Programming Languages and Compilers (CS 421)

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https://courses.engr.illinois.edu/cs421/fa2017/CS421A

Based in part on slides by Mattox Beckman, as updated by Vikram Adve, Gul Agha, and Elsa L Gunter

Example

- Ambiguous grammar:
  <exp> ::= 0 | 1 | <exp> + <exp>
  | <exp> * <exp>
- String with more then one parse:
  0 + 1 + 0
  1 * 1 + 1
- Source of ambiuity: associativity and precedence

Disambiguating a Grammar

- Given ambiguous grammar G, with start symbol S, find a grammar G' with same start symbol, such that
  language of G = language of G'
- Not always possible
- No algorithm in general

Disambiguating a Grammar

- Idea: Each non-terminal represents all strings having some property
- Identify these properties (often in terms of things that can't happen)
- Use these properties to inductively guarantee every string in language has a unique parse

Steps to Grammar Disambiguation

- Identify the rules and a smallest use that display ambiguity
- Decide which parse to keep; why should others be thrown out?
- What syntactic restrictions on subexpressions are needed to throw out the bad (while keeping the good)?
- Add a new non-terminal and rules to describe this set of restricted subexpressions (called stratifying, or refactoring)
- Replace old rules to use new non-terminals
- Rinse and repeat

Two Major Sources of Ambiguity

- Lack of determination of operator precedence
- Lack of determination of operator assoicativity
- Not the only sources of ambiguity

How to Enforce Associativity

- Have at most one recursive call per production
- When two or more recursive calls would be natural leave right-most one for right assoicativity, left-most one for left assoiciativity

Example

- <Sum> ::= 0 | 1 | <Sum> + <Sum>
  | (<Sum>)
- Becomes
  - <Sum> ::= <Num> | <Num> + <Sum>
  - <Num> ::= 0 | 1 | (<Sum>)

Operator Precedence

- Operators of highest precedence evaluated first (bind more tightly).
- Precedence for infix binary operators given in following table
- Needs to be reflected in grammar

Predence in Grammar

- Higher precedence translates to longer derivation chain
- Example:

<exp> ::= 0 | 1 | <exp> + <exp>
| <exp> * <exp>

- Becomes

<exp> ::= <mult_exp>
| <exp> + <mult_exp>
<mult_exp> ::= <id> | <mult_exp> * <id>
<id> ::= 0 | 1

Disambiguating a Grammar

- <exp>::= 0|1| b<exp> | <exp>a
- | <exp>m<exp>
- Want a has higher precedence than b, which in turn has higher precedence than m, and such that m associates to the left.

Disambiguating a Grammar

- <exp>::= 0|1| b<exp> | <exp>a
- | <exp>m<exp>
- Want a has higher precedence than b, which in turn has higher precedence than m, and such that m associates to the left.
- <exp> ::= <exp> m <not m> | <not m>
- <not m> ::= b <not m> | <not b m>
- <not b m> ::= <not b m>a | 0 | 1

Disambiguating a Grammar – Take 2

- <exp>::= 0|1| b<exp> | <exp>a
  | <exp>m<exp>
- Want b has higher precedence than m, which in turn has higher precedence than a, and such that m associates to the right.

Disambiguating a Grammar – Take 2

- <exp>::= 0|1| b<exp> | <exp>a
  | <exp>m<exp>
- Want b has higher precedence than m, which in turn has higher precedence than a, and such that m associates to the right.
- <exp> ::=
  <no a m> | <no m> m <no a>| <exp> a
- <no a> ::= <no a m> | <no a m> m <no a>
- <no m> ::= <no a m> | <exp> a
- <no a m> ::= b <no a m> | 0 | 1

LR Parsing

- Read tokens left to right (L)
- Create a rightmost derivation (R)
- How is this possible?
- Start at the bottom (left) and work your way up
- Last step has only one non-terminal to be replaced so is right-most
- Working backwards, replace mixed strings by non-terminals
- Always proceed so that there are no non-terminals to the right of the string to be replaced

LR Parsing Tables

- Build a pair of tables, Action and Goto, from the grammar
  - This is the hardest part, we omit here
  - Rows labeled by states
  - For Action, columns labeled by terminals and "end-of-tokens" marker
    - (more generally strings of terminals of fixed length)
  - For Goto, columns labeled by non-terminals

Action and Goto Tables

- Given a state and the next input, Action table says either
  - **shift** and go to state *n*, or
  - **reduce** by production *k* (explained in a bit)
  - **accept** or **error**
- Given a state and a non-terminal, Goto table says
  - go to state *m*

LR(i) Parsing Algorithm

- Based on push-down automata
- Uses states and transitions (as recorded in Action and Goto tables)
- Uses a stack containing states, terminals and non-terminals

LR(i) Parsing Algorithm

0. Insure token stream ends in special "end-of-tokens" symbol
1. Start in state 1 with an empty stack
2. Push **state**(1) onto stack
3. Look at next *i* tokens from token stream (*toks*) (don't remove yet)
4. If top symbol on stack is **state**(*n*), look up action in Action table at (*n*, *toks*)

LR(i) Parsing Algorithm

5. If action = **shift** *m*,
   a) Remove the top token from token stream and push it onto the stack
   b) Push **state**(*m*) onto stack
   c) Go to step 3

LR(i) Parsing Algorithm

6. If action = **reduce** *k* where production *k* is E ::= u
   a) Remove 2 * length(u) symbols from stack (u and all the interleaved states)
   b) If new top symbol on stack is **state**(*m*), look up new state *p* in Goto(*m*,E)
   c) Push E onto the stack, then push **state**(*p*) onto the stack
   d) Go to step 3

LR(i) Parsing Algorithm

7. If action = **accept**
   - Stop parsing, return success
8. If action = **error**,
   - Stop parsing, return failure

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

= ● (0 + 1) + 0 shift

LR(i) Parsing Algorithm

0. Insure token stream ends in special "end-of-tokens" symbol
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<Sum> =>

= (<Sum> ● + 1) + 0 shift
=> (0 ● + 1) + 0 reduce
= (● 0 + 1) + 0 shift
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Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

= (<Sum> + ● 1) + 0 shift
= (<Sum> ● + 1) + 0 shift
=> (0 ● + 1) + 0 reduce
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Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

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Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=> (<Sum> + <Sum> ●) + 0 reduce
=> (<Sum> + 1 ●) + 0 reduce
= (<Sum> + ● 1) + 0 shift
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<Sum> =>

= (<Sum> ●) + 0 shift
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Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

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=> <Sum> + 0 ● reduce
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Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> => <Sum> + <Sum > ● reduce
=> <Sum> + 0 ● reduce
= <Sum> + ● 0 shift
= <Sum> ● + 0 shift
=> (<Sum>) ● + 0 reduce
= (<Sum> ●) + 0 shift
=> (<Sum> + <Sum> ●) + 0 reduce
=> (<Sum> + 1 ●) + 0 reduce
= (<Sum> + ● 1) + 0 shift
= (<Sum> ● + 1) + 0 shift
=> (0 ● + 1) + 0 reduce
= (● 0 + 1) + 0 shift
= ● (0 + 1) + 0 shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> ● => <Sum> + <Sum > ● reduce
=> <Sum> + 0 ● reduce
= <Sum> + ● 0 shift
= <Sum> ● + 0 shift
=> (<Sum>) ● + 0 reduce
= (<Sum> ●) + 0 shift
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LR(i) Parsing Algorithm

7. If action = **accept**
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Example

(0 + 1) + 0

Example

(0 + 1) + 0

Example

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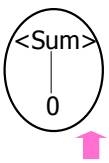
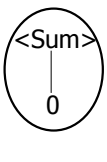
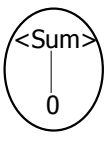
Example



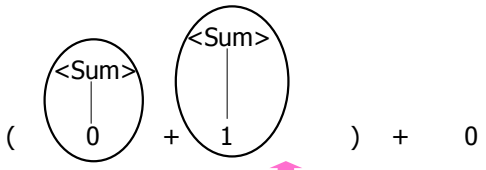
Example



Example



Example



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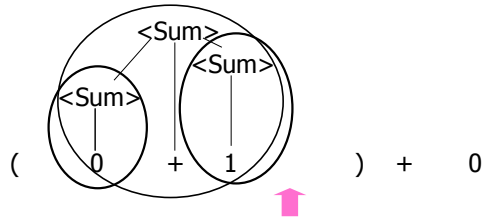

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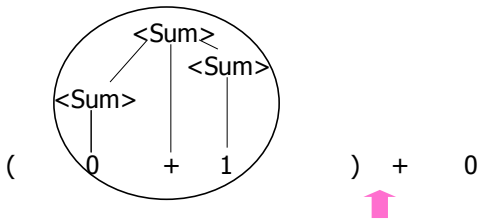

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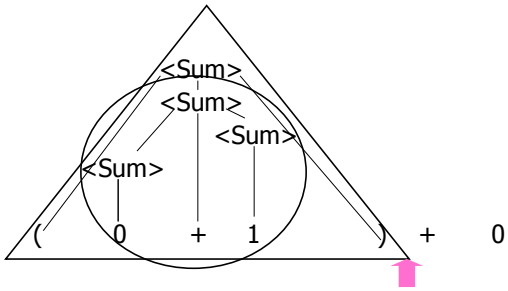

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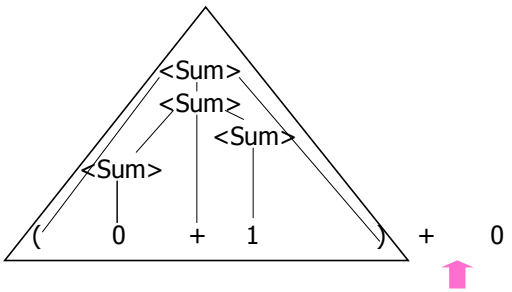

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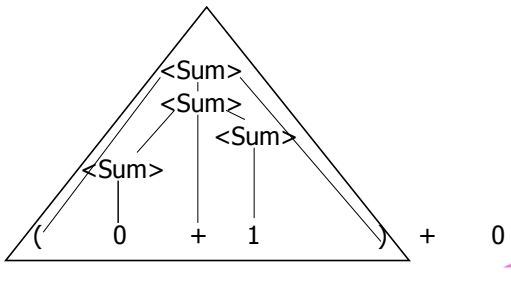

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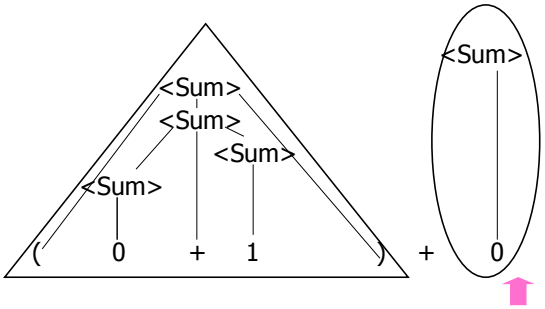

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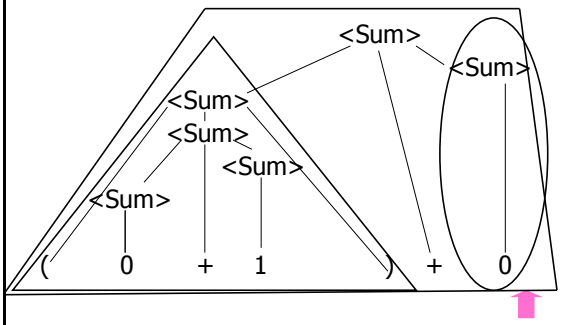

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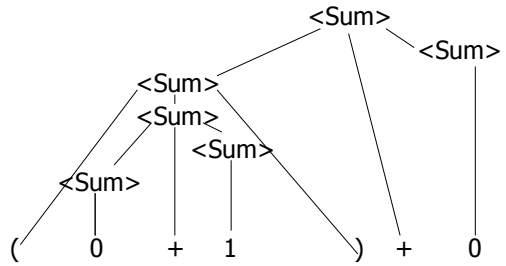

Shift-Reduce Conflicts

- **Problem**: can't decide whether the action for a state and input character should be **shift** or **reduce**
- Caused by ambiguity in grammar
- Usually caused by lack of associativity or precedence information in grammar

Example: <Sum> = 0 | 1 | (<Sum>)
| <Sum> + <Sum>

● 0 + 1 + 0	shift
-> 0 ● + 1 + 0	reduce
-> <Sum> ● + 1 + 0	shift
-> <Sum> + ● 1 + 0	shift
-> <Sum> + 1 ● + 0	reduce
-> <Sum> + <Sum> ● + 0	

Example - cont

- **Problem:** shift or reduce?
- You can shift-shift-reduce-reduce or reduce-shift-shift-reduce
- Shift first - right associative
- Reduce first- left associative

Reduce - Reduce Conflicts

- **Problem:** can't decide between two different rules to reduce by
- Again caused by ambiguity in grammar
- **Symptom:** RHS of one production suffix of another
- Requires examining grammar and rewriting it
- Harder to solve than shift-reduce errors

Example

- S ::= A | aB A ::= abc B ::= bc

  ● abc shift
  a ● bc shift
  ab ● c shift
  abc ●

- Problem: reduce by B ::= bc then by S ::= aB, or by A::= abc then S::A?

Recursive Descent Parsing

- Recursive descent parsers are a class of parsers derived fairly directly from BNF grammars
- A recursive descent parser traces out a parse tree in top-down order, corresponding to a left-most derivation (LL - left-to-right scanning, leftmost derivation)

Recursive Descent Parsing

- Each nonterminal in the grammar has a subprogram associated with it; the subprogram parses all phrases that the nonterminal can generate
- Each nonterminal in right-hand side of a rule corresponds to a recursive call to the associated subprogram

Recursive Descent Parsing

- Each subprogram must be able to decide how to begin parsing by looking at the left-most character in the string to be parsed
  - May do so directly, or indirectly by calling another parsing subprogram
- Recursive descent parsers, like other top-down parsers, cannot be built from left-recursive grammars
  - Sometimes can modify grammar to suit

Sample Grammar

<expr> ::= <term> | <term> + <expr>
| <term> - <expr>

<term> ::= <factor> | <factor> * <term>
| <factor> / <term>

<factor> ::= <id> | (<expr>)

Tokens as OCaml Types

- + - * / ( ) <id>
- Becomes an OCaml datatype

```
type token =
    Id_token of string
  | Left_parenthesis | Right_parenthesis
  | Times_token | Divide_token
  | Plus_token | Minus_token
```


Parse Trees as Datatypes

<expr> ::= <term> | <term> + <expr>
| <term> - <expr>

```
type expr =
    Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
```


Parse Trees as Datatypes

<term> ::= <factor> | <factor> * <term>
| <factor> / <term>

```
and term =
    Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
```


Parse Trees as Datatypes

<factor> ::= <id> | (<expr>)

```
and factor =
    Id_as_Factor of string
  | Parenthesized_Expr_as_Factor of expr
```


Parsing Lists of Tokens

- Will create three mutually recursive functions:
  - expr : token list -> (expr * token list)
  - term : token list -> (term * token list)
  - factor : token list -> (factor * token list)
- Each parses what it can and gives back parse and remaining tokens

Parsing an Expression

<expr> ::= <term> [(+ | -) <expr>]

```
let rec expr tokens =
  (match term tokens
    with ( term_parse , tokens_after_term) ->
      (match tokens_after_term
        with( Plus_token  :: tokens_after_plus) ->
```

Parsing an Expression

<expr> ::= <term> [(+ | -) <expr>]

```
let rec expr tokens =
  (match term tokens
    with ( term_parse , tokens_after_term) ->
     (match tokens_after_term
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```


Parsing a Plus Expression

<expr> ::= <term> [(+ | -) <expr>]

```
let rec expr tokens =
  (match term tokens
    with ( term_parse , tokens_after_term) ->
     (match tokens_after_term
      with ( Plus_token  :: tokens_after_plus) ->
```


Parsing a Plus Expression

<expr> ::= <term> [(+ | -) <expr>]

```
let rec expr tokens =
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    with ( term_parse , tokens_after_term) ->
     (match tokens_after_term
      with ( Plus_token  :: tokens_after_plus) ->
```


Parsing a Plus Expression

<expr> ::= <term> [(+ | -) <expr>]

```
let rec expr tokens =
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    with ( term_parse , tokens_after_term) ->
     (match tokens_after_term
      with ( Plus_token  :: tokens_after_plus) ->
```


Parsing a Plus Expression

<expr> ::= <term> + <expr>

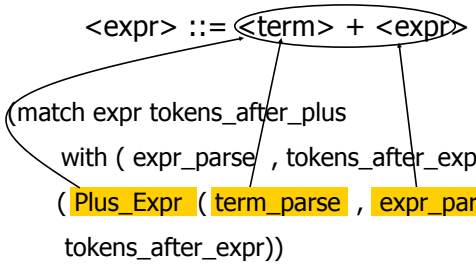
```
(match expr tokens_after_plus
    with ( expr_parse  , tokens_after_expr) ->
    ( Plus_Expr  ( term_parse  ,  expr_parse ),
     tokens_after_expr))
```


Parsing a Plus Expression

<expr> ::= <term> + <expr>

```
(match expr tokens_after_plus
    with ( expr_parse  , tokens_after_expr) ->
    ( Plus_Expr  ( term_parse  ,  expr_parse ),
     tokens_after_expr))
```

Building Plus Expression Parse Tree

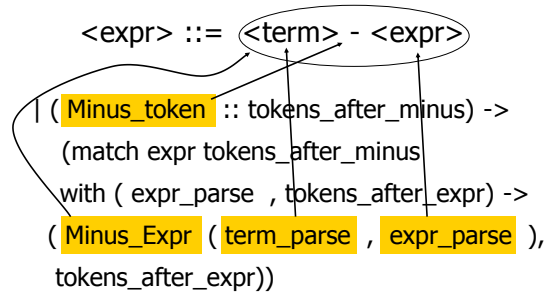


Parsing a Minus Expression

<expr> ::= <term> - <expr>

| (Minus_token :: tokens_after_minus) ->
(match expr tokens_after_minus
with (expr_parse , tokens_after_expr) ->
(Minus_Expr (term_parse , expr_parse),
tokens_after_expr))

Parsing a Minus Expression



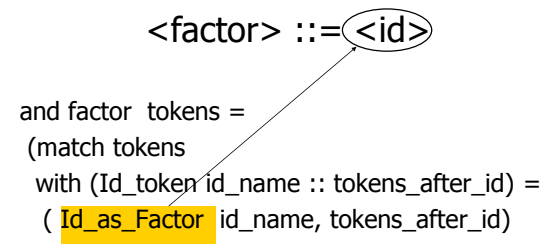
Parsing an Expression as a Term

<expr> ::= <term>

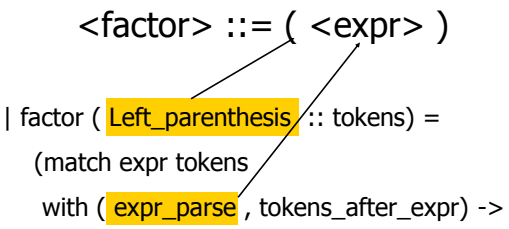
| _ -> (Term_as_Expr term_parse ,
tokens_after_term)))

- Code for **term** is same except for replacing addition with multiplication and subtraction with division

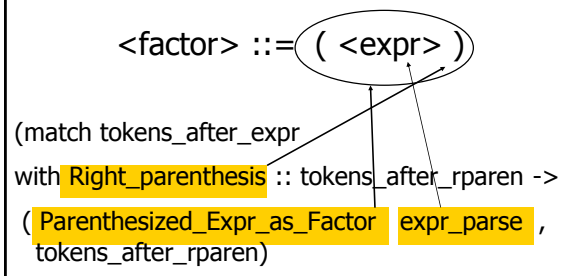
Parsing Factor as Id



Parsing Factor as Parenthesized Expression



Parsing Factor as Parenthesized Expression



Error Cases

- What if no matching right parenthesis?

```
 | _ -> raise (Failure "No matching
rparen") ))
```

- What if no leading id or left parenthesis?

```
| _ -> raise (Failure "No id or lparen" ));;
```

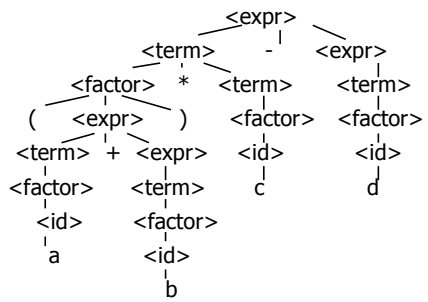

(a + b) * c - d

```
expr [Left_parenthesis; Id_token "a";
  Plus_token; Id_token "b";
  Right_parenthesis; Times_token;
  Id_token "c"; Minus_token;
  Id_token "d"];;
```


(a + b) * c - d

```
- : expr * token list =
(Minus_Expr
 (Mult_Term
  (Parenthesized_Expr_as_Factor
   (Plus_Expr
    (Factor_as_Term (Id_as_Factor "a"),
     Term_as_Expr (Factor_as_Term
  (Id_as_Factor "b")))),
   Factor_as_Term (Id_as_Factor "c")),
  Term_as_Expr (Factor_as_Term (Id_as_Factor
  "d"))),
 [])
```

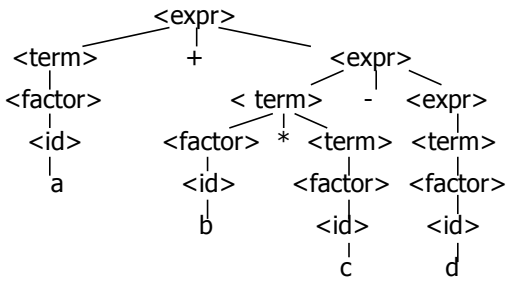

(a + b) * c – d



a + b * c – d

```
# expr [Id_token "a"; Plus_token; Id_token "b";
  Times_token; Id_token "c"; Minus_token;
       Id_token "d"];;
- : expr * token list =
(Plus_Expr
 (Factor_as_Term (Id_as_Factor "a"),
  Minus_Expr
   (Mult_Term (Id_as_Factor "b", Factor_as_Term
  (Id_as_Factor "c")),
    Term_as_Expr (Factor_as_Term (Id_as_Factor
  "d")))),
 [])
```

a + b * c – d



(a + b * c - d

expr [Left_parenthesis; Id_token "a"; Plus_token; Id_token "b"; Times_token; Id_token "c"; Minus_token; Id_token "d"];;

Exception: Failure "No matching rparen".

Can't parse because it was expecting a right parenthesis but it got to the end without finding one

a + b) * c – d (

expr [Id_token "a"; Plus_token; Id_token "b"; Right_parenthesis; Times_token; Id_token "c"; Minus_token; Id_token "d"; Left_parenthesis];;

- : expr * token list =

(Plus_Expr
(Factor_as_Term (Id_as_Factor "a"),
Term_as_Expr (Factor_as_Term (Id_as_Factor "b"))),
[Right_parenthesis; Times_token; Id_token "c"; Minus_token; Id_token "d"; Left_parenthesis])

Parsing Whole String

- Q: How to guarantee whole string parses?
- A: Check returned tokens empty

```
let parse tokens  =
   match expr tokens
    with (expr_parse, []) -> expr_parse
    | _ -> raise (Failure "No parse");;
```

- Fixes <expr> as start symbol

Streams in Place of Lists

- More realistically, we don't want to create the entire list of tokens before we can start parsing
- We want to generate one token at a time and use it to make one step in parsing
- Can use (token * (unit -> token)) or (token * (unit -> token option))
  in place of token list

Problems for Recursive-Descent Parsing

- Left Recursion:
  A ::= Aw
  translates to a subroutine that loops forever
- Indirect Left Recursion:
  A ::= Bw
  B ::= Av
  causes the same problem

Problems for Recursive-Descent Parsing

- Parser must always be able to choose the next action based only only the very next token
- Pairwise Disjointedness Test: Can we always determine which rule (in the non-extended BNF) to choose based on just the first token

Pairwise Disjointedness Test

- For each rule
  A ::= *y*

  Calculate

  FIRST (*y*) =

  {a | *y* =>* a*w*} ∪ {ε | if y =>* ε}
- For each pair of rules A ::= *y* and A ::= *z*, require FIRST(*y*) ∩ FIRST(*z*) = { }

Example

Grammar:
<S> ::= <A> a <B> b
<A> ::= <A> b | b
<B> ::= a <B> | a

FIRST (<A> b) = {b}
FIRST (b) = {b}
Rules for <A> not pairwise disjoint

Eliminating Left Recursion

- Rewrite grammar to shift left recursion to right recursion
  - Changes associativity
- Given

<expr> ::= <expr> + <term> and
<expr> ::= <term>
- Add new non-terminal <e> and replace above rules with

<expr> ::= <term><e>
<e> ::= + <term><e> | ε

Factoring Grammar

- Test too strong: Can't handle
  <expr> ::= <term> [(+ | -) <expr>]
- Answer: Add new non-terminal and replace above rules by
  <expr> ::= <term><e>
  <e> ::= + <term><e>
  <e> ::= - <term><e>
  <e> ::= ε
- You are delaying the decision point

Example

Both <A> and <B> have problems:

<S> ::= <A> a <B> b
<A> ::= <A> b | b
<B> ::= a <B> | a

Transform grammar to:

<S> ::= <A> a <B> b
<A> ::-= b<A1>
<A1> :: b<A1> | ε
<B> ::= a<B1>
<B1> ::= a<B1> | ε

Semantics

- Expresses the meaning of syntax
- Static semantics
  - Meaning based only on the form of the expression without executing it
  - Usually restricted to type checking / type inference

Dynamic semantics

- Method of describing meaning of executing a program
- Several different types:
  - Operational Semantics
  - Axiomatic Semantics
  - Denotational Semantics

Dynamic Semantics

- Different languages better suited to different types of semantics
- Different types of semantics serve different purposes

Operational Semantics

- Start with a simple notion of machine
- Describe how to execute (implement) programs of language on virtual machine, by describing how to execute each program statement (ie, following the *structure* of the program)
- Meaning of program is how its execution changes the state of the machine
- Useful as basis for implementations

Axiomatic Semantics

- Also called Floyd-Hoare Logic
- Based on formal logic (first order predicate calculus)
- Axiomatic Semantics is a logical system built from *axioms* and *inference rules*
- Mainly suited to simple imperative programming languages

Axiomatic Semantics

- Used to formally prove a property (*post-condition*) of the *state* (the values of the program variables) after the execution of program, assuming another property (*pre-condition*) of the state before execution
- Written :

  {Precondition} Program {Postcondition}
- Source of idea of *loop invariant*

Denotational Semantics

- Construct a function $\mathcal{M}$ assigning a mathematical meaning to each program construct
- Lambda calculus often used as the range of the meaning function
- Meaning function is compositional: meaning of construct built from meaning of parts
- Useful for proving properties of programs

Natural Semantics

- Aka Structural Operational Semantics, aka "Big Step Semantics"
- Provide value for a program by rules and derivations, similar to type derivations
- Rule conclusions look like

$$(C, m) \Downarrow m'$$

or

$$(E, m) \Downarrow v$$

Simple Imperative Programming Language

- $I \in$ *Identifiers*
- $N \in$ *Numerals*
- B ::= true | false | B & B | B or B | not B | $E < E$ | $E = E$
- E::= N | I | $E + E$ | $E * E$ | $E - E$ | $- E$
- C::= skip | C;C | I ::= E | if B then C else C fi | while B do C od

Natural Semantics of Atomic Expressions

- Identifiers: $(I,m) \Downarrow m(I)$
- Numerals are values: $(N,m) \Downarrow N$
- Booleans: (true,m) $\Downarrow$ true
  (false ,m) $\Downarrow$ false

Booleans:

$$\frac{(B, m) \Downarrow \text{false}}{(B \,\&\, B', m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{true} \quad (B', m) \Downarrow b}{(B \,\&\, B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(B \text{ or } B', m) \Downarrow \text{true}} \qquad \frac{(B, m) \Downarrow \text{false} \quad (B', m) \Downarrow b}{(B \text{ or } B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(\text{not } B, m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{false}}{(\text{not } B, m) \Downarrow \text{true}}$$

Relations

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \sim V = b}{(E \sim E', m) \Downarrow b}$$

- By $U \sim V = b$, we mean does (the meaning of) the relation ~ hold on the meaning of U and V
- May be specified by a mathematical expression/equation or rules matching U and V

Arithmetic Expressions

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \; op \; V = N}{(E \; op \; E', m) \Downarrow N}$$

where N is the specified value for $U \; op \; V$

Commands

Skip: $(\text{skip}, m) \Downarrow m$

Assignment: $$\frac{(E,m) \Downarrow V}{(I::=E,m) \Downarrow m[I <\text{-- } V]}$$

Sequencing: $$\frac{(C,m) \Downarrow m' \quad (C',m') \Downarrow m''}{(C;C', m) \Downarrow m''}$$

If Then Else Command

$$\frac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

$$\frac{(B,m) \Downarrow \text{false} \quad (C',m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

While Command

$$\frac{(B,m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m}$$

$$\frac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m' \quad (\text{while } B \text{ do } C \text{ od}, m') \Downarrow m''}{(\text{while } B \text{ do C od}, m) \Downarrow m''}$$

Example: If Then Else Rule

$$\frac{}{(\text{if } x > 5 \text{ then } y:= 2 + 3 \text{ else } y:=3 + 4 \text{ fi}, \{x \text{ -> } 7\}) \Downarrow ?}$$

Example: If Then Else Rule

$$\frac{\frac{}{(x > 5, \{x \text{ -> } 7\}) \Downarrow ?}}{(\text{if } x > 5 \text{ then } y:= 2 + 3 \text{ else } y:=3 + 4 \text{ fi}, \{x \text{ -> } 7\}) \Downarrow ?}$$

Example: Arith Relation

$$\frac{\dfrac{(x,\{x\text{->}7\})\Downarrow ?\quad (5,\{x\text{->}7\})\Downarrow ?}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow ?}}{\text{(if } x > 5 \text{ then y:= 2 + 3 else y:=3 + 4 fi, } \{x \text{ -> } 7\}) \Downarrow ?}\quad ? > ? = ?$$

Example: Identifier(s)

$$\frac{\dfrac{(x,\{x\text{->}7\})\Downarrow 7\quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow ?}}{\text{(if } x > 5 \text{ then y:= 2 + 3 else y:=3 + 4 fi, } \{x \text{ -> } 7\}) \Downarrow ?}\quad 7 > 5 = \text{true}$$

Example: Arith Relation

$$\frac{\dfrac{(x,\{x\text{->}7\})\Downarrow 7\quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow \text{true}}}{\text{(if } x > 5 \text{ then y:= 2 + 3 else y:=3 + 4 fi, } \{x \text{ -> } 7\}) \Downarrow ?}\quad 7 > 5 = \text{true}$$

Example: If Then Else Rule

$$\frac{\dfrac{(x,\{x\text{->}7\})\Downarrow 7\quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow \text{true}}\qquad \dfrac{}{(y:= 2 + 3,\ \{x\text{-> } 7\} \Downarrow ?}\ .}{\text{(if } x > 5 \text{ then y:= 2 + 3 else y:=3 + 4 fi, } \{x \text{ -> } 7\}) \Downarrow ?}\quad 7 > 5 = \text{true}$$

Example: Assignment

$$\frac{\dfrac{(x,\{x\text{->}7\})\Downarrow 7\quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow \text{true}}\qquad \dfrac{(2+3,\ \{x\text{->}7\})\Downarrow ?}{(y:= 2 + 3,\ \{x\text{-> } 7\} \Downarrow ?}}{\text{(if } x > 5 \text{ then y:= 2 + 3 else y:=3 + 4 fi, } \{x \text{ -> } 7\}) \Downarrow ?}\quad 7 > 5 = \text{true}$$

Example: Arith Op

$$\frac{\dfrac{(x,\{x\text{->}7\})\Downarrow 7\quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow \text{true}}\qquad \dfrac{(2+3,\ \{x\text{->}7\})\Downarrow ?}{(y:= 2 + 3,\ \{x\text{-> } 7\} \Downarrow ?}\ .}{\text{(if } x > 5 \text{ then y:= 2 + 3 else y:=3 + 4 fi, } \{x \text{ -> } 7\}) \Downarrow ?}\quad \dfrac{(2,\{x\text{->}7\})\Downarrow ?\quad (3,\{x\text{->}7\}) \Downarrow ?}{7 > 5 = \text{true}}\quad ? + ? = ?$$

Example: Numerals

$$\dfrac{\dfrac{(x,\{x\to 7\})\Downarrow 7 \quad (5,\{x\to 7\})\Downarrow 5}{7 > 5 = \text{true}} \quad \dfrac{\dfrac{(2,\{x\to 7\})\Downarrow 2 \quad (3,\{x\to 7\})\Downarrow 3}{2+3 = 5} \quad (2+3,\{x\to 7\})\Downarrow ?}{(y:= 2+3,\{x\to 7\})\Downarrow ?}}{(x > 5,\{x\to 7\})\Downarrow \text{true}}$$

$$(\text{if } x > 5 \text{ then } y:= 2+3 \text{ else } y:=3+4 \text{ fi},\ \{x\to 7\}) \Downarrow ?$$

Example: Arith Op

$$\dfrac{\dfrac{(x,\{x\to 7\})\Downarrow 7 \quad (5,\{x\to 7\})\Downarrow 5}{7 > 5 = \text{true}} \quad \dfrac{\dfrac{(2,\{x\to 7\})\Downarrow 2 \quad (3,\{x\to 7\})\Downarrow 3}{2+3 = 5} \quad (2+3,\{x\to 7\})\Downarrow 5}{(y:= 2+3,\{x\to 7\})\Downarrow ?}}{(x > 5,\{x\to 7\})\Downarrow \text{true}}$$

$$(\text{if } x > 5 \text{ then } y:= 2+3 \text{ else } y:=3+4 \text{ fi},\ \{x\to 7\}) \Downarrow\ ?$$

Example: Assignment

$$\dfrac{\dfrac{(x,\{x\to 7\})\Downarrow 7 \quad (5,\{x\to 7\})\Downarrow 5}{7 > 5 = \text{true}} \quad \dfrac{\dfrac{(2,\{x\to 7\})\Downarrow 2 \quad (3,\{x\to 7\})\Downarrow 3}{2+3 = 5} \quad (2+3,\{x\to 7\})\Downarrow 5}{(y:= 2+3,\{x\to 7\})\Downarrow \{x\to 7, y\to 5\}}}{(x > 5,\{x\to 7\})\Downarrow \text{true}}$$

$$(\text{if } x > 5 \text{ then } y:= 2+3 \text{ else } y:=3+4 \text{ fi},\ \{x\to 7\}) \Downarrow ?$$

Example: If Then Else Rule

$$\dfrac{\dfrac{(x,\{x\to 7\})\Downarrow 7 \quad (5,\{x\to 7\})\Downarrow 5}{7 > 5 = \text{true}} \quad \dfrac{\dfrac{(2,\{x\to 7\})\Downarrow 2 \quad (3,\{x\to 7\})\Downarrow 3}{2+3 = 5} \quad (2+3,\{x\to 7\})\Downarrow 5}{(y:= 2+3,\{x\to 7\})\Downarrow \{x\to 7, y\to 5\}}}{(x > 5,\{x\to 7\})\Downarrow \text{true}}$$

$$(\text{if } x > 5 \text{ then } y:= 2+3 \text{ else } y:=3+4 \text{ fi},\ \{x\to 7\}) \Downarrow\ \{x\to 7, y\to 5\}$$

Let in Command

$$\dfrac{(E,m) \Downarrow v \quad (C, m[I\leftarrow v]) \Downarrow m'}{(\text{let } I = E \text{ in } C,\ m) \Downarrow m''}$$

Where $m''(y) = m'(y)$ for $y \neq I$ and $m''(I) = m(I)$ if $m(I)$ is defined, and $m''(I)$ is undefined otherwise

Example

$$\dfrac{(5,\{x\to 17\}) \Downarrow 5 \quad \dfrac{(x,\{x\to 5\}) \Downarrow 5 \quad (3,\{x\to 5\}) \Downarrow 3}{(x+3,\{x\to 5\}) \Downarrow 8} \quad (x:=x+3,\{x\to 5\}) \Downarrow \{x\to 8\}}{(\text{let } x = 5 \text{ in } (x:=x+3),\ \{x\to 17\}) \Downarrow ?}$$

Example

$$\dfrac{(5,\{x\text{->}17\}) \Downarrow 5 \quad \dfrac{(x,\{x\text{->}5\}) \Downarrow 5 \quad (3,\{x\text{->}5\}) \Downarrow 3}{(x+3,\{x\text{->}5\}) \Downarrow 8}}{(x:=x+3,\{x\text{->}5\}) \Downarrow \{x\text{->}8\}}}{(\text{let } x = 5 \text{ in } (x:=x+3), \{x \text{ -> } 17\}) \Downarrow \{x\text{->}17\}}$$

Comment

- Simple Imperative Programming Language introduces variables *implicitly* through assignment
- The let-in command introduces scoped variables *explictly*
- Clash of constructs apparent in awkward semantics

Interpretation Versus Compilation

- A **compiler** from language L1 to language L2 is a program that takes an L1 program and for each piece of code in L1 generates a piece of code in L2 of same meaning
- An **interpreter** of L1 in L2 is an L2 program that executes the meaning of a given L1 program
- Compiler would examine the body of a loop once; an interpreter would examine it every time the loop was executed

Interpreter

- An *Interpreter* represents the operational semantics of a language L1 (source language) in the language of implementation L2 (target language)
- Built incrementally
  - Start with literals
  - Variables
  - Primitive operations
  - Evaluation of expressions
  - Evaluation of commands/declarations

Interpreter

- Takes abstract syntax trees as input
  - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
  - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next "state"
  - To get final value, put in a loop

Natural Semantics Example

- compute_exp (Var(v), m) = look_up v m
- compute_exp (Int(n), _) = Num (n)
- …
- compute_com(IfExp(b,c1,c2),m) =
  if compute_exp (b,m) = Bool(true)
  then compute_com (c1,m)
  else compute_com (c2,m)

Natural Semantics Example

- compute_com(While(b,c), m) =
  if compute_exp (b,m) = Bool(false)
  then m
  else compute_com
  (While(b,c), compute_com(c,m))

- May fail to terminate - exceed stack limits
- Returns no useful information then