Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve, Gul Agha, and Elsa L Gunter

Interpreter

- Takes abstract syntax trees as input
  - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
  - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next "state"
  - To get final value, put in a loop

Natural Semantics Example

- compute_exp (Var(v), m) = look_up v m
- compute_exp (Int(n), _) = Num (n)
- …
- compute_com(IfExp(b,c1,c2),m) =

  if compute_exp (b,m) = Bool(true)

  then compute_com (c1,m)

  else compute_com (c2,m)

Natural Semantics Example

- compute_com(While(b,c), m) =

    if compute_exp (b,m) = Bool(false)

    then m

    else compute_com

        (While(b,c), compute_com(c,m))

- May fail to terminate - exceed stack limits
- Returns no useful information then

Transition Semantics

- Form of operational semantics
- Describes how each program construct transforms machine state by *transitions*
- Rules look like

$$(C, m) \dashrightarrow (C', m') \quad \text{or} \quad (C,m) \dashrightarrow m'$$

- *C*, *C*' is code remaining to be executed
- m, m' represent the state/store/memory/environment
  - Partial mapping from identifiers to values
  - Sometimes *m* (or *C*) not needed
- Indicates exactly one step of computation

Expressions and Values

- *C*, *C'* used for commands; *E*, *E' for* expressions; *U*,*V* for values
- Special class of expressions designated as *values*
  - Eg 2, 3 are values, but 2+3 is only an expression
- Memory only holds values
  - Other possibilities exist

Evaluation Semantics

- Transitions successfully stops when E/C is a value/memory
- Evaluation fails if no transition possible, but not at value/memory
- Value/memory is the final *meaning* of original expression/command (in the given state)
- Coarse semantics: final value / memory
- More fine grained: whole transition sequence

Simple Imperative Programming Language

- $I \in Identifiers$
- $N \in Numerals$
- B ::= true | false | B & B | B or B | not B | E < E | E = E
- E::= N | I | E + E | E * E | E - E | - E
- C::= skip | C;C | I ::= E
  | if B then C else C fi | while B do C od

Transitions for Expressions

- Numerals are values

- Boolean values = {true, false}

- Identifiers: $(I,m) \dashrightarrow (m(I), m)$

Boolean Operations:

- Operators: (short-circuit)

(false & B, m) --> (false,m)

(true & B, m) --> (B,m)

$$\frac{(B, m) \dashrightarrow (B'', m)}{(B \,\&\, B', m) \dashrightarrow (B'' \,\&\, B', m)}$$

(true or B, m) --> (true,m)

(false or B, m) --> (B,m)

$$\frac{(B, m) \dashrightarrow (B'', m)}{(B \text{ or } B', m) \dashrightarrow (B'' \text{ or } B', m)}$$

(not true, m) --> (false,m)

(not false, m) --> (true,m)

$$\frac{(B, m) \dashrightarrow (B', m)}{(\text{not } B, m) \dashrightarrow (\text{not } B', m)}$$

Relations

$$\frac{(E,\ m) \dashrightarrow (E'',m)}{(E \sim E',\ m) \dashrightarrow (E''{\sim}E',m)}$$

$$\frac{(E,\ m) \dashrightarrow (E',m)}{(V \sim E,\ m) \dashrightarrow (V{\sim}E',m)}$$

$(U \sim V,\ m)$ --> (true,m) or (false,m)

depending on whether $U \sim V$ holds or not

Arithmetic Expressions

$$\frac{(E, m) \dashrightarrow (E'', m)}{(E \; op \; E', m) \dashrightarrow (E'' \; op \; E', m)}$$

$$\frac{(E, m) \dashrightarrow (E', m)}{(V \; op \; E, m) \dashrightarrow (V \; op \; E', m)}$$

$$(U \; op \; V, m) \dashrightarrow (N, m)$$

where N is the specified value for $U \; op \; V$

Commands - in English

- skip means done evaluating
- When evaluating an assignment, evaluate the expression first
- If the expression being assigned is already a value, update the memory with the new value for the identifier
- When evaluating a sequence, work on the first command in the sequence first
- If the first command evaluates to a new memory (ie completes), evaluate remainder with new memory

Commands

$$(\text{skip}, m) \dashrightarrow m$$

$$\frac{(E,m) \dashrightarrow (E',m)}{(I::=E,m) \dashrightarrow (I::=E',m)}$$

$$(I::=V,m) \dashrightarrow m[I \leftarrow V]$$

$$\frac{(C,m) \dashrightarrow (C'',m')}{(C;C', m) \dashrightarrow (C'';C',m')} \qquad \frac{(C,m) \dashrightarrow m'}{(C;C', m) \dashrightarrow (C',m')}$$

If Then Else Command - in English

- If the boolean guard in an if_then_else is true, then evaluate the first branch
- If it is false, evaluate the second branch
- If the boolean guard is not a value, then start by evaluating it first.

If Then Else Command

$$(\text{if true then } C \text{ else } C' \text{ fi}, m) \dashrightarrow (C, m)$$

$$(\text{if false then } C \text{ else } C' \text{ fi}, m) \dashrightarrow (C', m)$$

$$\frac{(B,m) \dashrightarrow (B',m)}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \dashrightarrow (\text{if } B' \text{ then } C \text{ else } C' \text{ fi}, m)}$$

While Command

(while *B* do *C* od, *m*) -->

(if *B* then *C*; while *B* do *C* od else skip fi, m)

In English: Expand a While into a test of the boolean guard, with the true case being to do the body and then try the while loop again, and the false case being to stop.

Example Evaluation

- First step:

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7})

--> ?

Example Evaluation

- First step:

$$\frac{(x > 5,\ \{x \text{ -> } 7\}) \text{ --> } ?}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\ \{x \text{ -> } 7\}) \text{ --> } ?}$$

Example Evaluation

- First step:

$$\frac{\dfrac{(x,\{x \to 7\}) \longrightarrow (7, \{x \to 7\})}{(x > 5, \{x \to 7\}) \longrightarrow ?}}{\begin{array}{c}(\text{if } x > 5 \text{ then } y:= 2 + 3 \text{ else } y:=3 + 4 \text{ fi},\\ \{x \to 7\})\\ \longrightarrow ?\end{array}}$$

Example Evaluation

- First step:

$$\dfrac{\dfrac{(x,\{x \text{ -> } 7\}) \text{ --> } (7, \{x \text{ -> } 7\})}{(x > 5, \{x \text{ -> } 7\}) \text{ --> } (7 > 5, \{x \text{ -> } 7\})}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \{x \text{ -> } 7\}) \text{ --> } ?}$$

Example Evaluation

- First step:

$$\dfrac{\dfrac{(x,\{x \rightarrow 7\}) \longrightarrow (7, \{x \rightarrow 7\})}{(x > 5, \{x \rightarrow 7\}) \longrightarrow (7 > 5, \{x \rightarrow 7\})}}{\begin{array}{c}(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \\ \{x \rightarrow 7\}) \\ \longrightarrow (\text{if } 7 > 5 \text{ then } y{:=}2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \ \{x \rightarrow \\ 7\})\end{array}}$$

Example Evaluation

- Second Step:

$$\frac{(7 > 5, \{x \to 7\}) \dashrightarrow (\text{true}, \{x \to 7\})}{(\text{if } 7 > 5 \text{ then } y{:=}2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \{x \to 7\})}$$

--> (if true then y:=2 + 3 else y:=3 + 4 fi, {x -> 7})

- Third Step:

(if true then y:=2 + 3 else y:=3 + 4 fi, {x -> 7})

-->(y:=2+3, {x->7})

Example Evaluation

- Fourth Step:

$$\frac{(2+3, \{x\text{-}>7\}) \text{ --> } (5, \{x \text{ -> } 7\})}{(y{:=}2+3, \{x\text{-}>7\}) \text{ --> } (y{:=}5, \{x\text{-}>7\})}$$

- Fifth Step:

$$(y{:=}5, \{x\text{-}>7\}) \text{ --> } \{y \text{ -> } 5, x \text{ -> } 7\}$$

Example Evaluation

- Bottom Line:

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi, {x -> 7})

--> (if 7 > 5 then y:=2 + 3 else y:=3 + 4 fi, {x -> 7})

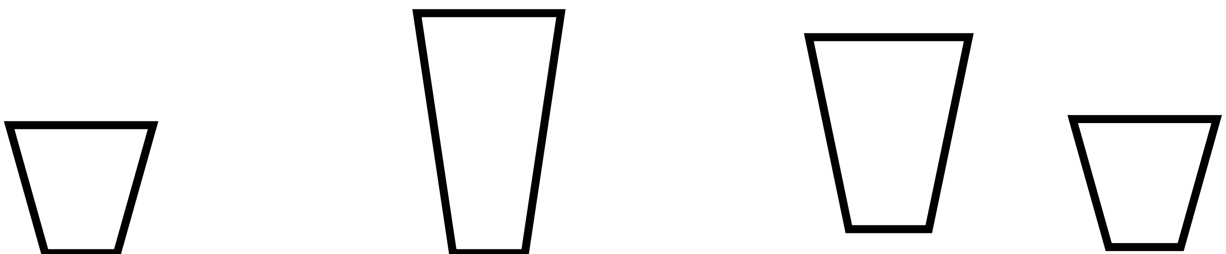
-->(if true then y:=2 + 3 else y:=3 + 4 fi, {x -> 7})

-->(y:=2+3, {x->7})

--> (y:=5, {x->7}) --> {y -> 5, x -> 7}

Transition Semantics Evaluation

- A sequence of steps with trees of justification for each step



$(C_1,m_1) \dashrightarrow (C_2,m_2) \dashrightarrow (C_3,m_3) \dashrightarrow \ldots \dashrightarrow m$

- Let -->* be the transitive closure of -->
- Ie, the smallest transitive relation containing -->

Adding Local Declarations

- Add to expressions:
- E ::= … | let I = E in E' | fun I -> E | E E'
- fun I -> E is a value
- Could handle local binding using state, but have assumption that evaluating expressions doesn't alter the environment
- We will use substitution here instead
- **Notation**: E [E' / I] means replace all free occurrence of I by E' in E

Call-by-value (Eager Evaluation)

$$(\text{let } I = V \text{ in } E,\ m) \dashrightarrow (E[V/\ I\ ], m)$$

$$\frac{(E,\ m) \dashrightarrow (E'', m)}{(\text{let } I = E \text{ in } E',\ m) \dashrightarrow (\text{let } I = E'' \text{ in } E')}$$

$$((\text{fun } I \text{ -> } E)\ V,\ m) \dashrightarrow (E[V\ /\ I\ ], m)$$

$$\frac{(E',\ m) \dashrightarrow (E'', m)}{((\text{fun } I \text{ -> } E)\ E',\ m) \dashrightarrow ((\text{fun } I \text{ -> } E)\ \mathrm{E}'',\ m)}$$

Call-by-name (Lazy Evaluation)

- (let I = E in E', m) --> (E' [E / I],m)

- ((fun I -> E') E, m) --> (E' [E / I],m)

- Question: Does it make a difference?
- It can depending on the language

Church-Rosser Property

- Church-Rosser Property: If $E \dashrightarrow^* E_1$ and $E \dashrightarrow^* E_2$, if there exists a value V such that $E_1 \dashrightarrow^* V$, then $E_2 \dashrightarrow^* V$
- Also called **confluence** or **diamond property**
- Example:

$$\begin{tikzcd}
 & E = 2 + 3 + 4 \arrow[dl] \arrow[dr] & \\
E_1 = 5 + 4 \arrow[dr] & & E_2 = 2 + 7 \arrow[dl] \\
 & V = 9 &
\end{tikzcd}$$

Does It always Hold?

- No. Languages with side-effects tend not be Church-Rosser with the combination of call-by-name and call-by-value
- Alonzo Church and Barkley Rosser proved in 1936 the λ-calculus does have it
- Benefit of Church-Rosser: can check equality of terms by evaluating them (Given evaluation strategy might not terminate, though)