Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Example Regular Expressions

- **(0∨1)*1**
  - The set of all strings of **0**'s and **1**'s ending in 1, **{1, 01, 11,…}**
- **a*b(a*)**
  - The set of all strings of a's and b's with exactly one b
- **((01) ∨(10))***
  - You tell me
- Regular expressions (equivalently, regular grammars) important for lexing, breaking strings into recognized words

Regular Grammars

- Subclass of BNF (covered in detail sool)
- Only rules of form
  <nonterminal>::=<terminal><nonterminal> or
  <nonterminal>::=<terminal> or
  <nonterminal>::= ε
- Defines same class of languages as regular expressions
- Important for writing lexers (programs that convert strings of characters into strings of tokens)
- Close connection to nondeterministic finite state automata – nonterminals ≅ states; rule ≅ edge

Example

- Regular grammar:
  <Balanced> ::= ε
  <Balanced> ::= 0<OneAndMore>
  <Balanced> ::= 1<ZeroAndMore>
  <OneAndMore> ::= 1<Balanced>
  <ZeroAndMore> ::= 0<Balanced>
- Generates even length strings where every initial substring of even length has same number of 0's as 1's

Example: Lexing

- Regular expressions good for describing lexemes (words) in a programming language
  - Identifier = (a ∨ b ∨ … ∨ z ∨ A ∨ B ∨ … ∨ Z) (a ∨ b ∨ … ∨ z ∨ A ∨ B ∨ … ∨ Z ∨ 0 ∨ 1 ∨ … ∨ 9)*
  - Digit = (0 ∨ 1 ∨ … ∨ 9)
  - Number = 0 ∨ (1 ∨ … ∨ 9)(0 ∨ … ∨ 9)* ∨ ~ (1 ∨ … ∨ 9)(0 ∨ … ∨ 9)*
  - Keywords: if = if, while = while,…

Implementing Regular Expressions

- Regular expressions reasonable way to generate strings in language
- Not so good for recognizing when a string is in language
- Problems with Regular Expressions
  - which option to choose,
  - how many repetitions to make
- Answer: finite state automata
- Should have seen in CS374

Lexing

- Different syntactic categories of “words”: tokens

Example:

- Convert sequence of characters into sequence of strings, integers, and floating point numbers.
- "asd 123 jkl 3.14" will become:

  [String "asd"; Int 123; String "jkl"; Float 3.14]

Lex, ocamllex

- Could write the reg exp, then translate to DFA by hand
  - A lot of work
- Better: Write program to take reg exp as input and automatically generates automata
- Lex is such a program
- ocamllex version for ocaml

How to do it

- To use regular expressions to parse our input we need:
  - Some way to identify the input string — call it a lexing buffer
  - Set of regular expressions,
  - Corresponding set of actions to take when they are matched.

How to do it

- The lexer will take the regular expressions and generate a state machine.
- The state machine will take our lexing buffer and apply the transitions...
- If we reach an accepting state from which we can go no further, the machine will perform the appropriate action.

Mechanics

- Put table of reg exp and corresponding actions (written in ocaml) into a file <*filename*>.mll
- Call

  ocamllex <*filename*>.mll
- Produces Ocaml code for a lexical analyzer in file <*filename*>.ml

Sample Input

```
rule main = parse
 ['0'-'9']+ { print_string "Int\n"}
 | ['0'-'9']+'.'['0'-'9']+ { print_string "Float\n"}
 | ['a'-'z']+ { print_string "String\n"}
 | _ { main lexbuf }
 {
 let newlexbuf = (Lexing.from_channel stdin) in
 print_string "Ready to lex.\n";
 main newlexbuf
}
```

General Input

{ *header* }
let *ident* = *regexp* ...
rule *entrypoint* [*arg1*... *argn*] = parse
        regexp { *action* }
    | ...
    | *regexp* { *action* }
and *entrypoint* [*arg1*... *argn*] =
   parse ...and ...
{ *trailer* }

Ocamllex Input

- *header* and *trailer* contain arbitrary ocaml code put at top an bottom of <*filename*>.ml
- let *ident* = *regexp* ... Introduces *ident* for use in later regular expressions

Ocamllex Input

- <*filename*>.ml contains one lexing function per *entrypoint*
  - Name of function is name given for *entrypoint*
  - Each entry point becomes an Ocaml function that takes *n*+1 arguments, the extra implicit last argument being of type Lexing.lexbuf
- *arg1*... *argn* are for use in *action*

Ocamllex Regular Expression

- Single quoted characters for letters: 'a'
- _: (underscore) matches any letter
- Eof: special "end_of_file" marker
- Concatenation same as usual
- "*string*": concatenation of sequence of characters
- e_1 | e_2 : choice - what was $e_1 \vee e_2$

Ocamllex Regular Expression

- [c_1 - c_2]: choice of any character between first and second inclusive, as determined by character codes
- [^c_1 - c_2]: choice of any character NOT in set
- *e**: same as before
- *e*+: same as *e e**
- *e*?: option - was $e_1 \vee \varepsilon$

Ocamllex Regular Expression

- e_1 # e_2: the characters in e_1 but not in e_2; e_1 and e_2 must describe just sets of characters
- *ident*: abbreviation for earlier reg exp in let *ident* = *regexp*
- e_1 as *id*: binds the result of e_1 to *id* to be used in the associated *action*

Ocamllex Manual

- More details can be found at

http://caml.inria.fr/pub/docs/manual-ocaml/lexyacc.html

Example : test.mll

```
{ type result = Int of int | Float of float |
  String of string }
let digit = ['0'-'9']
let digits = digit +
let lower_case = ['a'-'z']
let upper_case = ['A'-'Z']
let letter = upper_case | lower_case
let letters = letter +
```

Example : test.mll

```
rule main = parse
   (digits)'.'digits as f  { Float (float_of_string f) }
 | digits as n             { Int (int_of_string n) }
 | letters as s            { String s}
 | _ { main lexbuf }
 { let newlexbuf = (Lexing.from_channel stdin) in
 print_string "Ready to lex.";
 print_newline ();
 main newlexbuf  }
```

Example

```
# #use "test.ml";;
…
val main : Lexing.lexbuf -> result = <fun>
val __ocaml_lex_main_rec : Lexing.lexbuf -> int ->
  result = <fun>
Ready to lex.
hi there 234 5.2
- : result = String "hi"
```

What happened to the rest?!?

Example

```
# let b = Lexing.from_channel stdin;;
# main b;;
hi 673 there
- : result = String "hi"
# main b;;
- : result = Int 673
# main b;;
- : result = String "there"
```

Your Turn

- Work on ML4
  - Add a few keywords
  - Implement booleans and unit
  - Implement Ints and Floats
  - Implement identifiers

Problem

- How to get lexer to look at more than the first token at one time?
- One Answer: *action* tells it to -- recursive calls
- Side Benefit: can add “state” into lexing
- Note: already used this with the _ case
- Mainly useful when you can make your lexer be your parser
  - OCamlyacc parser needs tokens one at a time

Example

```
rule main = parse
  (digits) '.' digits as f { Float
  (float_of_string f) :: main lexbuf}
 | digits as n          { Int (int_of_string n) ::
   main lexbuf }
 | letters as s         { String s :: main lexbuf}
 | eof                   { [] }
 | _                     { main lexbuf }
```

Example Results

Ready to lex.

hi there 234 5.2

- : result list = [String "hi"; String "there"; Int 234; Float 5.2]

#

Used Ctrl-d to send the end-of-file signal

Dealing with comments

First Attempt

```
let open_comment = "(*"
let close_comment = "*)"
rule main = parse
  (digits) '.' digits as f { Float (float_of_string
  f) :: main lexbuf}
 | digits as n          { Int (int_of_string n) ::
   main lexbuf }
 | letters as s         { String s :: main lexbuf}
```

Dealing with comments

```
 | open_comment        { comment  lexbuf}
 | eof               { [] }
 | _ { main lexbuf }
and comment = parse
   close_comment      { main lexbuf }
 | _                 { comment lexbuf }
```

Dealing with nested comments

```
rule main = parse ...
 | open_comment          { comment 1 lexbuf}
 | eof                 { [] }
 | _ { main lexbuf }
and comment depth = parse
   open_comment          { comment (depth+1)
   lexbuf }
 | close_comment        { if depth = 1
                           then main lexbuf
                          else comment (depth - 1) lexbuf }
 | _                    { comment depth lexbuf }
```

Dealing with nested comments

```
rule main = parse
   (digits) '.' digits as f { Float (float_of_string f) ::
   main lexbuf}
 | digits as n         { Int (int_of_string n) :: main
   lexbuf }
 | letters as s        { String s :: main lexbuf}
 | open_comment          { (comment 1 lexbuf}
 | eof                 { [] }
 | _ { main lexbuf }
```

Dealing with nested comments

```
and comment depth = parse
   open_comment        { comment (depth+1) lexbuf }
 | close_comment       { if depth = 1
                           then main lexbuf
                          else comment (depth - 1) lexbuf }
 | _                   { comment depth lexbuf }
```

Types of Formal Language Descriptions

- Regular expressions, regular grammars
- Context-free grammars, BNF grammars, syntax diagrams
- Finite state automata
- Whole family more of grammars and automata – covered in automata theory

Sample Grammar

- Language: Parenthesized sums of 0's and 1's
- <Sum> ::= 0
- <Sum >::= 1
- <Sum> ::= <Sum> + <Sum>
- <Sum> ::= (<Sum>)

BNF Grammars

- Start with a set of characters, **a,b,c,…**
  - We call these *terminals*
- Add a set of different characters, **X,Y,Z, …**
  - We call these *nonterminals*
- One special nonterminal **S** called *start symbol*

BNF Grammars

- BNF rules (aka *productions*) have form

  X ::= *y*

  where **X** is any nonterminal and *y* is a string of terminals and nonterminals
- BNF *grammar* is a set of BNF rules such that every nonterminal appears on the left of some rule

Sample Grammar

- Terminals: 0 1 + ( )
- Nonterminals: <Sum>
- Start symbol = <Sum>

- <Sum> ::= 0
- <Sum >::= 1
- <Sum> ::= <Sum> + <Sum>
- <Sum> ::= (<Sum>)
- Can be abbreviated as

<Sum> ::= 0 | 1
| <Sum> + <Sum> | (<Sum>)

BNF Deriviations

- Given rules

X::= *y***Z***w* and **Z**::=*v*

we may replace **Z** by *v* to say

X => *y***Z***w* => *yvw*

- Sequence of such replacements called *derivation*
- Derivation called *right-most* if always replace the right-most non-terminal

BNF Derivations

- Start with the start symbol:

<Sum> =>

BNF Derivations

- Pick a non-terminal

<Sum> =>

BNF Derivations

- Pick a rule and substitute:
  - <Sum> ::= <Sum> + <Sum>

<Sum> => <Sum> + <Sum >

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >

BNF Derivations

- Pick a rule and substitute:
  - <Sum> ::= (<Sum>)

<Sum> => <Sum> + <Sum >
      => (<Sum>) + <Sum>

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
      => (<Sum>) + <Sum>

BNF Derivations

- Pick a rule and substitute:
  - <Sum> ::= <Sum> + <Sum>

<Sum> => <Sum> + <Sum >
      => (<Sum>) + <Sum>
      => (<Sum> + <Sum>) + <Sum>

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
      => (<Sum>) + <Sum>
      => (<Sum> + <Sum>) + <Sum>

BNF Derivations

- Pick a rule and substitute:
  - <Sum >::= 1

<Sum> => <Sum> + <Sum >
      => (<Sum>) + <Sum>
      => (<Sum> + <Sum>) + <Sum>
      => (<Sum> + 1) + <Sum>

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
      => (<Sum>) + <Sum>
      => (<Sum> + <Sum>) + <Sum>
      => (<Sum> + 1) + <Sum>

BNF Derivations

- Pick a rule and substitute:
  - <Sum >::= 0

```
<Sum> => <Sum> + <Sum >
      => ( <Sum> ) + <Sum>
      => ( <Sum> + <Sum> ) + <Sum>
      => ( <Sum> + 1 ) + <Sum>
      => ( <Sum> + 1 ) + 0
```

BNF Derivations

- Pick a non-terminal:

```
<Sum> => <Sum> + <Sum >
      => ( <Sum> ) + <Sum>
      => ( <Sum> + <Sum> ) + <Sum>
      => ( <Sum> + 1 ) + <Sum>
      => ( <Sum> + 1 ) + 0
```

BNF Derivations

- Pick a rule and substitute
  - <Sum> ::= 0

```
<Sum> => <Sum> + <Sum >
      => ( <Sum> ) + <Sum>
      => ( <Sum> + <Sum> ) + <Sum>
      => ( <Sum> + 1 ) + <Sum>
      => ( <Sum> + 1 ) 0
      => ( 0 + 1 ) + 0
```

BNF Derivations

- (0 + 1) + 0 is generated by grammar

```
<Sum> => <Sum> + <Sum >
      => ( <Sum> ) + <Sum>
      => ( <Sum> + <Sum> ) + <Sum>
      => ( <Sum> + 1 ) + <Sum>
      => ( <Sum> + 1 ) + 0
      => ( 0 + 1 ) + 0
```

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

```
<Sum> =>
```

BNF Semantics

- The meaning of a BNF grammar is the set of all strings consisting only of terminals that can be derived from the Start symbol

Regular Grammars

- Subclass of BNF
- Only rules of form <nonterminal>::=<terminal><nonterminal> or <nonterminal>::=<terminal> or <nonterminal>::= ε
- Defines same class of languages as regular expressions
- Important for writing lexers (programs that convert strings of characters into strings of tokens)
- Close connection to nondeterministic finite state automata – nonterminals ≅ states; rule ≅ edge

Example

- Regular grammar:

  <Balanced> ::= ε
  <Balanced> ::= 0<OneAndMore>
  <Balanced> ::= 1<ZeroAndMore>
  <OneAndMore> ::= 1<Balanced>
  <ZeroAndMore> ::= 0<Balanced>
- Generates even length strings where every initial substring of even length has same number of 0's as 1's

Extended BNF Grammars

- Alternatives: allow rules of from X::=*y*|*z*
  - Abbreviates X::= *y*, X::= *z*
- Options: X::=*y*[*v*]*z*
  - Abbreviates X::=*yvz*, X::=*yz*
- Repetition: X::=*y*{*v*}**z*
  - Can be eliminated by adding new nonterminal V and rules X::=*yz*, X::=*y*V*z*, V::=*v*, V::=*v*V

Parse Trees

- Graphical representation of derivation
- Each node labeled with either non-terminal or terminal
- If node is labeled with a terminal, then it is a leaf (no sub-trees)
- If node is labeled with a non-terminal, then it has one branch for each character in the right-hand side of rule used to substitute for it

Example

- Consider grammar:

```
<exp>  ::= <factor>
         |  <factor> +  <factor>
<factor>  ::=  <bin>
           |  <bin>  *  <exp>
<bin> ::=  0  | 1
```

- Problem: Build parse tree for 1 * 1 + 0 as an <exp>

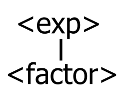
Example cont.

- 1 * 1 + 0:   <exp>

<exp> is the start symbol for this parse tree

Example cont.

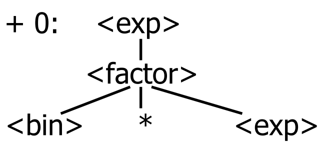
- 1 * 1 + 0:



Use rule: <exp> ::= <factor>

Example cont.

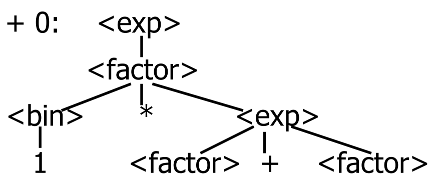
- 1 * 1 + 0:



Use rule: <factor> ::= <bin> * <exp>

Example cont.

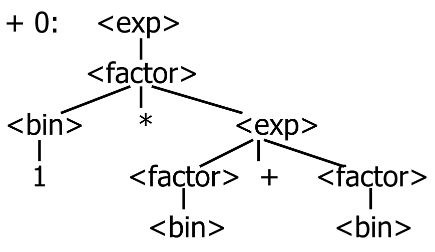
- 1 * 1 + 0:



Use rules: <bin> ::= 1 and
<exp> ::= <factor> + <factor>

Example cont.

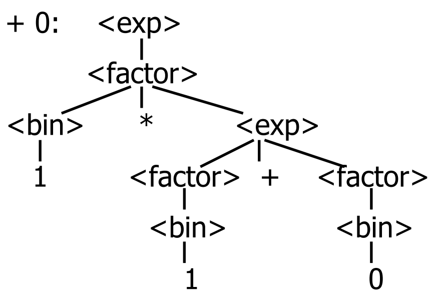
- 1 * 1 + 0:



Use rule: <factor> ::= <bin>

Example cont.

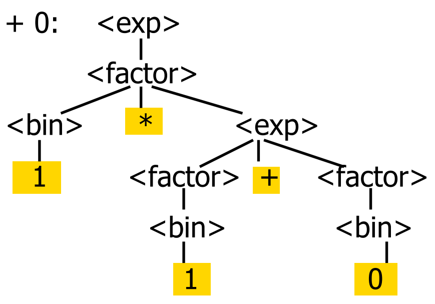
- 1 * 1 + 0:



Use rules: <bin> ::= 1 | 0

Example cont.

- 1 * 1 + 0:



Fringe of tree is string generated by grammar

Your Turn: 1 * 0 + 0 * 1

Parse Tree Data Structures

- Parse trees may be represented by OCaml datatypes
- One datatype for each nonterminal
- One constructor for each rule
- Defined as mutually recursive collection of datatype declarations

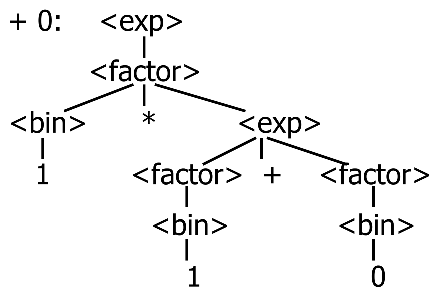
Example

- Recall grammar:
  <exp> ::= <factor> | <factor> + <factor>
  <factor> ::= <bin> | <bin> * <exp>
  <bin> ::= 0 | 1
- type exp = Factor2Exp of factor

```
type exp = Factor2Exp of factor
             | Plus of factor * factor
and factor = Bin2Factor of bin
                | Mult of bin * exp
and bin = Zero | One
```

Example cont.

- 1 * 1 + 0:



Example cont.

- Can be represented as

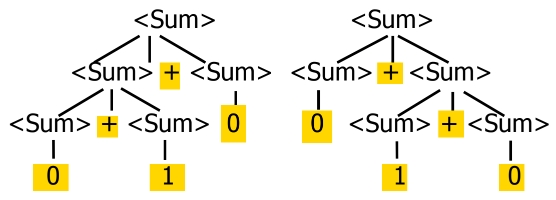
```
Factor2Exp
(Mult(One,
        Plus(Bin2Factor One,
               Bin2Factor Zero)))
```

Ambiguous Grammars and Languages

- A BNF grammar is *ambiguous* if its language contains strings for which there is more than one parse tree
- If all BNF’s for a language are ambiguous then the language is *inherently ambiguous*

Example: Ambiguous Grammar

- 0 + 1 + 0



Example

- What is the result for:

  3 + 4 * 5 + 6

Example

- What is the result for:

  3 + 4 * 5 + 6

- Possible answers:
  - 41 = ((3 + 4) * 5) + 6
  - 47 = 3 + (4 * (5 + 6))
  - 29 = (3 + (4 * 5)) + 6 = 3 + ((4 * 5) + 6)
  - 77 = (3 + 4) * (5 + 6)

Example

- What is the value of:

  7 – 5 – 2

Example

- What is the value of:

  7 – 5 – 2

- Possible answers:
  - In Pascal, C++, SML assoc. left

    7 – 5 – 2 = (7 – 5) – 2 = 0
  - In APL, associate to right

    7 – 5 – 2 = 7 – (5 – 2) = 4

Two Major Sources of Ambiguity

- Lack of determination of operator precedence
- Lack of determination of operator assoicativity

- Not the only sources of ambiguity