Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve, Gul Agha, and Elsa L Gunter

BNF Grammars

- Start with a set of characters, **a**,**b**,**c**,...
  - We call these *terminals*
- Add a set of different characters, **X**,**Y**,**Z**,...
  - We call these *nonterminals*
- One special nonterminal **S** called *start symbol*

BNF Grammars

- BNF rules (aka *productions*) have form

  $$\mathbf{X} ::= y$$

  where **X** is any nonterminal and *y* is a string of terminals and nonterminals
- BNF *grammar* is a set of BNF rules such that every nonterminal appears on the left of some rule

Sample Grammar

- Terminals: 0 1 + ( )
- Nonterminals: <Sum>
- Start symbol = <Sum>

- <Sum> ::= 0
- <Sum >::= 1
- <Sum> ::= <Sum> + <Sum>
- <Sum> ::= (<Sum>)
- Can be abbreviated as

<Sum> ::= 0 | 1
                | <Sum> + <Sum> | ( )

BNF Deriviations

- Given rules

$$\mathbf{X} ::= y\mathbf{Z}w \text{ and } \mathbf{Z} ::= v$$

we may replace **Z** by *v* to say

$$\mathbf{X} => y\mathbf{Z}w => yvw$$

- Sequence of such replacements called *derivation*
- Derivation called *right-most* if always replace the right-most non-terminal

BNF Semantics

- The meaning of a BNF grammar is the set of all strings consisting only of terminals that can be derived from the Start symbol

BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Start with the start symbol:

<Sum> =>

BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a non-terminal

<Sum> =>

BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a rule and substitute:
  - <Sum> ::= <Sum> + <Sum>

<Sum> => <Sum> + <Sum >

BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >

BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a rule and substitute:
  - <Sum> ::= (<Sum>)

<Sum> => <Sum> + <Sum >

=> (<Sum>) + <Sum>

BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >

=> (<Sum>) + <Sum>

BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a rule and substitute:
  - <Sum> ::= <Sum> + <Sum>

<Sum> => <Sum> + <Sum >

=> (<Sum>) + <Sum>

=> (<Sum> + <Sum>) + <Sum>

BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>

BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a rule and substitute:
  - <Sum >::= 1

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>

BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>

BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a rule and substitute:
  - <Sum >::= 0

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) + 0

BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) + 0

BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a rule and substitute
  - <Sum> ::= 0

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) 0
=> (0 + 1) + 0

BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- (0 + 1) + 0 is generated by grammar

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) + 0
=> (0 + 1) + 0

Regular Grammars

- Subclass of BNF
- Only rules of form
  <nonterminal>::=<terminal><nonterminal> or
  <nonterminal>::=<terminal> or
  <nonterminal>::=ε
- Defines same class of languages as regular expressions
- Important for writing lexers (programs that convert strings of characters into strings of tokens)

Example

- Regular grammar:
  <Balanced> ::= ε
  <Balanced> ::= 0<OneAndMore>
  <Balanced> ::= 1<ZeroAndMore>
  <OneAndMore> ::= 1<Balanced>
  <ZeroAndMore> ::= 0<Balanced>
- Generates even length strings where every initial substring of even length has same number of 0's as 1's

Extended BNF Grammars

- Alternatives: allow rules of from $X::=y|z$
  - Abbreviates $X::= y$, $X::= z$
- Options: $X::=y[v]z$
  - Abbreviates $X::=yvz$, $X::=yz$
- Repetition: $X::=y\{v\}^*z$
  - Can be eliminated by adding new nonterminal V and rules $X::=yz$, $X::=yVz$, $V::=v$, $V::=vV$

Parse Trees

- Graphical representation of derivation
- Each node labeled with either non-terminal or terminal
- If node is labeled with a terminal, then it is a leaf (no sub-trees)
- If node is labeled with a non-terminal, then it has one branch for each character in the right-hand side of rule used to substitute for it

Example

- Consider grammar:

  <exp> ::= <factor>

  | <factor> + <factor>

  <factor> ::= <bin>

  | <bin> * <exp>

  <bin> ::= 0 | 1

- Problem: Build parse tree for 1 * 1 + 0 as an <exp>

Example cont.

- 1 * 1 + 0: <exp>

<exp> is the start symbol for this parse tree

Example cont.

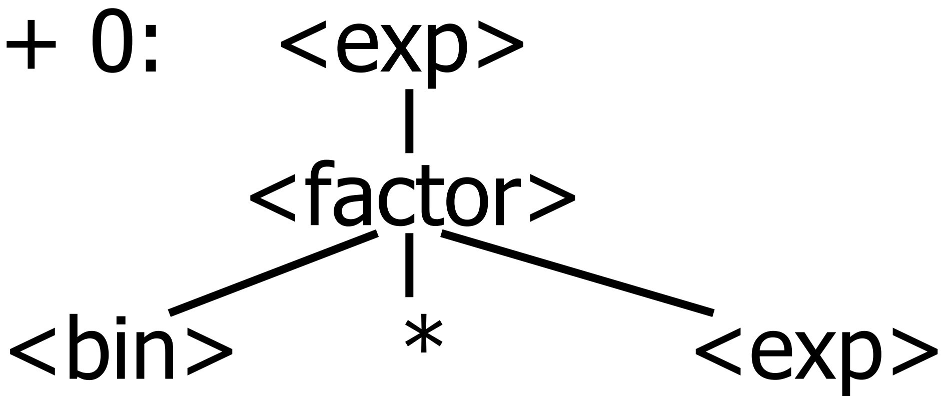
- 1 * 1 + 0: <exp>

<exp>
|
<factor>

Use rule: <exp> ::= <factor>

Example cont.

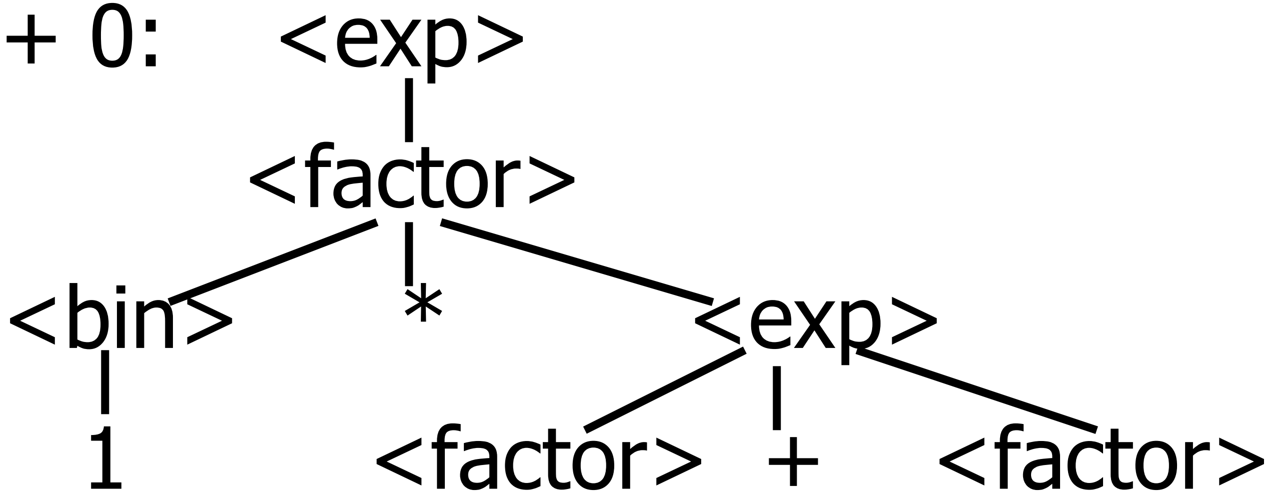
- 1 * 1 + 0:



Use rule: <factor> ::= <bin> * <exp>

Example cont.

- 1 * 1 + 0:

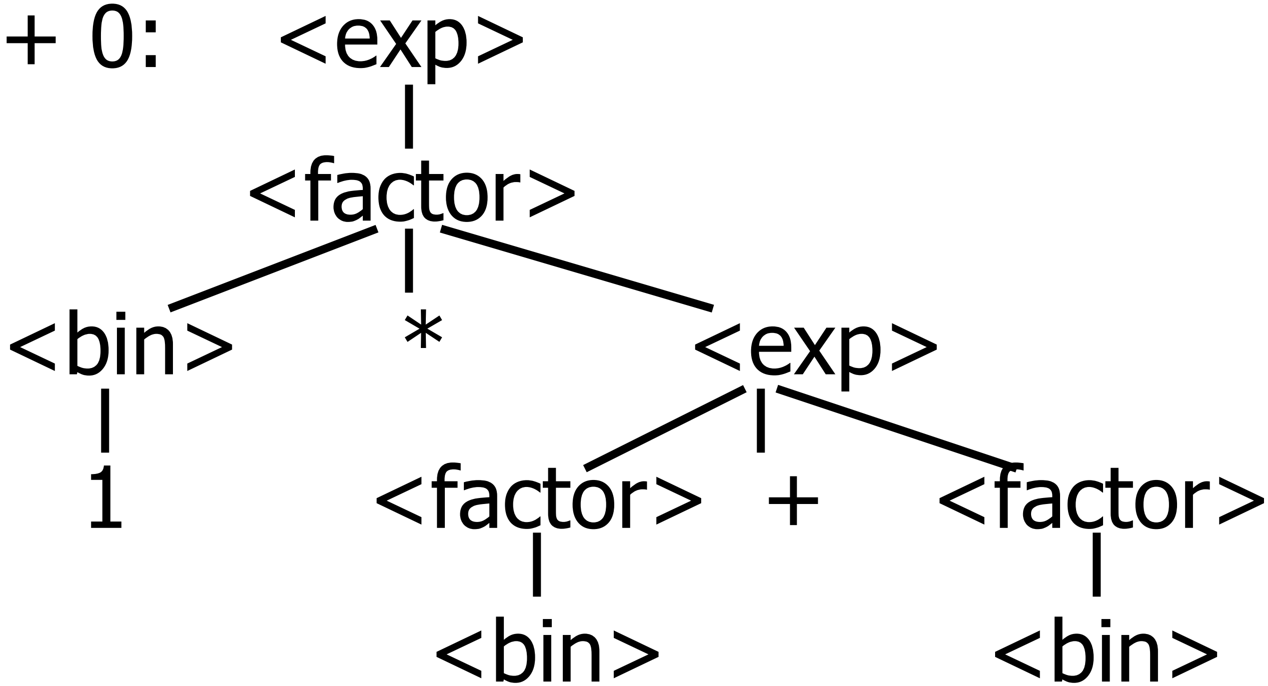


Use rules: <bin> ::= 1 and

<exp> ::= <factor> + <factor>

Example cont.

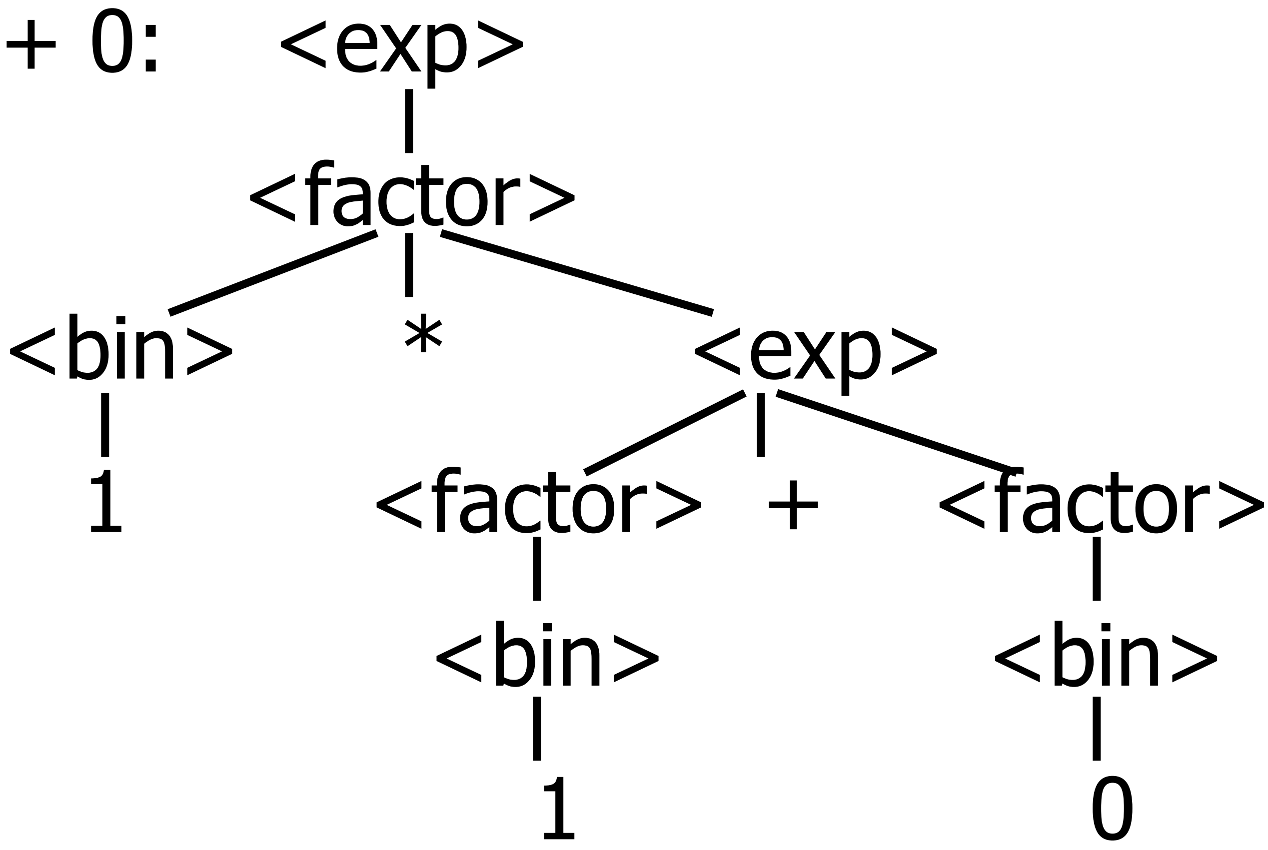
- 1 * 1 + 0:



Use rule: <factor> ::= <bin>

Example cont.

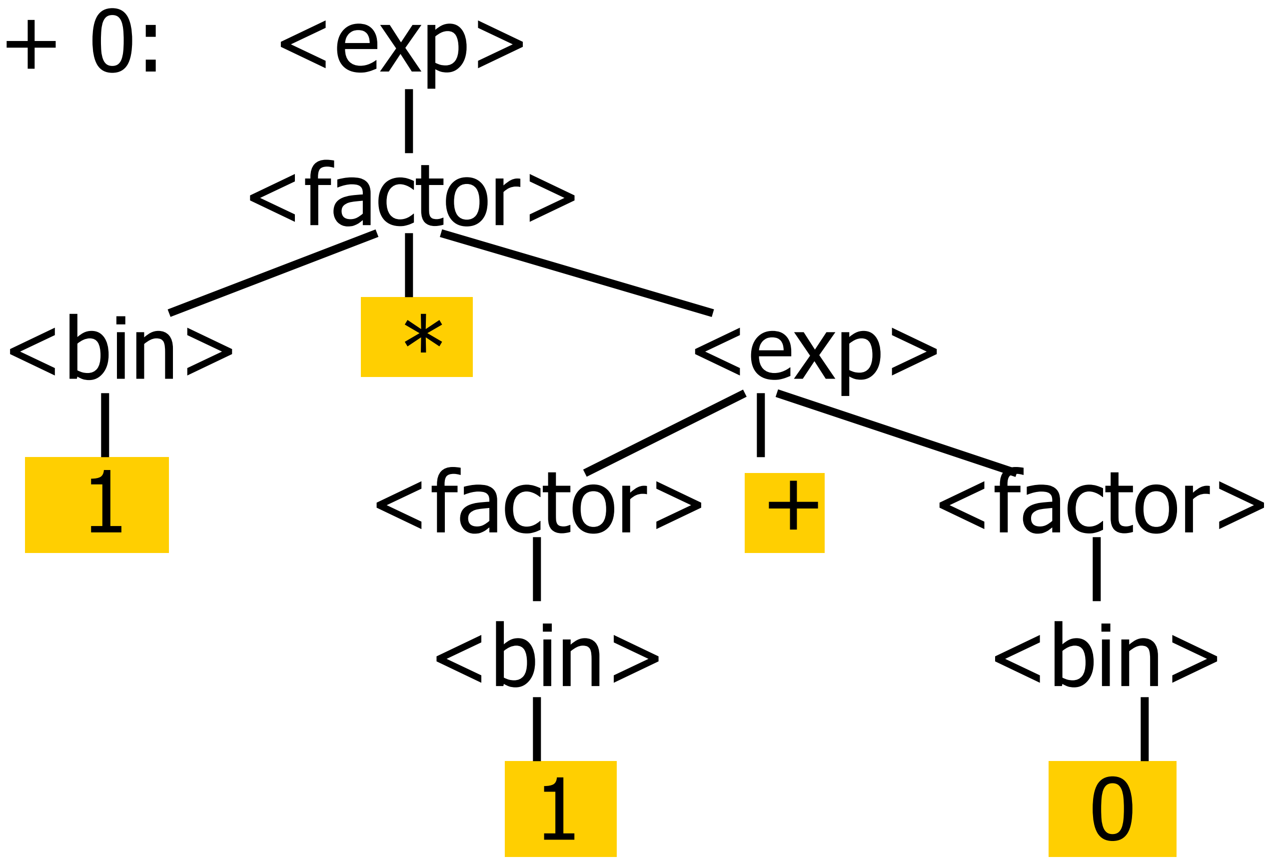
- 1 * 1 + 0:



Use rules: <bin> ::= 1 | 0

Example cont.

- 1 * 1 + 0:



Fringe of tree is string generated by grammar

Your Turn: 1 * 0 + 0 * 1

<exp> ::= <factor>
| <factor> + <factor>
<factor> ::= <bin>
| <bin> * <exp>
<bin> ::= 0 | 1

Parse Tree Data Structures

- Parse trees may be represented by OCaml datatypes
- One datatype for each nonterminal
- One constructor for each rule
- Defined as mutually recursive collection of datatype declarations

Example

- Recall grammar:

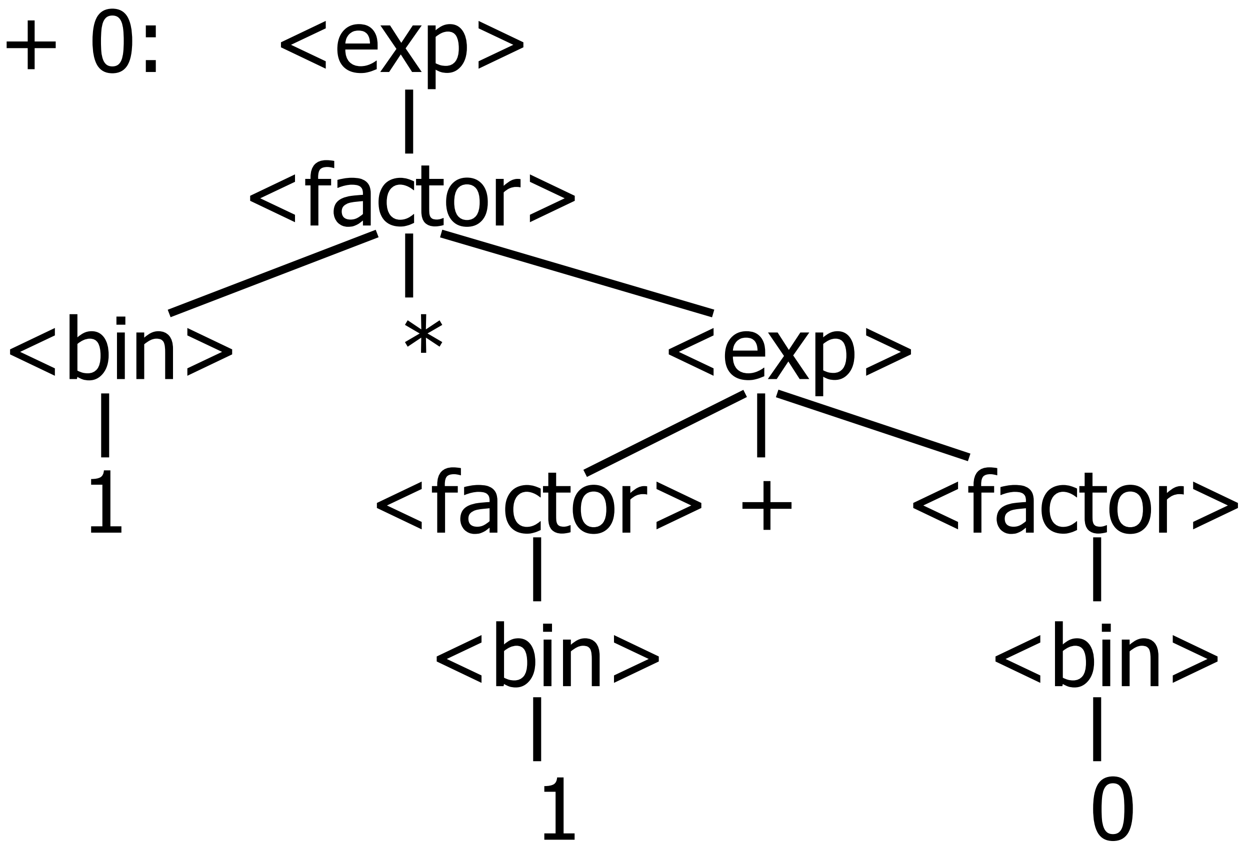
```
<exp>  ::= <factor>  |  <factor> +  <factor>
<factor>  ::=  <bin> |  <bin>  *  <exp>
<bin> ::=  0  | 1
```

- ```
  type exp = Factor2Exp of factor
           | Plus of factor * factor
    and factor = Bin2Factor of bin
               | Mult of bin * exp
    and bin = Zero | One
  ```
  ```

Example cont.

```
type exp = Factor2Exp of factor
         | Plus of factor * factor
  and factor = Bin2Factor of bin
             | Mult of bin * exp
  and bin = Zero | One
```

- 1 * 1 + 0:



Example cont.

- Can be represented as

```
Factor2Exp
(Mult(One,
          Plus(Bin2Factor One,
                  Bin2Factor Zero)))
```

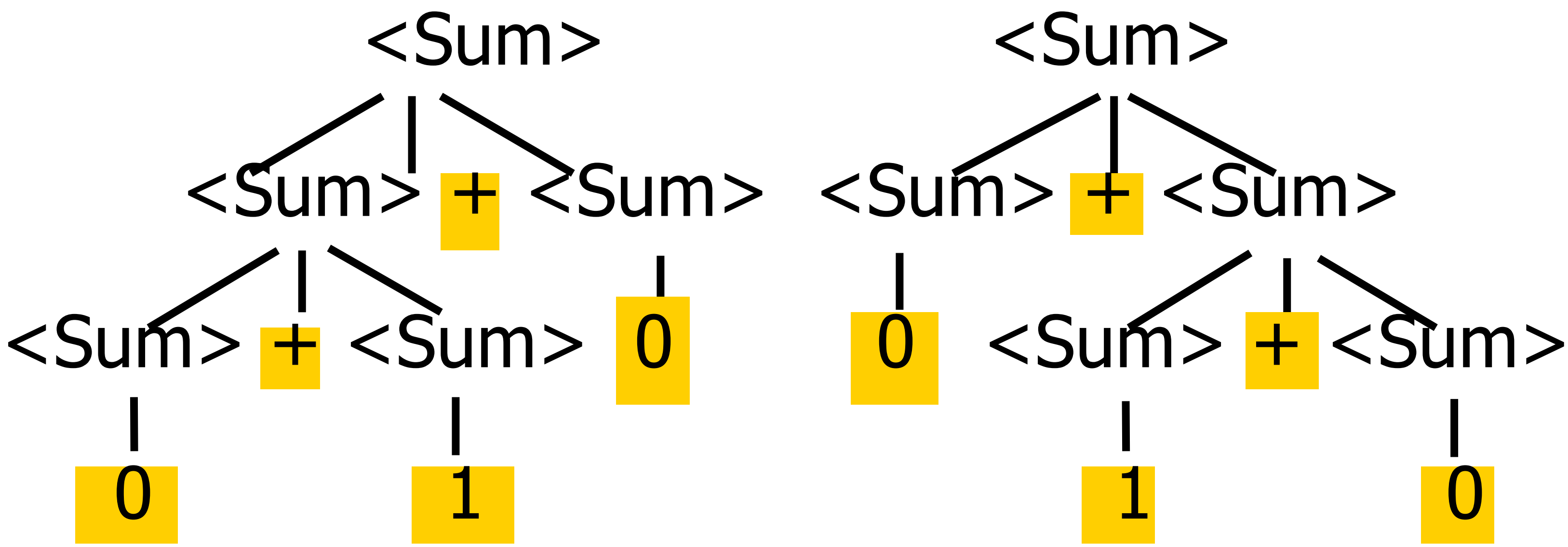
Ambiguous Grammars and Languages

- A BNF grammar is *ambiguous* if its language contains strings for which there is more than one parse tree

- If all BNF's for a language are ambiguous then the language is *inherently ambiguous*

Example: Ambiguous Grammar

- 0 + 1 + 0



Example

- What is the result for:

$$3 + 4 * 5 + 6$$

Example

- What is the result for:

$$3 + 4 * 5 + 6$$

- Possible answers:
  - $41 = ((3 + 4) * 5) + 6$
  - $47 = 3 + (4 * (5 + 6))$
  - $29 = (3 + (4 * 5)) + 6 = 3 + ((4 * 5) + 6)$
  - $77 = (3 + 4) * (5 + 6)$

Example

- What is the value of:

$$7 - 5 - 2$$

Example

- What is the value of:

$$7 - 5 - 2$$

- Possible answers:
  - In Pascal, C++, SML assoc. left

    $7 - 5 - 2 = (7 - 5) - 2 = 0$
  - In APL, associate to right

    $7 - 5 - 2 = 7 - (5 - 2) = 4$

Two Major Sources of Ambiguity

- Lack of determination of operator precedence
- Lack of determination of operator assoicativity

- Not the only sources of ambiguity

Disambiguating a Grammar

- Given ambiguous grammar G, with start symbol S, find a grammar G’ with same start symbol, such that

  language of G = language of G’

- Not always possible
- No algorithm in general

Disambiguating a Grammar

- Idea: Each non-terminal represents all strings having some property
- Identify these properties (often in terms of things that can't happen)
- Use these properties to inductively guarantee every string in language has a unique parse

Steps to Grammar Disambiguation

- Identify the rules and a smallest use that display ambiguity
- Decide which parse to keep; why should others be thrown out?
- What syntactic restrictions on subexpressions are needed to throw out the bad (while keeping the good)?
- Add a new non-terminal and rules to describe this set of restricted subexpressions (called stratifying, or refactoring)
- Replace old rules to use new non-terminals
- Rinse and repeat

Example

- Ambiguous grammar:
  <exp> ::= 0 | 1 | <exp> + <exp>
  | <exp> * <exp>
- String with more then one parse:

  0 + 1 + 0

  1 * 1 + 1
- Source of ambiuity: associativity and precedence

Two Major Sources of Ambiguity

- Lack of determination of operator precedence
- Lack of determination of operator assoicativity

- Not the only sources of ambiguity

How to Enforce Associativity

- Have at most one recursive call per production

- When two or more recursive calls would be natural leave right-most one for right assoicativity, left-most one for left assoiciativity

Example

- <Sum> ::= 0 | 1 | <Sum> + <Sum>
  | (<Sum>)
- Becomes
  - <Sum> ::= <Num> | <Num> + <Sum>
  - <Num> ::= 0 | 1 | (<Sum>)

Operator Precedence

- Operators of highest precedence evaluated first (bind more tightly).
- Precedence for infix binary operators given in following table
- Needs to be reflected in grammar

Precedence Table - Sample

	Fortan	Pascal	C/C++	Ada	SML
highest	**	*, /, div, mod	++, --	**	div, mod, /, *
	*, /	+, -	*, /, %	*, /, mod	+, -, ^
	+, -		+, -	+, -	::

First Example Again

- In any above language, 3 + 4 * 5 + 6 = 29
- In APL, all infix operators have same precedence
  - Thus we still don't know what the value is (handled by associativity)
- How do we handle precedence in grammar?

Predence in Grammar

- Higher precedence translates to longer derivation chain
- Example:

<exp> ::= 0 | 1 | <exp> + <exp>
| <exp> * <exp>

- Becomes

<exp> ::= <mult_exp>
| <exp> + <mult_exp>
<mult_exp> ::= <id> | <mult_exp> * <id>
<id> ::= 0 | 1

Parser Code

- <*grammar*>.ml defines one parsing function per entry point
- Parsing function takes a lexing function (lexer buffer to token) and a lexer buffer as arguments
- Returns semantic attribute of corresponding entry point

Ocamlyacc Input

- File format:

```
%{
    <header>
%}
    <declarations>
%%
    <rules>
%%
    <trailer>
```

Ocamlyacc <*header*>

- Contains arbitrary Ocaml code
- Typically used to give types and functions needed for the semantic actions of rules and to give specialized error recovery
- May be omitted
- <*footer*> similar. Possibly used to call parser

Ocamlyacc <declarations>

- %token *symbol* … *symbol*
- Declare given symbols as tokens
- %token <*type*> *symbol* … *symbol*
- Declare given symbols as token constructors, taking an argument of type <*type*>
- %start *symbol* … *symbol*
- Declare given symbols as entry points; functions of same names in <*grammar*>.ml

Ocamlyacc <*declarations*>

- %type <*type*> *symbol … symbol*

  Specify type of attributes for given symbols. Mandatory for start symbols
- %left *symbol … symbol*
- %right *symbol … symbol*
- %nonassoc *symbol … symbol*

  Associate precedence and associativity to given symbols. Same line,same precedence; earlier line, lower precedence (broadest scope)

Ocamlyacc <*rules*>

- *nonterminal* :

  symbol ... *symbol* { *semantic_action* }

  | ...

  | *symbol* ... *symbol* { *semantic_action* }

  ;
- Semantic actions are arbitrary Ocaml expressions
- Must be of same type as declared (or inferred) for *nonterminal*
- Access semantic attributes (values) of symbols by position: $1 for first symbol, $2 to second …

Example - Base types

```
(* File: expr.ml *)
type expr =
    Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
and term =
    Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
and factor =
    Id_as_Factor of string
  | Parenthesized_Expr_as_Factor of expr
```

Example - Lexer (exprlex.mll)

```
{ (*open Exprparse*) }
let numeric = ['0' - '9']
let letter =['a' - 'z' 'A' - 'Z']
rule token = parse
  | "+" {Plus_token}
  | "-"  {Minus_token}
  | "*"  {Times_token}
  | "/"  {Divide_token}
  | "("  {Left_parenthesis}
  | ")"  {Right_parenthesis}
  | letter (letter|numeric|"_")* as id  {Id_token id}
  | [' ' '\t' '\n'] {token lexbuf}
  | eof {EOL}
```

Example - Parser (exprparse.mly)

```
%{ open Expr
%}
%token <string> Id_token
%token Left_parenthesis Right_parenthesis
%token Times_token Divide_token
%token Plus_token Minus_token
%token EOL
%start main
%type <expr> main
%%
```

Example - Parser (exprparse.mly)

```
expr:
   term
       { Term_as_Expr $1 }
 | term Plus_token expr
       { Plus_Expr ($1, $3) }
 | term Minus_token expr
       { Minus_Expr ($1, $3) }
```

Example - Base types

```
(* File: expr.ml *)
type expr =
   Term_as_Expr of term
 | Plus_Expr of (term * expr)
 | Minus_Expr of (term * expr)
```

Example - Parser (exprparse.mly)

```
term:
   factor
       { Factor_as_Term $1 }
 | factor Times_token term
       { Mult_Term ($1, $3) }
 | factor Divide_token term
       { Div_Term ($1, $3) }
```

Example - Base types

```
(* File: expr.ml *)
type expr =
    Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
and term =
    Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
```

Example - Parser (exprparse.mly)

```
factor:
    Id_token
        { Id_as_Factor $1 }
  | Left_parenthesis expr Right_parenthesis
        {Parenthesized_Expr_as_Factor $2 }
main:
  | expr EOL
        { $1 }
```

Example - Base types

```
(* File: expr.ml *)
type expr =
    Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
and term =
    Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
and factor =
    Id_as_Factor of string
  | Parenthesized_Expr_as_Factor of expr
```

Example - Using Parser

```
# #use "expr.ml";;
…
# #use "exprparse.ml";;
…
# #use "exprlex.ml";;
…
# let test s =
  let lexbuf = Lexing.from_string (s^"\n") in
      main token lexbuf;;
```

Example - Using Parser

```
# test "a + b";;
- : expr =
Plus_Expr
 (Factor_as_Term (Id_as_Factor "a"),
  Term_as_Expr
    (Factor_as_Term (Id_as_Factor "b"))
  )
```

Example - Base types

```
(* File: expr.ml *)
type expr =
    Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
and term =
    Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
and factor =
    Id_as_Factor of string
  | Parenthesized_Expr_as_Factor of expr
```

LR Parsing

- Read tokens left to right (L)
- Create a rightmost derivation (R)
- How is this possible?
- Start at the bottom (left) and work your way up
- Last step has only one non-terminal to be replaced so is right-most
- Working backwards, replace mixed strings by non-terminals
- Always proceed so that there are no non-terminals to the right of the string to be replaced

Example

$(\quad 0 \quad + \quad 1 \quad) \quad + \quad 0$

Example

(0 + 1) + 0

Example

(0 + 1) + 0

Example

<Sum>

(0 + 1) + 0

Example

<Sum>

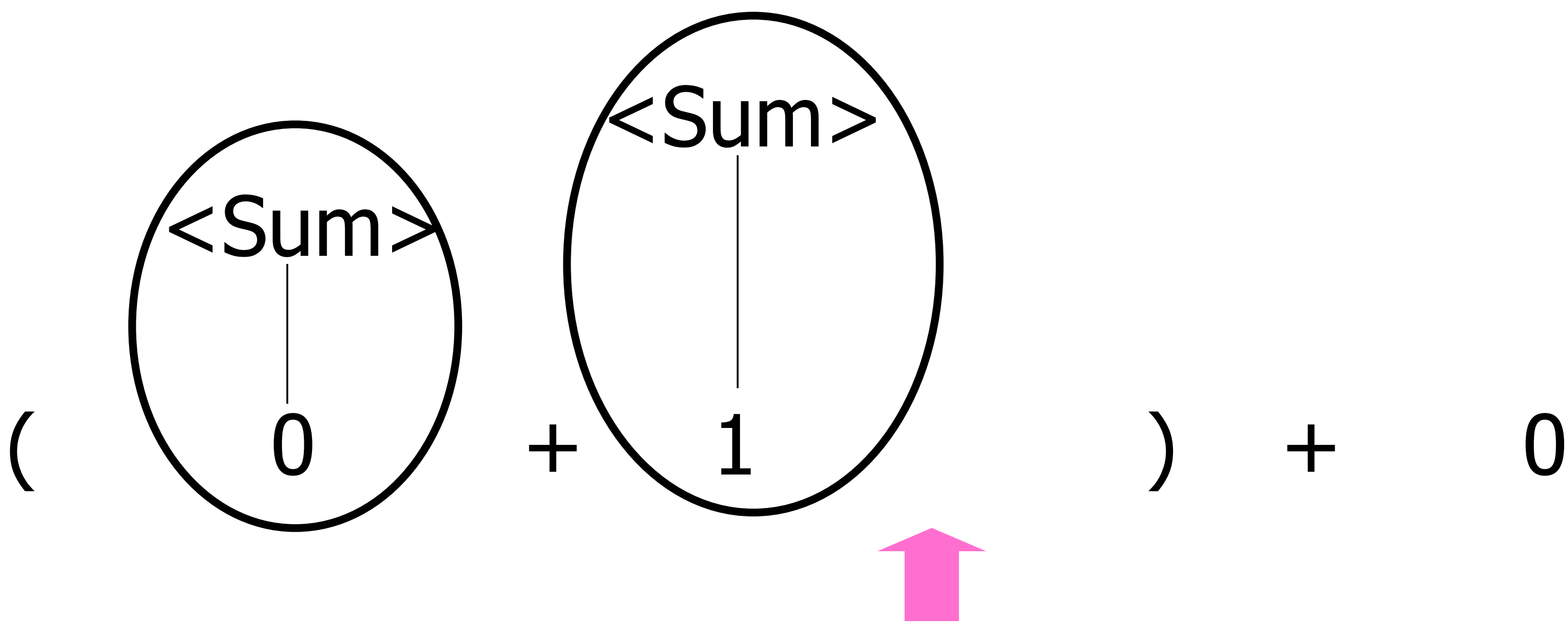
(0 + 1) + 0

Example

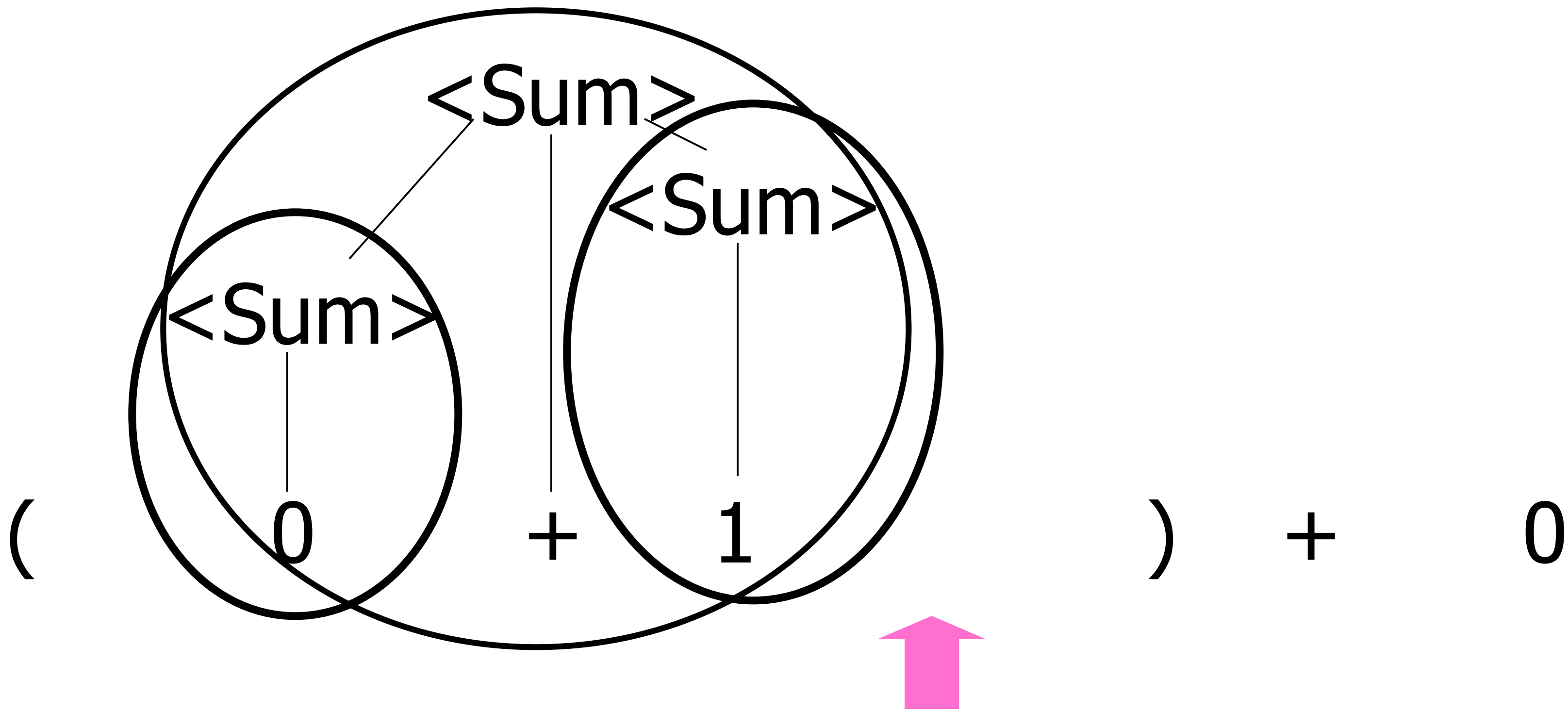
<Sum>

(0 + 1) + 0

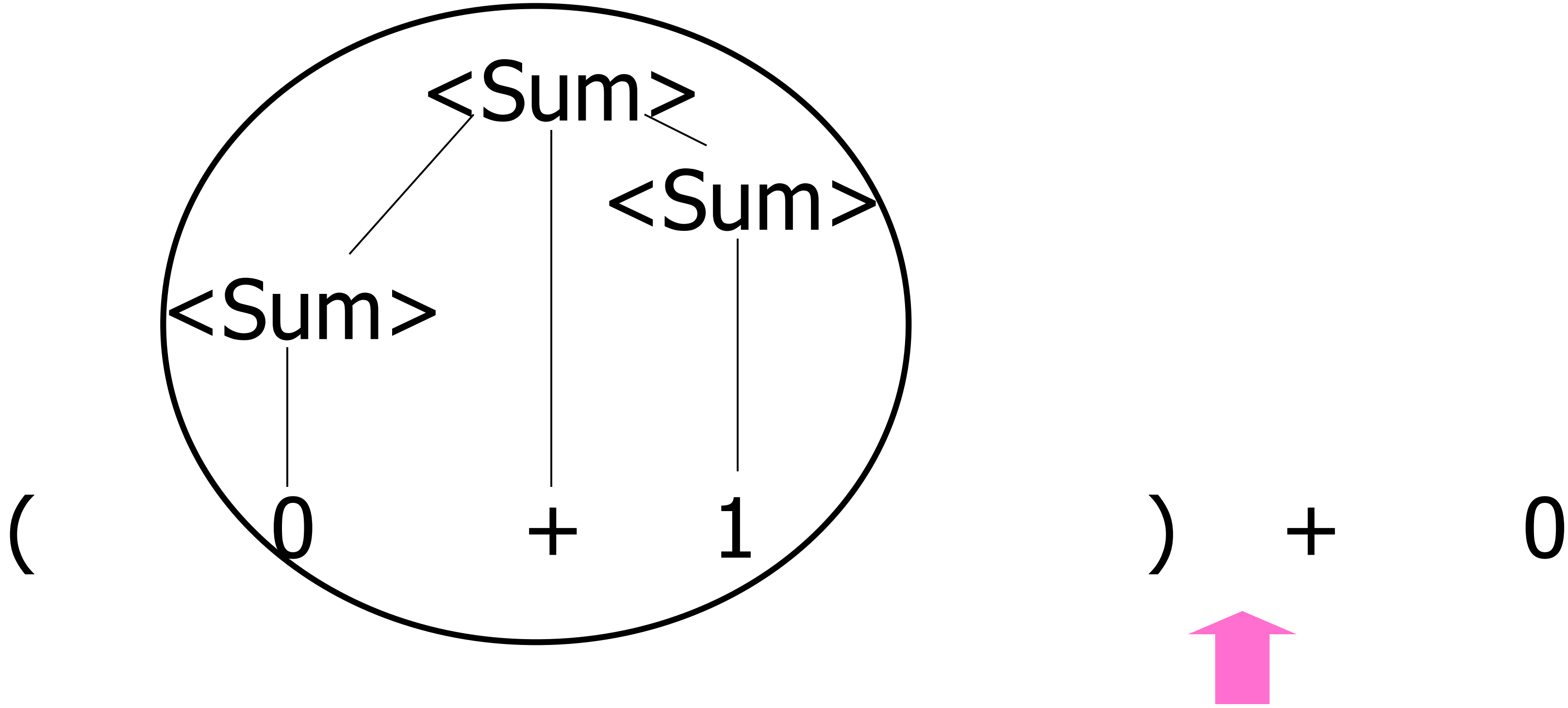
Example



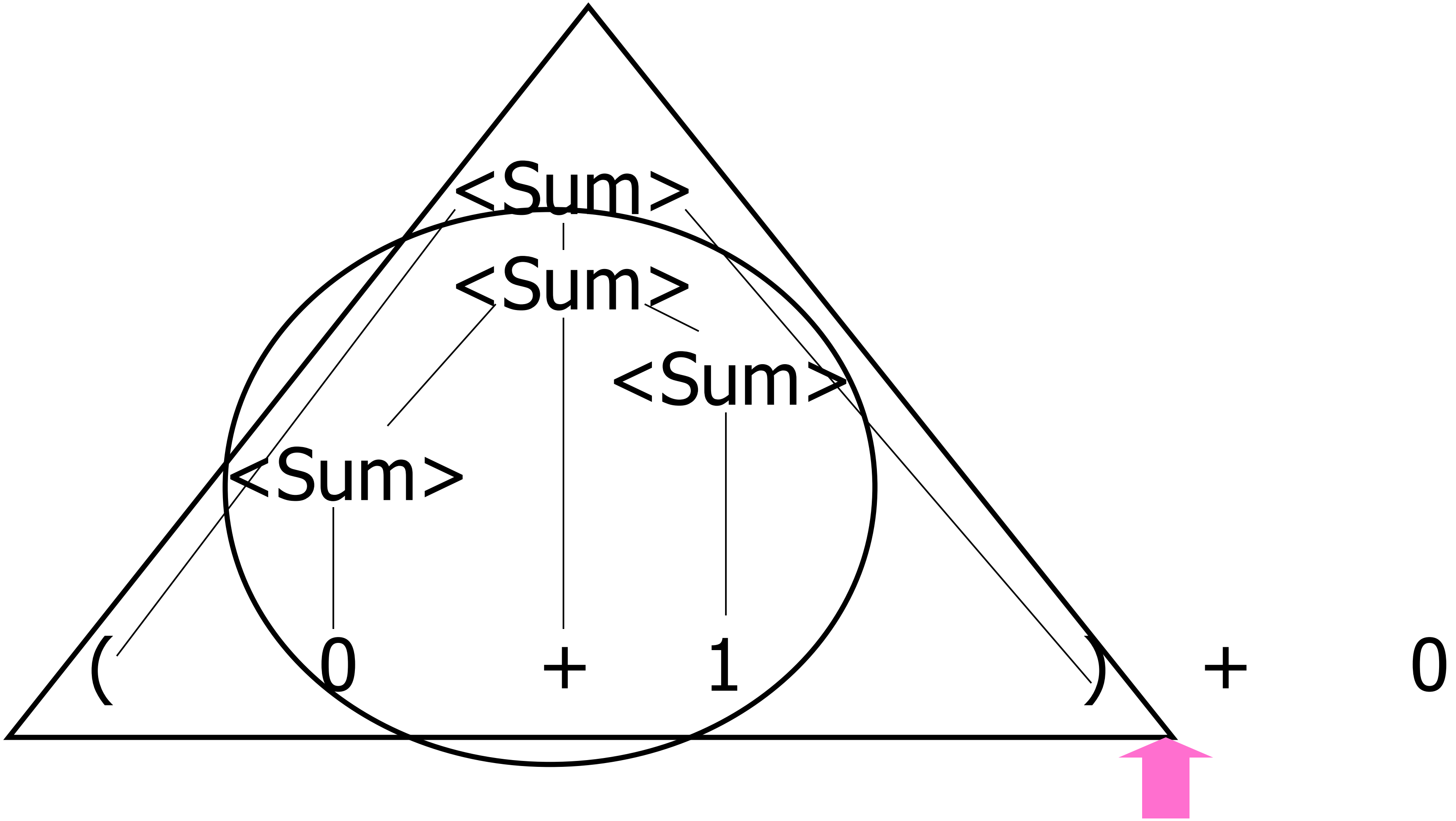
Example



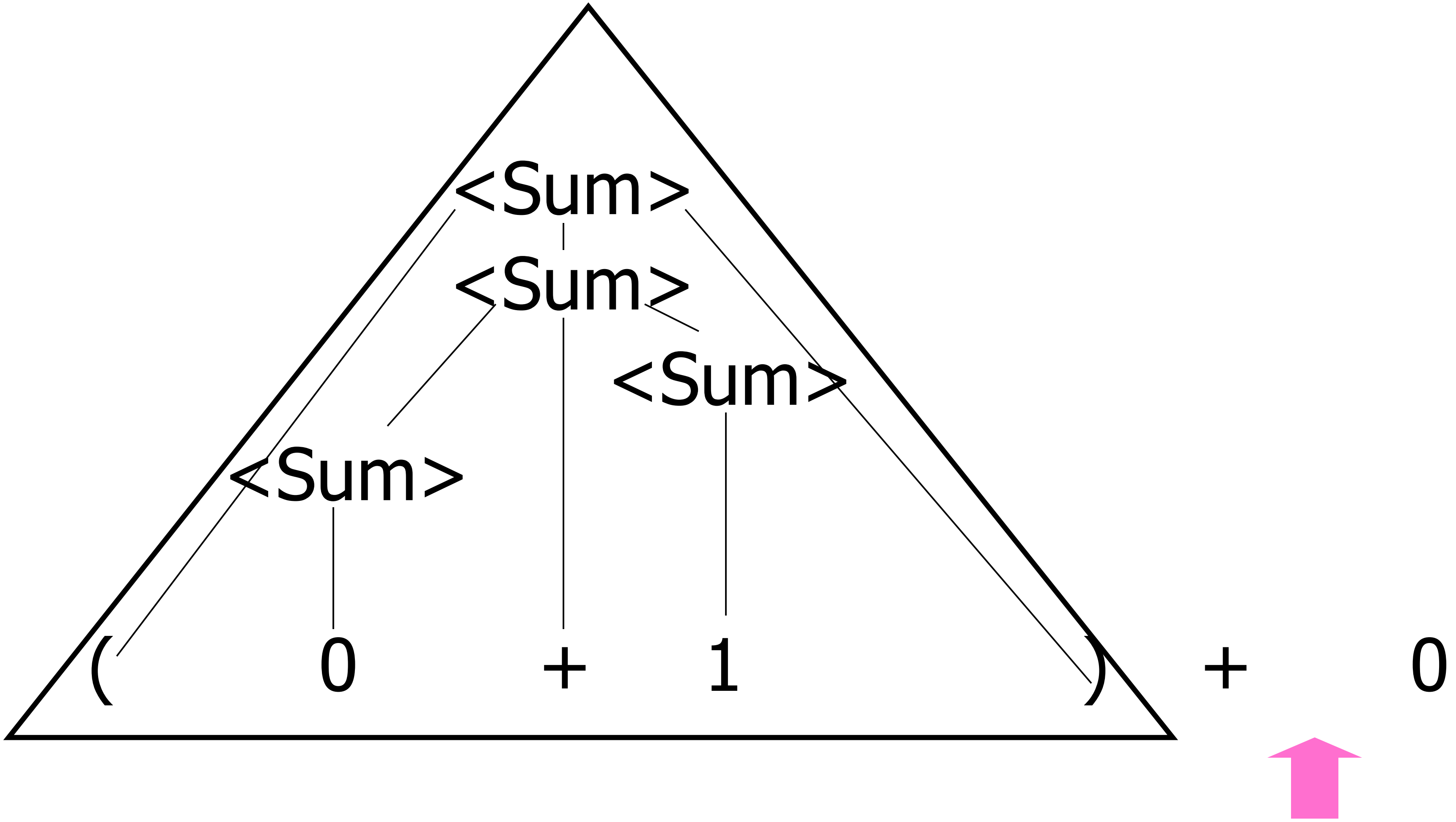
Example



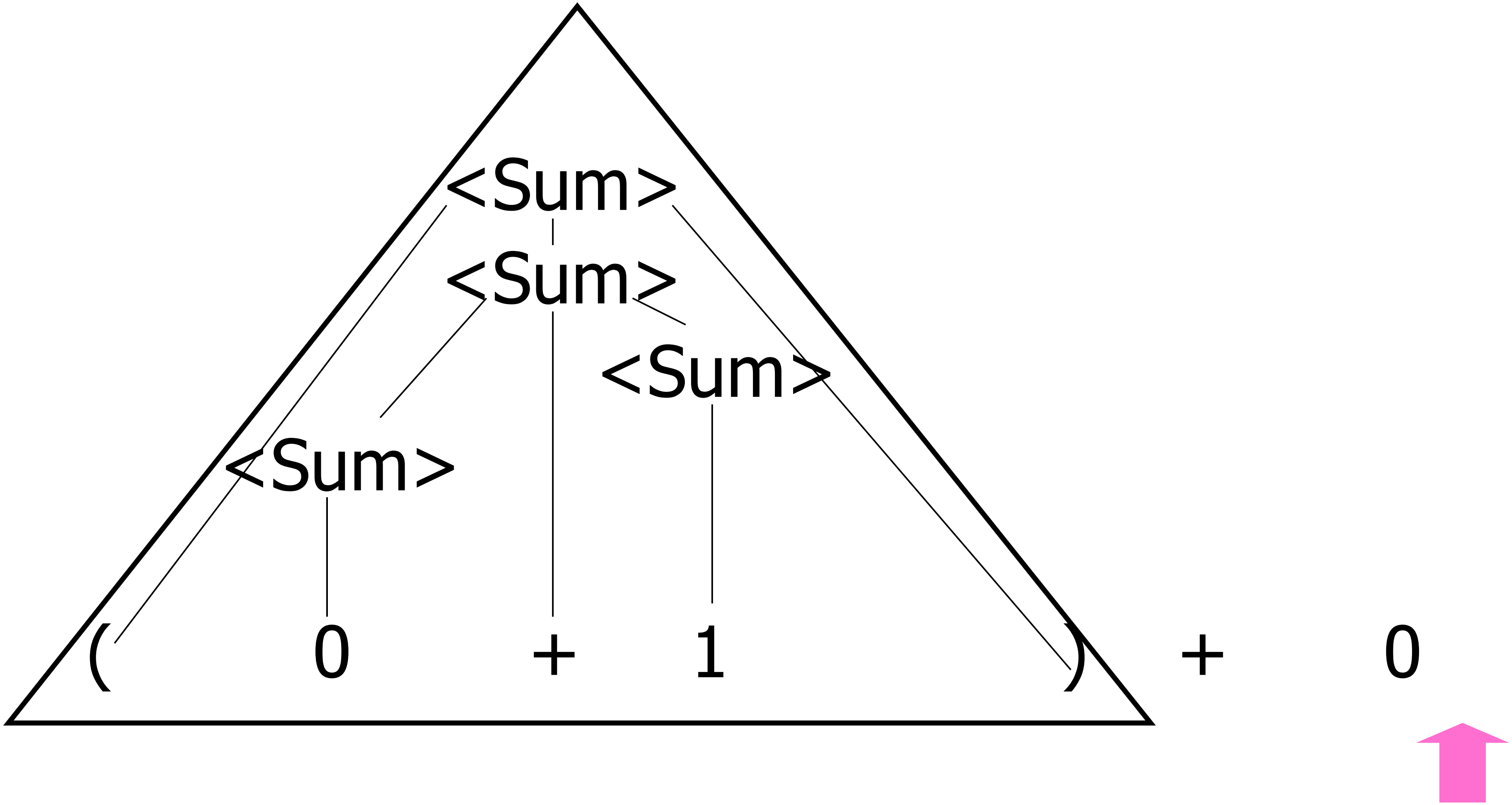
Example



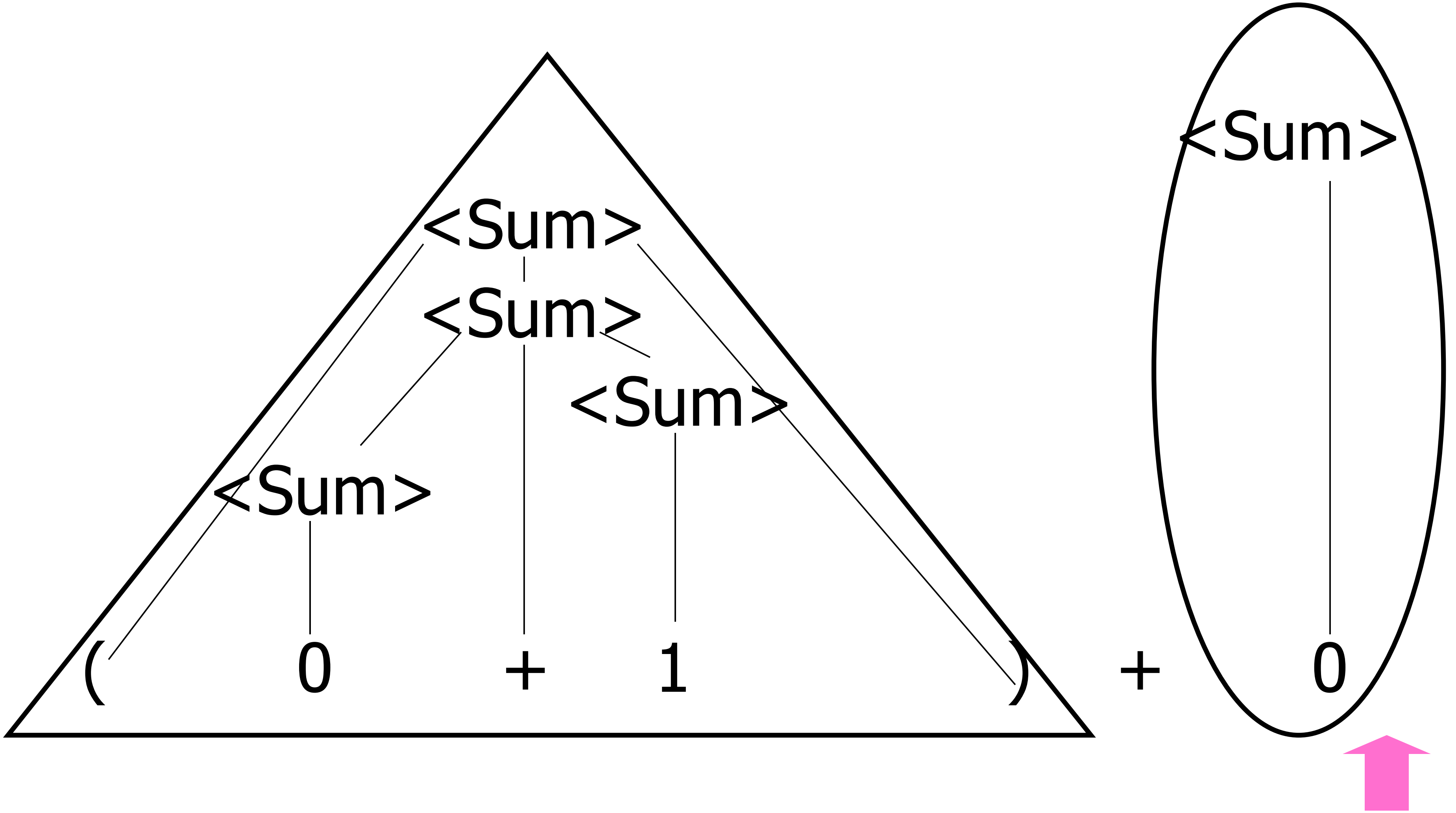
Example



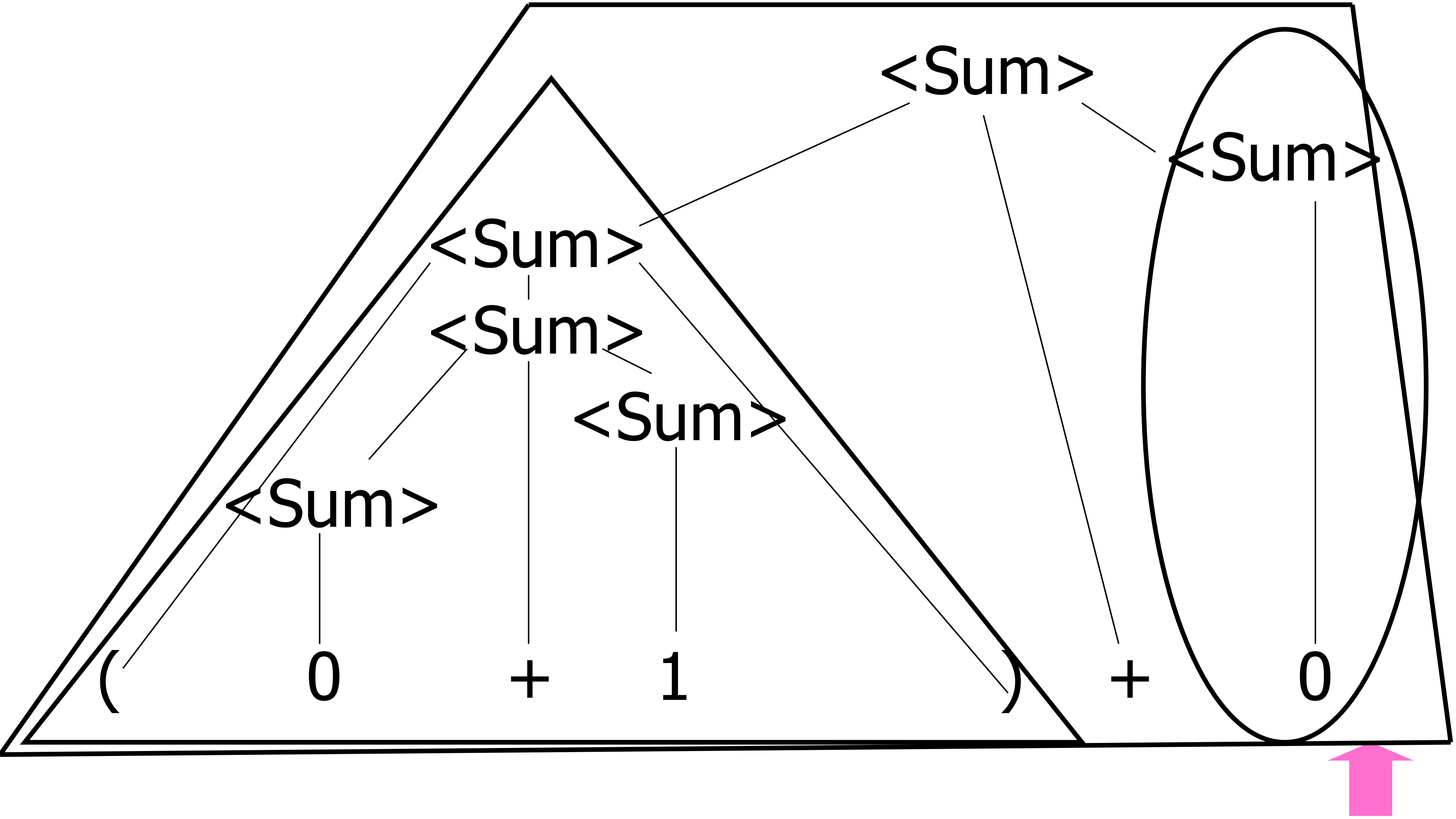
Example



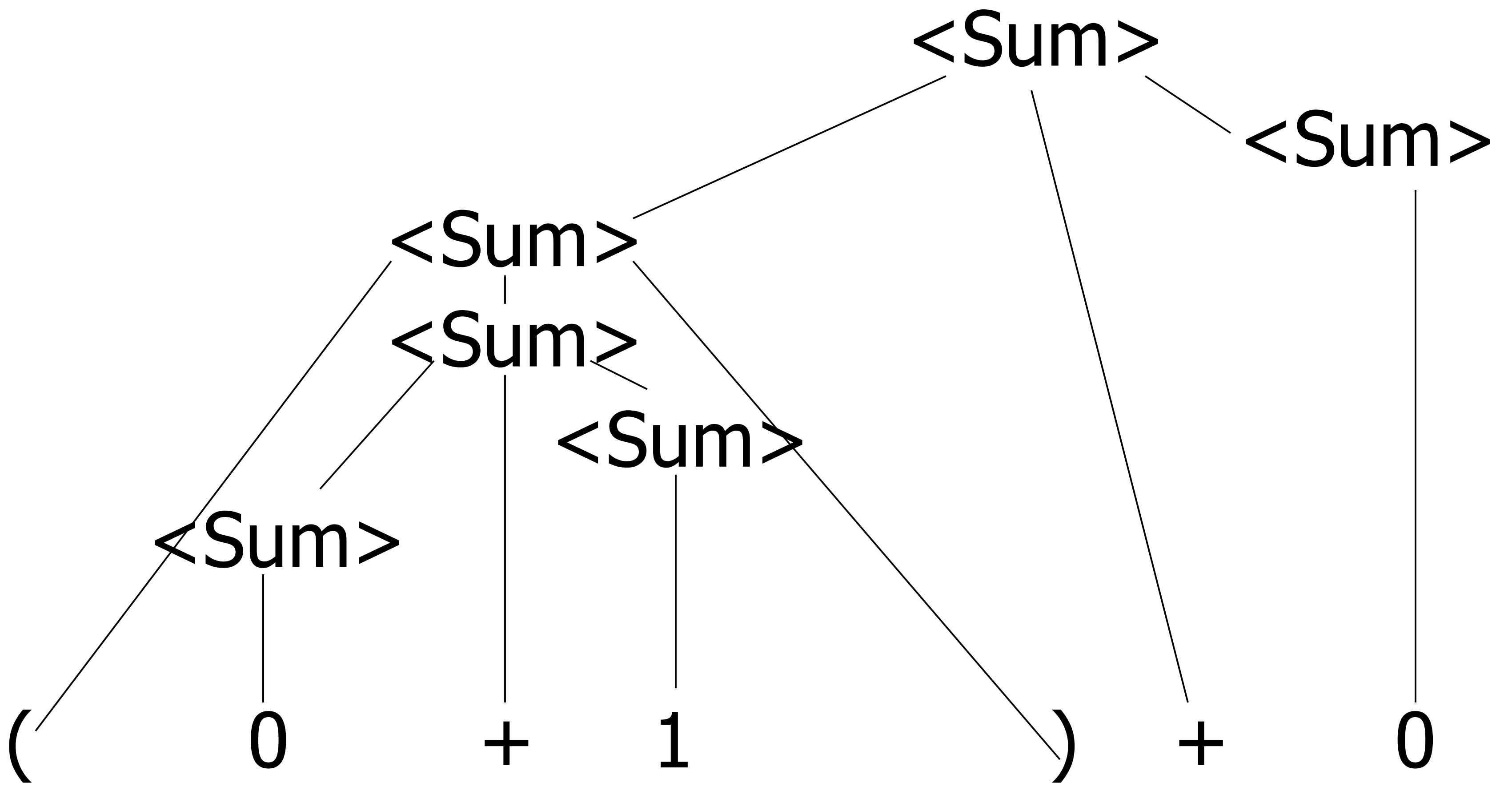
Example



Example



Example



LR Parsing Tables

- Build a pair of tables, Action and Goto, from the grammar
  - This is the hardest part, we omit here
  - Rows labeled by states
  - For Action, columns labeled by terminals and "end-of-tokens" marker
    - (more generally strings of terminals of fixed length)
  - For Goto, columns labeled by non-terminals

Action and Goto Tables

- Given a state and the next input, Action table says either
  - **shift** and go to state *n*, or
  - **reduce** by production *k* (explained in a bit)
  - **accept** or **error**
- Given a state and a non-terminal, Goto table says
  - go to state *m*

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

= ● (0 + 1) + 0 shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=> (0 ● + 1) + 0	reduce
= (● 0 + 1) + 0	shift
= ● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

= (<Sum> ● + 1) + 0 shift
=> (0 ● + 1) + 0 reduce
= (● 0 + 1) + 0 shift
= ● (0 + 1) + 0 shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=	(<Sum> + ● 1) + 0	shift
=	(<Sum> ● + 1) + 0	shift
=>	(0 ● + 1) + 0	reduce
=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=>	(<Sum> + 1 ●) + 0	reduce
=	(<Sum> + ● 1) + 0	shift
=	(<Sum> ● + 1) + 0	shift
=>	(0 ● + 1) + 0	reduce
=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=> (<Sum> + <Sum> ●) + 0 reduce
=> (<Sum> + 1 ●) + 0 reduce
= (<Sum> + ● 1) + 0 shift
= (<Sum> ● + 1) + 0 shift
=> (0 ● + 1) + 0 reduce
= (● 0 + 1) + 0 shift
= ● (0 + 1) + 0 shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=	(<Sum> ●) + 0	shift
=>	(<Sum> + <Sum> ●) + 0	reduce
=>	(<Sum> + 1 ●) + 0	reduce
=	(<Sum> + ● 1) + 0	shift
=	(<Sum> ● + 1) + 0	shift
=>	(0 ● + 1) + 0	reduce
=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=> (<Sum>) ● + 0 reduce
= (<Sum> ●) + 0 shift
=> (<Sum> + <Sum> ●) + 0 reduce
=> (<Sum> + 1 ●) + 0 reduce
= (<Sum> + ● 1) + 0 shift
= (<Sum> ● + 1) + 0 shift
=> (0 ● + 1) + 0 reduce
= (● 0 + 1) + 0 shift
= ● (0 + 1) + 0 shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=	<Sum> ● + 0	shift
=>	(<Sum>) ● + 0	reduce
=	(<Sum> ●) + 0	shift
=>	(<Sum> + <Sum> ●) + 0	reduce
=>	(<Sum> + 1 ●) + 0	reduce
=	(<Sum> + ● 1) + 0	shift
=	(<Sum> ● + 1) + 0	shift
=>	(0 ● + 1) + 0	reduce
=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=	<Sum> + ● 0	shift
=	<Sum> ● + 0	shift
=>	(<Sum>) ● + 0	reduce
=	(<Sum> ●) + 0	shift
=>	(<Sum> + <Sum> ●) + 0	reduce
=>	(<Sum> + 1 ●) + 0	reduce
=	(<Sum> + ● 1) + 0	shift
=	(<Sum> ● + 1) + 0	shift
=>	(0 ● + 1) + 0	reduce
=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum>	=>	
	=> <Sum> + 0 ●	reduce
	= <Sum> + ● 0	shift
	= <Sum> ● + 0	shift
	=> (<Sum>) ● + 0	reduce
	= (<Sum> ●) + 0	shift
	=> (<Sum> + <Sum> ●) + 0	reduce
	=> (<Sum> + 1 ●) + 0	reduce
	= (<Sum> + ● 1) + 0	shift
	= (<Sum> ● + 1) + 0	shift
	=> (0 ● + 1) + 0	reduce
	= (● 0 + 1) + 0	shift
	= ● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum>	=>	<Sum> + <Sum > ●	reduce
	=>	<Sum> + 0 ●	reduce
	=	<Sum> + ● 0	shift
	=	<Sum> ● + 0	shift
	=>	(<Sum>) ● + 0	reduce
	=	(<Sum> ●) + 0	shift
	=>	(<Sum> + <Sum> ●) + 0	reduce
	=>	(<Sum> + 1 ●) + 0	reduce
	=	(<Sum> + ● 1) + 0	shift
	=	(<Sum> ● + 1) + 0	shift
	=>	(0 ● + 1) + 0	reduce
	=	(● 0 + 1) + 0	shift
	=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> ●	=> <Sum> + <Sum > ●	reduce
	=> <Sum> + 0 ●	reduce
	= <Sum> + ● 0	shift
	= <Sum> ● + 0	shift
	=> (<Sum>) ● + 0	reduce
	= (<Sum> ●) + 0	shift
	=> (<Sum> + <Sum> ●) + 0	reduce
	=> (<Sum> + 1 ●) + 0	reduce
	= (<Sum> + ● 1) + 0	shift
	= (<Sum> ● + 1) + 0	shift
	=> (0 ● + 1) + 0	reduce
	= (● 0 + 1) + 0	shift
	= ● (0 + 1) + 0	shift

LR(i) Parsing Algorithm

- Based on push-down automata
- Uses states and transitions (as recorded in Action and Goto tables)
- Uses a stack containing states, terminals and non-terminals

LR(i) Parsing Algorithm

0. Insure token stream ends in special “end-of-tokens” symbol
1. Start in state 1 with an empty stack
2. Push **state**(1) onto stack
3. Look at next *i* tokens from token stream (*toks*) (don’t remove yet)
4. If top symbol on stack is **state**(*n*), look up action in Action table at (*n*, *toks*)

LR(i) Parsing Algorithm

5. If action = **shift** *m*,
   a) Remove the top token from token stream and push it onto the stack
   b) Push **state**(*m*) onto stack
   c) Go to step 3

LR(i) Parsing Algorithm

6. If action = **reduce** *k* where production *k* is E ::= u
   a) Remove 2 * length(u) symbols from stack (u and all the interleaved states)
   b) If new top symbol on stack is **state**(*m*), look up new state *p* in Goto(*m*,E)
   c) Push E onto the stack, then push **state**(*p*) onto the stack
   d) Go to step 3

LR(i) Parsing Algorithm

7. If action = **accept**
   - Stop parsing, return success
8. If action = **error**,
   - Stop parsing, return failure

Adding Synthesized Attributes

- Add to each **reduce** a rule for calculating the new synthesized attribute from the component attributes
- Add to each non-terminal pushed onto the stack, the attribute calculated for it
- When performing a **reduce**,
  - gather the recorded attributes from each non-terminal popped from stack
  - Compute new attribute for non-terminal pushed onto stack

Shift-Reduce Conflicts

- **Problem**: can’t decide whether the action for a state and input character should be **shift** or **reduce**
- Caused by ambiguity in grammar
- Usually caused by lack of associativity or precedence information in grammar

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

● 0 + 1 + 0	shift
-> 0 ● + 1 + 0	reduce
-> <Sum> ● + 1 + 0	shift
-> <Sum> + ● 1 + 0	shift
-> <Sum> + 1 ● + 0	reduce
-> <Sum> + <Sum> ● + 0	

Example - cont

- **Problem:** shift or reduce?
- You can shift-shift-reduce-reduce or reduce-shift-shift-reduce
- Shift first - right associative
- Reduce first- left associative

Reduce - Reduce Conflicts

- **Problem:** can't decide between two different rules to reduce by
- Again caused by ambiguity in grammar
- **Symptom:** RHS of one production suffix of another
- Requires examining grammar and rewriting it
- Harder to solve than shift-reduce errors

Example

- S ::= A | aB   A ::= abc   B ::= bc

  ● abc   shift
  a ● bc   shift
  ab ● c   shift
  abc ●

- Problem: reduce by B ::= bc then by S ::= aB, or by A::= abc then S::A?