forward recursion, first call the function recursively on all recursive components, and then build final result from partial results
- Wait until whole structure has been traversed to start building answer

Forward Recursion: Examples

```
# let rec double_up list =
    match list
    with [ ] -> [ ]
      | (x :: xs) -> (x :: x :: double_up xs);;
val double_up : 'a list -> 'a list = <fun>

# let rec poor_rev list =
  match list
  with [] -> []
    | (x::xs) -> poor_rev xs @ [x];;
val poor_rev : 'a list -> 'a list = <fun>
```

Forward Recursion: Examples

```
# let rec double_up list =
    match list
    with [ ] -> [ ]
       | (x :: xs) -> (x :: x :: double_up xs);;
val double_up : 'a list -> 'a list = <fun>
```

Base Case | Operator | Recursive Call

```
# let rec poor_rev list =
  match list
  with [] -> []
     | (x::xs) -> poor_rev xs @ [x];;
val poor_rev : 'a list -> 'a list = <fun>
```

Base Case | Operator | Recursive Call

Encoding Forward Recursion with Fold

```
# let rec append list1 list2 = match list1 with
  [ ] -> list2 | x::xs -> x :: append xs list2;;
val append : 'a list -> 'a list -> 'a list = <fun>
```

Base Case | Operation | Recursive Call

```
# let append list1 list2 =
   fold_right (fun x y -> x :: y) list1 list2;;
val append : 'a list -> 'a list -> 'a list = <fun>
# append [1;2;3] [4;5;6];;
 - : int list = [1; 2; 3; 4; 5; 6]
```


Mapping Recursion

- Can use the higher-order recursive map function instead of direct recursion

```
# let doubleList list =
   List.map (fun x -> 2 * x) list;;
val doubleList : int list -> int list = <fun>
# doubleList [2;3;4];;
- : int list = [4; 6; 8]
```


Mapping Recursion

- Can use the higher-order recursive map function instead of direct recursion

```
# let doubleList list =
   List.map (fun x -> 2 * x) list;;
val doubleList : int list -> int list = <fun>
# doubleList [2;3;4];;
- : int list = [4; 6; 8]
```

- Same function, but no rec

Folding Recursion

- Another common form “folds” an operation over the elements of the structure

```
# let rec multList list = match list
  with [ ] -> 1
  | x::xs -> x * multList xs;;
val multList : int list -> int = <fun>
# multList [2;4;6];;
- : int = 48
```


Folding Recursion

- Another common form “folds” an operation over the elements of the structure

```
# let rec multList list = match list
  with [ ] -> 1
  | x::xs -> x * multList xs;;
val multList : int list -> int = <fun>
# multList [2;4;6];;
- : int = 48
```

- Computes (2 * (4 * (6 * 1)))

Folding Recursion

- multList folds to the right
- Same as:

```
# let multList list =
    List.fold_right
    (fun x -> fun p -> x * p)
    list 1;;
val multList : int list -> int = <fun>
# multList [2;4;6];;
- : int = 48
```


How long will it take?

- Remember the big-O notation from CS 225 and CS 374
- Question: given input of size *n*, how long to generate output?
- Express output time in terms of input size, omit constants and take biggest power

How long will it take?

Common big-O times:

- Constant time $O(1)$
  - input size doesn't matter
- Linear time $O(n)$
  - double input ⇒ double time
- Quadratic time $O(n^2)$
  - double input ⇒ quadruple time
- Exponential time $O(2^n)$
  - increment input ⇒ double time

Linear Time

- Expect most list operations to take linear time $O(n)$
- Each step of the recursion can be done in constant time
- Each step makes only one recursive call
- List example: multList, append
- Integer example: factorial

Quadratic Time

- Each step of the recursion takes time proportional to input
- Each step of the recursion makes only one recursive call.
- List example:

```
# let rec poor_rev list = match list
  with [] -> []
    | (x::xs) -> poor_rev xs @ [x];;
val poor_rev : 'a list -> 'a list = <fun>
```

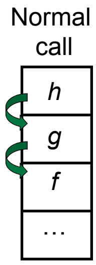

Exponential running time

- Poor worst-case running times on input of any size
- Each step of recursion takes constant time
- Each recursion makes two recursive calls
- Easy to write naïve code that is exponential for functions that can be linear

Exponential running time

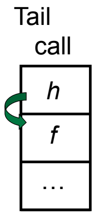
```
# let rec slow n =
      if n <= 1
      then 1
      else 1+slow (n-1) + slow(n-2);;
val slow : int -> int = <fun>
# List.map slow [1;2;3;4;5;6;7;8;9];;
- : int list = [1; 3; 5; 9; 15; 25; 41; 67; 109]
```


An Important Optimization



- When a function call is made, the return address needs to be saved to the stack so we know to where to return when the call is finished
- What if *f* calls *g* and *g* calls *h*, but calling *h* is the last thing *g* does (a *tail call*)?

An Important Optimization



- When a function call is made, the return address needs to be saved to the stack so we know to where to return when the call is finished
- What if *f* calls *g* and *g* calls *h*, but calling *h* is the last thing *g* does (a *tail call*)?
- Then *h* can return directly to *f* instead of *g*

Tail Recursion

- A recursive program is tail recursive if all recursive calls are tail calls
- Tail recursive programs may be optimized to be implemented as loops, thus removing the function call overhead for the recursive calls
- Tail recursion generally requires extra "accumulator" arguments to pass partial results
  - May require an auxiliary function

Tail Recursion - Example

```
# let rec rev_aux list revlist =
  match list with [ ] -> revlist
  | x :: xs -> rev_aux xs (x::revlist);;
val rev_aux : 'a list -> 'a list -> 'a list = <fun>

# let rev list = rev_aux list [ ];;
val rev : 'a list -> 'a list = <fun>
```

- What is its running time?

Comparison

- poor_rev [1,2,3] =
- (poor_rev [2,3]) @ [1] =
- ((poor_rev [3]) @ [2]) @ [1] =
- (((poor_rev [ ]) @ [3]) @ [2]) @ [1] =
- (([ ] @ [3]) @ [2]) @ [1]) =
- ([3] @ [2]) @ [1] =
- (3:: ([ ] @ [2])) @ [1] =
- [3,2] @ [1] =
- 3 :: ([2] @ [1]) =
- 3 :: (2:: ([ ] @ [1])) = [3, 2, 1]

Comparison

- rev [1,2,3] =
- rev_aux [1,2,3] [ ] =
- rev_aux [2,3] [1] =
- rev_aux [3] [2,1] =
- rev_aux [ ] [3,2,1] = [3,2,1]

Folding Functions over Lists

How are the following functions similar?

```
# let rec sumlist list = match list with
  [ ] -> 0 | x::xs -> x + sumlist xs;;
val sumlist : int list -> int = <fun>
# sumlist [2;3;4];;
- : int = 9
# let rec prodlist list = match list with
  [ ] -> 1 | x::xs -> x * prodlist xs;;
val prodlist : int list -> int = <fun>
# prodlist [2;3;4];;
- : int = 24
```


Folding

```
# let rec fold_left f a list = match list
  with [] -> a | (x :: xs) -> fold_left f (f a x) xs;;
val fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a =
  <fun>
```

fold_left f a [x_1; x_2;…;x_n] = f(…(f (f a x_1) x_2)…)x_n

```
# let rec fold_right f list b = match list
  with [ ] -> b | (x :: xs) -> f x (fold_right f xs b);;
val fold_right : ('a -> 'b -> 'b) -> 'a list -> 'b -> 'b =
  <fun>
```

fold_right f [x_1; x_2;…;x_n] b = f x_1(f x_2 (…(f x_n b)…))

Folding - Forward Recursion

```
# let sumlist list = fold_right (+) list 0;;
val sumlist : int list -> int = <fun>
# sumlist [2;3;4];;
- : int = 9
# let prodlist list = fold_right ( * ) list 1;;
val prodlist : int list -> int = <fun>
# prodlist [2;3;4];;
- : int = 24
```


Folding - Tail Recursion

```
# let rev list =
      fold_left
        (fun l -> fun x -> x :: l)    //comb op
        []              //accumulator cell
        list
```


Folding

- Can replace recursion by fold_right in any forward primitive recursive definition
  - Primitive recursive means it only recurses on immediate subcomponents of recursive data structure
- Can replace recursion by fold_left in any tail primitive recursive definition

Continuations

- A programming technique for all forms of "non-local" control flow:
  - non-local jumps
  - exceptions
  - general conversion of non-tail calls to tail calls
- Essentially it's a higher-order function version of GOTO

Continuations

- Idea: Use functions to represent the control flow of a program
- Method: Each procedure takes a function as an extra argument to which to pass its result; outer procedure "returns" no result
- Function receiving the result called a continuation
- Continuation acts as "accumulator" for work still to be done

Continuation Passing Style

- Writing procedures such that all procedure calls take a continuation to which to give (pass) the result, and return no result, is called continuation passing style (CPS)

Continuation Passing Style

- A compilation technique to implement non-local control flow, especially useful in interpreters.
- A formalization of non-local control flow in denotational semantics
- Possible intermediate state in compiling functional code

Why CPS?

- Makes order of evaluation explicitly clear
- Allocates variables (to become registers) for each step of computation
- Essentially converts functional programs into imperative ones
  - Major step for compiling to assembly or byte code
- Tail recursion easily identified
- Strict forward recursion converted to tail recursion
  - At the expense of building large closures in heap

Other Uses for Continuations

- CPS designed to preserve order of evaluation
- Continuations used to express order of evaluation
- Can be used to change order of evaluation
- Implements:
  - Exceptions and exception handling
  - Co-routines
  - (pseudo, aka green) threads

Example

- Simple reporting continuation:

```
# let report x = (print_int x; print_newline( ) );;
val report : int -> unit = <fun>
```

- Simple function using a continuation:

```
# let addk (a, b) k = k (a + b);;
val addk : int * int -> (int -> ’a) -> ’a = <fun>
# addk (22, 20) report;;
2
- : unit = ()
```


Simple Functions Taking Continuations

- Given a primitive operation, can convert it to pass its result forward to a continuation
- Examples:

```
# let subk (x, y) k = k(x + y);;
val subk : int * int -> (int -> 'a) -> 'a = <fun>
# let eqk (x, y) k = k(x = y);;
val eqk : 'a * 'a -> (bool -> 'b) -> 'b = <fun>
# let timesk (x, y) k = k(x * y);;
val timesk : int * int -> (int -> 'a) -> 'a = <fun>
```


Your turn now

Try Problem 7 on MP2

Try consk

Nesting Continuations

```
# let add_triple (x, y, z) = (x + y) + z;;
val add_triple : int * int * int -> int = <fun>
# let add_triple (x,y,z)=let p = x + y in p + z;;
val add_three : int -> int -> int -> int = <fun>
# let add_triple_k (x, y, z) k =
  addk (x, y) (fun p -> addk (p, z) k );;
val add_triple_k: int * int * int -> (int -> 'a) ->
  'a = <fun>
```


add_three: a different order

- # let add_triple (x, y, z) = x + (y + z);;
- How do we write add_triple_k to use a different order?

- let add_triple_k (x, y, z) k =