Programming Languages and Compilers (CS 421)

Elsa L Gunter
2112 SC, UIUC
http://courses.engr.illinois.edu/cs421

Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Disambiguating a Grammar

- <exp>::= 0|1| b<exp> | <exp>a
- | <exp>m<exp>
- Want a has higher precedence than b, which in turn has higher precedence than m, and such that m associates to the left.

Disambiguating a Grammar

- <exp>::= 0|1| b<exp> | <exp>a
- | <exp>m<exp>
- Want a has higher precedence than b, which in turn has higher precedence than m, and such that m associates to the left.
- <exp> ::= <exp> m <not m> | <not m>
- <not m> ::= b <not m> | <not b m>
- <not b m> ::= <not b m>a | 0 | 1

Disambiguating a Grammar – Take 2

- <exp>::= 0|1| b<exp> | <exp>a
  | <exp>m<exp>
- Want b has higher precedence than m, which in turn has higher precedence than a, and such that m associates to the right.

Disambiguating a Grammar – Take 2

- <exp>::= 0|1| b<exp> | <exp>a
  | <exp>m<exp>
- Want b has higher precedence than m, which in turn has higher precedence than a, and such that m associates to the right.
- <exp> ::=
  <no a m> | <not m> m <no a>| <exp> a
- <no a> ::= <no a m> | <no a m> m <no a>
- <not m> ::= <no a m> | <exp> a
- <no a m> ::= b <no a m> | 0 | 1

LR Parsing

- Read tokens left to right (L)
- Create a rightmost derivation (R)
- How is this possible?
- Start at the bottom (left) and work your way up
- Last step has only one non-terminal to be replaced so is right-most
- Working backwards, replace mixed strings by non-terminals
- Always proceed so that there are no non-terminals to the right of the string to be replaced

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

= ● (0 + 1) + 0 shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

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=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=>	(0 ● + 1) + 0	reduce
=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=	(<Sum> ● + 1) + 0	shift
=>	(0 ● + 1) + 0	reduce
=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=	(<Sum> + ● 1) + 0	shift
=	(<Sum> ● + 1) + 0	shift
=>	(0 ● + 1) + 0	reduce
=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=>	(<Sum> + 1 ●) + 0	reduce
=	(<Sum> + ● 1) + 0	shift
=	(<Sum> ● + 1) + 0	shift
=>	(0 ● + 1) + 0	reduce
=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=> (<Sum> + <Sum> ●) + 0 reduce
=> (<Sum> + 1 ●) + 0 reduce
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= <Sum> ● + 0 shift
=> (<Sum>) ● + 0 reduce
= (<Sum> ●) + 0 shift
=> (<Sum> + <Sum> ●) + 0 reduce
=> (<Sum> + 1 ●) + 0 reduce
= (<Sum> + ● 1) + 0 shift
= (<Sum> ● + 1) + 0 shift
=> (0 ● + 1) + 0 reduce
= (● 0 + 1) + 0 shift
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Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

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= <Sum> + ● 0 shift
= <Sum> ● + 0 shift
=> (<Sum>) ● + 0 reduce
= (<Sum> ●) + 0 shift
=> (<Sum> + <Sum> ●) + 0 reduce
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Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

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=> <Sum> + 0 ● reduce
= <Sum> + ● 0 shift
= <Sum> ● + 0 shift
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Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> => <Sum> + <Sum > ● reduce
=> <Sum> + 0 ● reduce
= <Sum> + ● 0 shift
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= ● (0 + 1) + 0 shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> ● => <Sum> + <Sum > ● reduce
=> <Sum> + 0 ● reduce
= <Sum> + ● 0 shift
= <Sum> ● + 0 shift
=> (<Sum>) ● + 0 reduce
= (<Sum> ●) + 0 shift
=> (<Sum> + <Sum> ●) + 0 reduce
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= (<Sum> ● + 1) + 0 shift
=> (0 ● + 1) + 0 reduce
= (● 0 + 1) + 0 shift
= ● (0 + 1) + 0 shift

Example

(0 + 1) + 0

Example

(0 + 1) + 0

Example

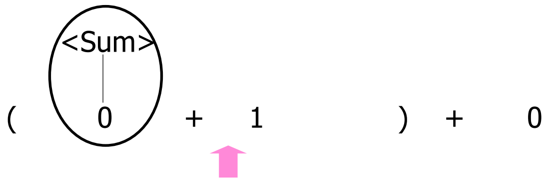
(0 + 1) + 0

Example

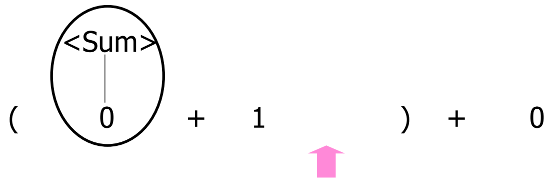
<Sum>

(0 + 1) + 0

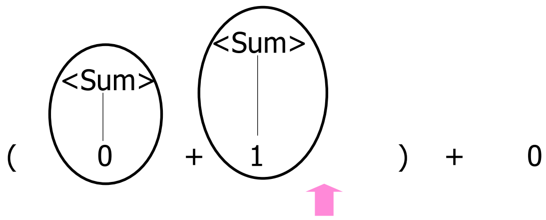
Example



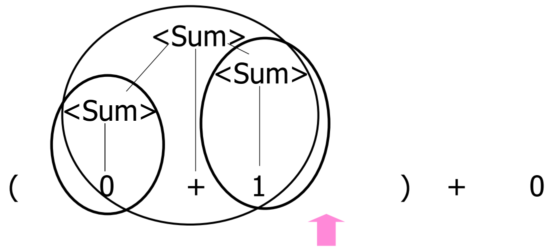
Example



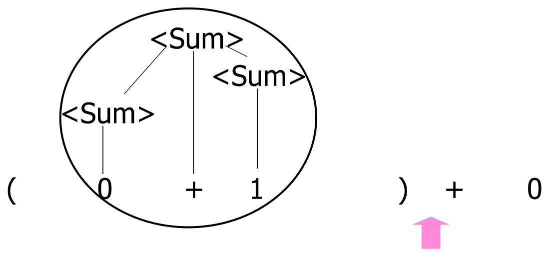
Example



Example



Example



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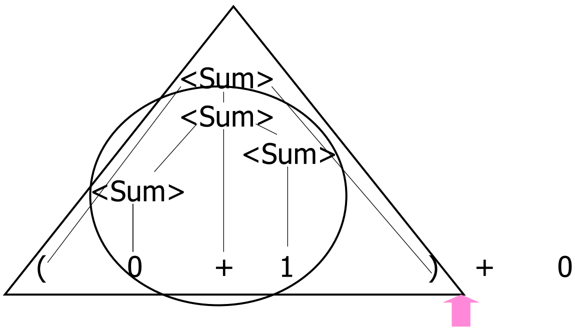
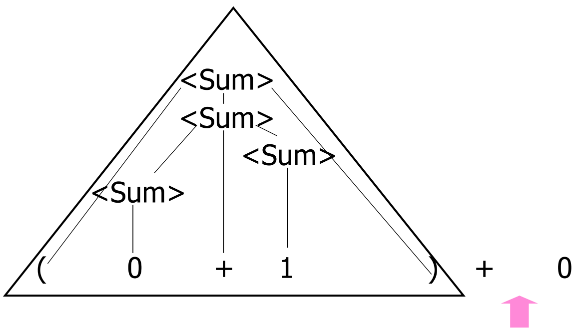
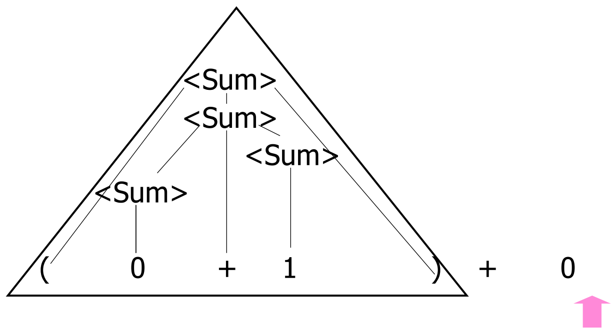


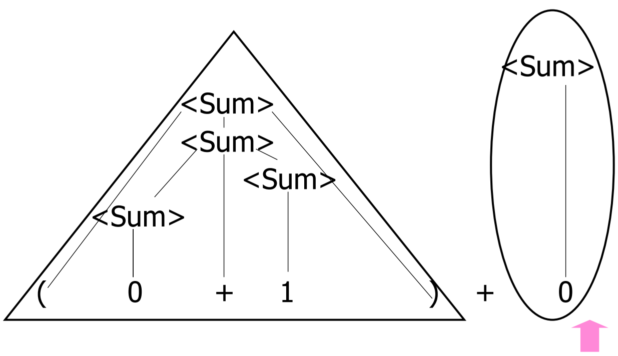
Example



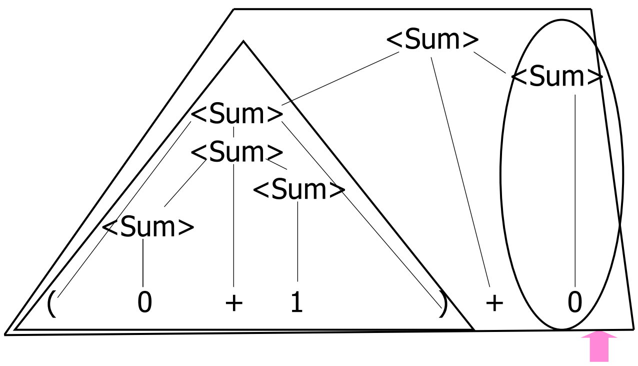
Example



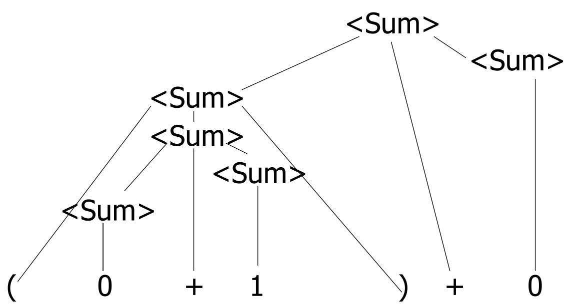
Example



Example



Example



LR Parsing Tables

- Build a pair of tables, Action and Goto, from the grammar
  - This is the hardest part, we omit here
  - Rows labeled by states
  - For Action, columns labeled by terminals and “end-of-tokens” marker
    - (more generally strings of terminals of fixed length)
  - For Goto, columns labeled by non-terminals

Action and Goto Tables

- Given a state and the next input, Action table says either
  - **shift** and go to state *n*, or
  - **reduce** by production *k* (explained in a bit)
  - **accept** or **error**
- Given a state and a non-terminal, Goto table says
  - go to state *m*

LR(i) Parsing Algorithm

- Based on push-down automata
- Uses states and transitions (as recorded in Action and Goto tables)
- Uses a stack containing states, terminals and non-terminals

LR(i) Parsing Algorithm

0. Insure token stream ends in special "end-of-tokens" symbol
1. Start in state 1 with an empty stack
2. Push **state**(1) onto stack
3. → Look at next *i* tokens from token stream (*toks*) (don't remove yet)
4. If top symbol on stack is **state**(*n*), look up action in Action table at (*n*, *toks*)

LR(i) Parsing Algorithm

5. If action = **shift** *m*,
   a) Remove the top token from token stream and push it onto the stack
   b) Push **state**(*m*) onto stack
   c) Go to step 3

LR(i) Parsing Algorithm

6. If action = **reduce** *k* where production *k* is E ::= u
   a) Remove 2 * length(u) symbols from stack (u and all the interleaved states)
   b) If new top symbol on stack is **state**(*m*), look up new state *p* in Goto(*m*,E)
   c) Push E onto the stack, then push **state**(*p*) onto the stack
   d) Go to step 3

LR(i) Parsing Algorithm

7. If action = **accept**
   - Stop parsing, return success
8. If action = **error**,
   - Stop parsing, return failure

Adding Synthesized Attributes

- Add to each **reduce** a rule for calculating the new synthesized attribute from the component attributes
- Add to each non-terminal pushed onto the stack, the attribute calculated for it
- When performing a **reduce**,
  - gather the recorded attributes from each non-terminal popped from stack
  - Compute new attribute for non-terminal pushed onto stack

Shift-Reduce Conflicts

- **Problem**: can't decide whether the action for a state and input character should be **shift** or **reduce**
- Caused by ambiguity in grammar
- Usually caused by lack of associativity or precedence information in grammar

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

● 0 + 1 + 0	shift
-> 0 ● + 1 + 0	reduce
-> <Sum> ● + 1 + 0	shift
-> <Sum> + ● 1 + 0	shift
-> <Sum> + 1 ● + 0	reduce
-> <Sum> + <Sum> ● + 0	

Example - cont

- **Problem:** shift or reduce?
- You can shift-shift-reduce-reduce or reduce-shift-shift-reduce
- Shift first - right associative
- Reduce first- left associative

Reduce - Reduce Conflicts

- **Problem:** can't decide between two different rules to reduce by
- Again caused by ambiguity in grammar
- **Symptom:** RHS of one production suffix of another
- Requires examining grammar and rewriting it
- Harder to solve than shift-reduce errors

Example

- S ::= A | aB A ::= abc B ::= bc

● abc	shift
a ● bc	shift
ab ● c	shift
abc ●	

- Problem: reduce by B ::= bc then by S ::= aB, or by A::= abc then S::A?

Recursive Descent Parsing

- Recursive descent parsers are a class of parsers derived fairly directly from BNF grammars
- A recursive descent parser traces out a parse tree in top-down order, corresponding to a left-most derivation (LL - left-to-right scanning, leftmost derivation)

Recursive Descent Parsing

- Each nonterminal in the grammar has a subprogram associated with it; the subprogram parses all phrases that the nonterminal can generate
- Each nonterminal in right-hand side of a rule corresponds to a recursive call to the associated subprogram

Recursive Descent Parsing

- Each subprogram must be able to decide how to begin parsing by looking at the left-most character in the string to be parsed
  - May do so directly, or indirectly by calling another parsing subprogram
- Recursive descent parsers, like other top-down parsers, cannot be built from left-recursive grammars
  - Sometimes can modify grammar to suit

Sample Grammar

<expr> ::= <term> | <term> + <expr>
| <term> - <expr>

<term> ::= <factor> | <factor> * <term>
| <factor> / <term>

<factor> ::= <id> | (<expr>)

Tokens as OCaml Types

- + - * / ( ) <id>
- Becomes an OCaml datatype

```
type token =
    Id_token of string
  | Left_parenthesis | Right_parenthesis
  | Times_token | Divide_token
  | Plus_token | Minus_token
```


Parse Trees as Datatypes

<expr> ::= <term> | <term> + <expr>
| <term> - <expr>

```
type expr =
    Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
```

Parse Trees as Datatypes

```
<term> ::= <factor> | <factor> *
  <term>
           | <factor> / <term>

and term =
    Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
```


Parse Trees as Datatypes

```
<factor> ::= <id> | ( <expr> )

and factor =
    Id_as_Factor of string
  | Parenthesized_Expr_as_Factor of expr
```


Parsing Lists of Tokens

- Will create three mutually recursive functions:
  - expr : token list -> (expr * token list)
  - term : token list -> (term * token list)
  - factor : token list -> (factor * token list)
- Each parses what it can and gives back parse and remaining tokens

Parsing an Expression

```
  <expr> ::= <term> [( + | - ) <expr> ]
let rec expr tokens =
  (match term tokens
   with ( term_parse , tokens_after_term) ->
     (match tokens_after_term
      with( Plus_token  :: tokens_after_plus) ->
```


Parsing an Expression

```
  <expr> ::= <term> [( + | - ) <expr> ]
let rec expr tokens =
  (match term tokens
   with ( term_parse , tokens_after_term) ->
     (match tokens_after_term
      with ( Plus_token  :: tokens_after_plus) ->
```


Parsing a Plus Expression

```
  <expr> ::= <term> [( + | - ) <expr> ]
let rec expr tokens =
  (match term tokens
   with ( term_parse , tokens_after_term) ->
     (match tokens_after_term
      with ( Plus_token  :: tokens_after_plus) ->
```

Parsing a Plus Expression

<expr> ::= <term> [(+ | -) <expr>]

let rec expr tokens =
  (match term tokens
    with (term_parse , tokens_after_term) ->
      (match tokens_after_term
        with (Plus_token :: tokens_after_plus) ->

Parsing a Plus Expression

<expr> ::= <term> [(+ | -) <expr>]

let rec expr tokens =
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      (match tokens_after_term
        with (Plus_token :: tokens_after_plus) ->

Parsing a Plus Expression

<expr> ::= <term> + <expr>

(match expr tokens_after_plus
  with (expr_parse , tokens_after_expr) ->
  (Plus_Expr (term_parse , expr_parse),
  tokens_after_expr))

Parsing a Plus Expression

<expr> ::= <term> + <expr>

(match expr tokens_after_plus
  with (expr_parse , tokens_after_expr) ->
  (Plus_Expr (term_parse , expr_parse),
  tokens_after_expr))

Building Plus Expression Parse Tree

<expr> ::= <term> + <expr>

(match expr tokens_after_plus
  with (expr_parse , tokens_after_expr) ->
  (Plus_Expr (term_parse , expr_parse),
  tokens_after_expr))

Parsing a Minus Expression

<expr> ::= <term> - <expr>

| (Minus_token :: tokens_after_minus) ->
  (match expr tokens_after_minus
  with (expr_parse , tokens_after_expr) ->
  (Minus_Expr (term_parse , expr_parse),
  tokens_after_expr))

Parsing a Minus Expression

<expr> ::= <term> - <expr>

| (Minus_token :: tokens_after_minus) ->
(match expr tokens_after_minus
with (expr_parse , tokens_after_expr) ->
(Minus_Expr (term_parse , expr_parse),
tokens_after_expr))

Parsing an Expression as a Term

<expr> ::= <term>

| _ -> (Term_as_Expr term_parse ,
tokens_after_term)))

- Code for **term** is same except for replacing addition with multiplication and subtraction with division

Parsing Factor as Id

<factor> ::= <id>

and factor tokens =
(match tokens
with (Id_token id_name :: tokens_after_id) =
(Id_as_Factor id_name, tokens_after_id)

Parsing Factor as Parenthesized Expression

<factor> ::= (<expr>)

| factor (Left_parenthesis :: tokens) =
(match expr tokens
with (expr_parse , tokens_after_expr) ->

Parsing Factor as Parenthesized Expression

<factor> ::= (<expr>)

(match tokens_after_expr
with Right_parenthesis :: tokens_after_rparen ->
(Parenthesized_Expr_as_Factor expr_parse ,
tokens_after_rparen)

Error Cases

- What if no matching right parenthesis?

  | _ -> raise (Failure "No matching rparen")))

- What if no leading id or left parenthesis?
  | _ -> raise (Failure "No id or lparen"));;

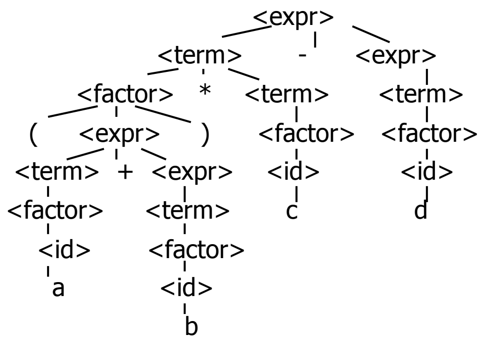
(a + b) * c - d

expr [Left_parenthesis; Id_token "a”;
Plus_token; Id_token "b”;
Right_parenthesis; Times_token;
Id_token "c”; Minus_token;
Id_token "d"];;

(a + b) * c - d

```
- : expr * token list =
(Minus_Expr
  (Mult_Term
    (Parenthesized_Expr_as_Factor
      (Plus_Expr
        (Factor_as_Term (Id_as_Factor "a"),
         Term_as_Expr (Factor_as_Term
   (Id_as_Factor "b")))),
      Factor_as_Term (Id_as_Factor "c")),
   Term_as_Expr (Factor_as_Term (Id_as_Factor
   "d"))),
 [])
```

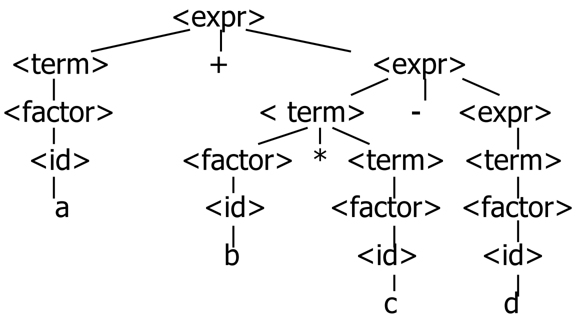

(a + b) * c – d



a + b * c – d

```
# expr [Id_token "a”; Plus_token; Id_token "b”;
  Times_token; Id_token "c”; Minus_token;
      Id_token "d"];;
- : expr * token list =
(Plus_Expr
  (Factor_as_Term (Id_as_Factor "a"),
   Minus_Expr
    (Mult_Term (Id_as_Factor "b", Factor_as_Term
   (Id_as_Factor "c")),
     Term_as_Expr (Factor_as_Term (Id_as_Factor
   "d")))),
 [])
```


a + b * c – d



(a + b * c - d

expr [Left_parenthesis; Id_token "a”;
Plus_token; Id_token "b”; Times_token;
Id_token "c”; Minus_token; Id_token "d"];;

Exception: Failure "No matching rparen".

Can’t parse because it was expecting a right parenthesis but it got to the end without finding one

a + b) * c – d (

expr [Id_token "a"; Plus_token; Id_token "b";
Right_parenthesis; Times_token; Id_token "c";
Minus_token; Id_token "d"; Left_parenthesis];;

```
- : expr * token list =
(Plus_Expr
 (Factor_as_Term (Id_as_Factor "a"),
  Term_as_Expr (Factor_as_Term (Id_as_Factor
  "b"))),
 [Right_parenthesis; Times_token; Id_token "c";
  Minus_token; Id_token "d"; Left_parenthesis])
```


Parsing Whole String

- Q: How to guarantee whole string parses?
- A: Check returned tokens empty

```
let parse tokens =
    match expr tokens
     with (expr_parse, []) -> expr_parse
     | _ -> raise (Failure "No parse");;
```

- Fixes <expr> as start symbol

Streams in Place of Lists

- More realistically, we don't want to create the entire list of tokens before we can start parsing
- We want to generate one token at a time and use it to make one step in parsing
- Can use (token * (unit -> token)) or (token * (unit -> token option))
  in place of token list

Problems for Recursive-Descent Parsing

- Left Recursion:
  A ::= Aw
  translates to a subroutine that loops forever
- Indirect Left Recursion:
  A ::= Bw
  B ::= Av
  causes the same problem

Problems for Recursive-Descent Parsing

- Parser must always be able to choose the next action based only only the very next token

- Pairwise Disjointedness Test: Can we always determine which rule (in the non-extended BNF) to choose based on just the first token

Pairwise Disjointedness Test

- For each rule
  A ::= *y*

Calculate

FIRST (*y*) =

{a | *y* =>* a*w*} ∪ {ε | if y =>* ε}

- For each pair of rules A ::= *y* and A ::= *z*, require FIRST(*y*) ∩ FIRST(*z*) = { }

Example

Grammar:

<S> ::= <A> a <B> b

<A> ::= <A> b | b

<B> ::= a <B> | a

FIRST (<A> b) = {b}

FIRST (b) = {b}

Rules for <A> not pairwise disjoint

Eliminating Left Recursion

- Rewrite grammar to shift left recursion to right recursion
  - Changes associativity
- Given

<expr> ::= <expr> + <term> and

<expr> ::= <term>

- Add new non-terminal <e> and replace above rules with

<expr> ::= <term><e>

<e> ::= + <term><e> | ε

Factoring Grammar

- Test too strong: Can't handle
  <expr> ::= <term> [(+ | -) <expr>]
- Answer: Add new non-terminal and replace above rules by
  <expr> ::= <term><e>
  <e> ::= + <term><e>
  <e> ::= - <term><e>
  <e> ::= ε
- You are delaying the decision point

Example

Both <A> and <B> have problems:	Transform grammar to:
<S> ::= <A> a <B> b	<S> ::= <A> a <B> b
<A> ::= <A> b | b	<A> ::-= b<A1>
<B> ::= a <B> | a	<A1> :: b<A1> | ε
	<B> ::= a<B1>
	<B1> ::= a<B1> | ε

Semantics

- Expresses the meaning of syntax
- Static semantics
  - Meaning based only on the form of the expression without executing it
  - Usually restricted to type checking / type inference

Dynamic semantics

- Method of describing meaning of executing a program
- Several different types:
  - Operational Semantics
  - Axiomatic Semantics
  - Denotational Semantics

Dynamic Semantics

- Different languages better suited to different types of semantics
- Different types of semantics serve different purposes

Operational Semantics

- Start with a simple notion of machine
- Describe how to execute (implement) programs of language on virtual machine, by describing how to execute each program statement (ie, following the *structure* of the program)
- Meaning of program is how its execution changes the state of the machine
- Useful as basis for implementations

Axiomatic Semantics

- Also called Floyd-Hoare Logic
- Based on formal logic (first order predicate calculus)
- Axiomatic Semantics is a logical system built from *axioms* and *inference rules*
- Mainly suited to simple imperative programming languages

Axiomatic Semantics

- Used to formally prove a property (*post-condition*) of the *state* (the values of the program variables) after the execution of program, assuming another property (*pre-condition*) of the state before execution
- Written :

  {Precondition} Program {Postcondition}
- Source of idea of *loop invariant*

Denotational Semantics

- Construct a function $\mathcal{M}$ assigning a mathematical meaning to each program construct
- Lambda calculus often used as the range of the meaning function
- Meaning function is compositional: meaning of construct built from meaning of parts
- Useful for proving properties of programs

Natural Semantics

- Aka Structural Operational Semantics, aka "Big Step Semantics"
- Provide value for a program by rules and derivations, similar to type derivations
- Rule conclusions look like

$$(C, m) \Downarrow m'$$

or

$$(E, m) \Downarrow v$$

Simple Imperative Programming Language

- $I \in$ *Identifiers*
- $N \in$ *Numerals*
- B ::= true | false | B & B | B or B | not B | $E < E$ | $E = E$
- $E ::= N \mid I \mid E + E \mid E * E \mid E - E \mid - E$
- C::= skip | $C;C$ | $I ::= E$ | if B then C else C fi | while B do C od

Natural Semantics of Atomic Expressions

- Identifiers: $(I,m) \Downarrow m(I)$
- Numerals are values: $(N,m) \Downarrow N$
- Booleans: $(\text{true},m) \Downarrow \text{true}$
  $(\text{false},m) \Downarrow \text{false}$

Booleans:

$$\frac{(B, m) \Downarrow \text{false}}{(B \,\&\, B', m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{true} \quad (B', m) \Downarrow b}{(B \,\&\, B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(B \text{ or } B', m) \Downarrow \text{true}} \qquad \frac{(B, m) \Downarrow \text{false} \quad (B', m) \Downarrow b}{(B \text{ or } B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(\text{not } B, m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{false}}{(\text{not } B, m) \Downarrow \text{true}}$$

Relations

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \sim V = b}{(E \sim E', m) \Downarrow b}$$

- By $U \sim V = b$, we mean does (the meaning of) the relation $\sim$ hold on the meaning of U and V
- May be specified by a mathematical expression/equation or rules matching U and V

Arithmetic Expressions

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \text{ op } V = N}{(E \text{ op } E', m) \Downarrow N}$$

where N is the specified value for U *op* V

Commands

Skip: $(\text{skip}, m) \Downarrow m$

Assignment: $$\frac{(E,m) \Downarrow V}{(I::=E,m) \Downarrow m[I <\!\!-\!\!- V]}$$

Sequencing: $$\frac{(C,m) \Downarrow m' \quad (C',m') \Downarrow m''}{(C;C', m) \Downarrow m''}$$

If Then Else Command

$$\frac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

$$\frac{(B,m) \Downarrow \text{false} \quad (C',m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

While Command

$$\frac{(B,m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m}$$

$$\frac{(B,m)\Downarrow\text{true} \quad (C,m)\Downarrow m' \quad (\text{while } B \text{ do } C \text{ od}, m')\Downarrow m''}{(\text{while } B \text{ do C od}, m) \Downarrow m''}$$

Example: If Then Else Rule

$$\frac{}{(\text{if x > 5 then y:= 2 + 3 else y:=3 + 4 fi, \{x -> 7\}}) \Downarrow ?}$$

Example: If Then Else Rule

$$\frac{\frac{}{(x > 5, \{x \to 7\})\Downarrow ?}}{(\text{if x > 5 then y:= 2 + 3 else y:=3 + 4 fi, \{x -> 7\}}) \Downarrow ?}$$

Example: Arith Relation

$$\frac{\frac{? > ? = ? \quad (x,\{x\to 7\})\Downarrow ? \quad (5,\{x\to 7\})\Downarrow ?}{(x > 5, \{x \to 7\})\Downarrow ?}}{(\text{if x > 5 then y:= 2 + 3 else y:=3 + 4 fi, \{x -> 7\}}) \Downarrow ?}$$

Example: Identifier(s)

$$\frac{\frac{7 > 5 = \text{true} \quad (x,\{x\to 7\})\Downarrow 7 \quad (5,\{x\to 7\})\Downarrow 5}{(x > 5, \{x \to 7\})\Downarrow ?}}{(\text{if x > 5 then y:= 2 + 3 else y:=3 + 4 fi, \{x -> 7\}}) \Downarrow ?}$$

Example: Arith Relation

7 > 5 = true

(x,{x->7})⇓7   (5,{x->7})⇓5

(x > 5, {x -> 7})⇓true

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7}) ⇓ ?

Example: If Then Else Rule

7 > 5 = true

(x,{x->7})⇓7   (5,{x->7})⇓5   (y:= 2 + 3, {x-> 7}

(x > 5, {x -> 7})⇓true   ⇓ ?   .

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7}) ⇓ ?

Example: Assignment

7 > 5 = true   (2+3, {x->7})⇓?

(x,{x->7})⇓7   (5,{x->7})⇓5   (y:= 2 + 3, {x-> 7}

(x > 5, {x -> 7})⇓true   ⇓ ?

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7}) ⇓ ?

Example: Arith Op

? + ? = ?

(2,{x->7})⇓?   (3,{x->7}) ⇓?

7 > 5 = true   (2+3, {x->7})⇓?

(x,{x->7})⇓7   (5,{x->7})⇓5   (y:= 2 + 3, {x-> 7}

(x > 5, {x -> 7})⇓true   ⇓?   .

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7}) ⇓ ?

Example: Numerals

2 + 3 = 5

(2,{x->7})⇓2   (3,{x->7}) ⇓3

7 > 5 = true   (2+3, {x->7})⇓?

(x,{x->7})⇓7   (5,{x->7})⇓5   (y:= 2 + 3, {x-> 7}

(x > 5, {x -> 7})⇓true   ⇓ ?

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7}) ⇓ ?

Example: Arith Op

2 + 3 = 5

(2,{x->7})⇓2   (3,{x->7}) ⇓3

7 > 5 = true   (2+3, {x->7})⇓5

(x,{x->7})⇓7   (5,{x->7})⇓5   (y:= 2 + 3, {x-> 7}

(x > 5, {x -> 7})⇓true   ⇓?

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7}) ⇓ ?

Example: Assignment

2 + 3 = 5
(2,{x->7})⇓2   (3,{x->7}) ⇓3
7 > 5 = true   (2+3, {x->7})⇓5
(x,{x->7})⇓7   (5,{x->7})⇓5   (y:= 2 + 3, {x-> 7}
(x > 5, {x -> 7})⇓true   ⇓ {x->7, y->5}
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓?

Example: If Then Else Rule

2 + 3 = 5
(2,{x->7})⇓2   (3,{x->7}) ⇓3
7 > 5 = true   (2+3, {x->7})⇓5
(x,{x->7})⇓7   (5,{x->7})⇓5   (y:= 2 + 3, {x-> 7}
(x > 5, {x -> 7})⇓true   ⇓ {x->7, y->5}
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ {x->7, y->5}

Let in Command

(*E*,*m*) ⇓v (*C*,*m*[*I*<-*v*]) ⇓ *m'*
(let *I* = *E* in *C*, *m*) ⇓ *m''*

Where *m''* (*y*) = *m'* (*y*) for *y*≠ *I* and
m'' (*I*) = *m* (*I*) if *m*(*I*) is defined,
and *m''* (*I*) is undefined otherwise

Example

(x,{x->5}) ⇓ 5   (3,{x->5}) ⇓ 3
(x+3,{x->5}) ⇓ 8
(5,{x->17}) ⇓ 5   (x:=x+3,{x->5}) ⇓ {x->8}
(let x = 5 in (x:=x+3), {x -> 17}) ⇓ ?

Example

(x,{x->5}) ⇓ 5   (3,{x->5}) ⇓ 3
(x+3,{x->5}) ⇓ 8
(5,{x->17}) ⇓ 5   (x:=x+3,{x->5}) ⇓ {x->8}
(let x = 5 in (x:=x+3), {x -> 17}) ⇓ {x->17}

Comment

- Simple Imperative Programming Language introduces variables *implicitly* through assignment
- The let-in command introduces scoped variables *explictly*
- Clash of constructs apparent in awkward semantics

Interpretation Versus Compilation

- A **compiler** from language L1 to language L2 is a program that takes an L1 program and for each piece of code in L1 generates a piece of code in L2 of same meaning
- An **interpreter** of L1 in L2 is an L2 program that executes the meaning of a given L1 program
- Compiler would examine the body of a loop once; an interpreter would examine it every time the loop was executed

Interpreter

- An *Interpreter* represents the operational semantics of a language L1 (source language) in the language of implementation L2 (target language)
- Built incrementally
  - Start with literals
  - Variables
  - Primitive operations
  - Evaluation of expressions
  - Evaluation of commands/declarations

Interpreter

- Takes abstract syntax trees as input
  - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
  - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next "state"
  - To get final value, put in a loop

Natural Semantics Example

- compute_exp (Var(v), m) = look_up v m
- compute_exp (Int(n), _) = Num (n)
- …
- compute_com(IfExp(b,c1,c2),m) =

  if compute_exp (b,m) = Bool(true)
  then compute_com (c1,m)
  else compute_com (c2,m)

Natural Semantics Example

- compute_com(While(b,c), m) =

  if compute_exp (b,m) = Bool(false)
  then m
  else compute_com
  (While(b,c), compute_com(c,m))

- May fail to terminate - exceed stack limits
- Returns no useful information then