# Solutions for Sample Questions for Midterm 1 (CS 421 Fall 2017)

On the actual midterm, you will have plenty of space to put your answers. The actual midterm will likely have no more than 9 questions plus one extra credit question. In addition to questions of the kind asked below, you should expect to see questions from your MPs and WA1 on the exams.
Some of these questions may be reused for the exam.

1. Given the following OCAML code:
   > **let x = 3;;**
   > **let f y = x + y;;**
   > **let x = 5;;**
   > **let z = f 2;;**
   > **let x = "hi";;**

   What value will **z** have? Will the last declaration (**let x = "hi";**) cause a type error?
   What is the value of **x** after this code has been executed?

**Solution:**
   z is bound to 5
   let x = "hi" will not cause a type error
   x is bound to "hi"

2. What environment is in effect after each declaration in the code in Problem 1?
   What is the step-by-step formal evaluation of **f 2** in the fourth declaration, starting from the environment after the third declaration?

**Solution:**
   > **let x = 3;;**
   > **{x → 3}**
   >
   > **let f y = x + y;;**
   > **{f → <y → x+y, {x → 3}>, x → 3}**
   >
   > **let x = 5;;**
   > **{x → 5} + {f → <y → x+y, {x → 3}>, x → 3} =**
   > **{x → 5, f → <y → x+y, {x → 3}>}**
   >
   > **let z = f 2;;**
   > **Eval(f 2, {x → 5, f → <y → x+y, {x → 3}>} =**
   > **App(Eval(f, {x → 5, f → <y → x+y, {x → 3}>}),**
   > **        Eval(2, {x → 5, f → <y → x+y, {x → 3}>}))) =**
   > **App(<y → x+y, {x → 3}>,2) =**
   > **Eval(x+y, {y→2} + {x → 3}) =**
   > **Eval(x, {y→2, x → 3}) + Eval(y, {y→2, x → 3}) =3 + 2 = 5**
   >
   > **{z → 5, x → 5, f → <y → x+y, {x → 3}>}**

let x = "hi";;
{x → "hi"} + {z → 5, x → 5, f → <y → x+y, {x → 3}>} =
{x → "hi", z → 5,  f → <y → x+y, {x → 3}>}

3. What is the effect of each of the following pieces of code?
    a. **(fun x -> (print_string "a"; x + 2)) (print_string "b"; 4);;**
       **It prints ba and returns 6**
    b. **let f = (print_string "a"; fun x -> x + 2) in f (print_string "b"; 4);;**
       **It prints ab and returns 6**
    c. **let f = fun g -> (print_string "a"; g 2) in f (fun x -> print_string "b"; 4 + x);;**
       **It prints ab and returns 6**
4. Consider the following two OCaml functions, **loop1** and **loop2**:
       **let rec loop1 () = loop1(); ()**
       **let rec loop2 () = loop2();;**
       **val loop1 : unit -> unit = <fun>**
       **val loop2 : unit -> 'a = <fun>**
   Suppose you were to run **loop1();;** and **loop2();;** in OCaml, (pressing CTRL + C after at
   least a minute to terminate infinite loops when necessary).
    a. For each program, what behavior would you expect to see?
    b. What is the difference between loop1 and loop2?
    c. For each program state if it is:
        i.   recursive,
        **ii.**  forward recursive,
        **iii.** tail-recursive.
**Solution:**
    a. The first program generates a stack overflow, while the second program runs
       indefinitely.
    b. Because **loop1** is not tail-recursive, each new recursive call must push a new
       activation record onto the stack, hence the stack overflow, but since **loop2** is tail-
       recursive, each new activation record may overwrite the previous call, and thus the
       stack does not grow. It should also be observed that **loop2** has a more general type
       (**:unit -> 'a**) than that of **loop1:unit -> unit**, and hence may be used in places
       where other return types beside unit are required. (Of course, it had better never
       actually be applied.)
    c. Both programs are recursive, and in fact forward recursive, but **loop2** is tail-
       recursive while **loop1** is not


5. Write an OCAML function **pair_up** that takes first a function, then an input list and
   returns a list of pairs of an element from input list (the second argument), paired with the
   result of applying the first argument to that element.  What is the OCAML type of
   **pair_up**?  What is the result of the following expressions:
    a. **pair_up (fun x -> x + 3) [6;4;1];;**
    b. **pair_up ((fun x -> "Hi, "^x), ["John"; "Mary";"Dana"]);;**
    c. **pair_up (fun x -> x *. 2.0);;**

**Solution:**

```
let rec  pair_up  f  l =
    (match l with [] -> []
        | x :: xs ->  (x, f x)::pair_up f xs)
    alternately  let pair_up f = List.map (fun x -> (x, f x))

    pair_up : ('a -> 'b)  -> 'a list -> ('a * 'b) list
```

   a.  [(6, 9); (4,7); (1,4)];;
   b.  type error
   c.  A function of type float list -> (float * float) list that returns a list of pairs of an element from the input list paired with twice itself.

6. Write an Ocaml function **palindrome :string list -> unit** that first prints the strings in the list from left to right, followed by printing them right to left, recursing over the list only once. (Potential extra credit problem: Do this using each of **List.fold_right** and **List.fold_left** but no explicit use of **let rec**.)

**Solution:**

```
let rec palindrome l =
    match l with [] -> ()
        | s::ss -> (print_string s; palindrome ss; print_string s);;

let rec palindrome l =
    List.fold_right
    (fun s -> fun print_middle -> (fun () -> (print_string s; print_middle (); print_string s)))
    l
    (fun () -> ())
    ();;

let palindrome l =
    List.fold_left
    (fun print_middle -> fun s ->  (print_string s; fun () -> (print_middle (print_string s))))
    (fun () -> ())
    l
    ();;
```

7. Using **fold_right : ('a -> 'b -> 'b) -> 'a list -> 'b -> 'b**, but without using explicit recursion, write a function **concat** : ' a list list -> 'a list that appends all the lists in the input list of lists, preserving the order of elements. You may use the append function **@.**

**Solution:** let concat lst = List.fold_right (@) lst [];;

8. Write an Ocaml function **list_print : string list -> unit** that prints all the strings in a list from left to right:
   a.  using tail recursion, but no higher order functions,
   b.  using **fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a** but no explicit recursion.

**Solution:**

a. let rec list_print lst = match lst with [ ] -> ( ) | s::ss -> (print_string s; list_print ss);;
b. let list_print lst = List.fold_left (fun ( ) -> fun s -> print_string s) () lst;;

9. Put the following function in full continuation passing style:
    **let rec  sum_odd  n = if n <= 0 then 0 else ((2 * n) – 1) + sum_odd (n – 1);;**
    Primitive operations ( +, - , *, <=) do not have to be converted to CPS, but all procedure calls must be.
**Solution:**
    let add_k a b k = k(a + b)
    let minus_k a b k = k(a - b)
    let times_k a b k = k(a * b)
    let leq_k a b k = k(a <= b)


    let rec  sum_odd_k n k =
       leq_k n 0 (fun b ->  if  b then k 0
                     else minus_k n 1
                     (fun d -> sum_odd_k d
                     (fun r -> times_k 2 n
                      (fun t -> minus_k t 1
                       (fun m -> add_k m r k )))))

10. Write the definition of an OCAML variant type **reg_exp** to express abstract syntax trees for regular expressions over a base character set of booleans.  Thus, a boolean is a **reg_exp,** epsilon is a **reg_exp**, the concatenation of two **reg_exp**'s is a **reg_exp,** the "choice" of two **reg_exp**'s is a **reg_exp**, and the Kleene star of a **reg_exp** is a **reg_exp**.
**Solution:**
type reg_exp =
  Char of bool
 | Epsilon
 | Concat of (reg_exp * reg_exp)
 | Choice of (reg_exp * reg_exp)
 | Kleene_star of reg_exp
 | Paren of reg_exp     (* I would accept it with this case missing *)


11. Given the following OCAML datatype:
                **type int_seq = Null  | Snoc of (int_seq * int)**
    write a tail-recursive function in OCAML **all_pos : int_seq -> bool** that returns **true** if every integer in the input **int_seq** to which **all_pos** is applied is strictly greater than 0 and **false** otherwise.  Thus **all_pos (Snoc(Snoc(Snoc(Null,3),5),7))** should returns **true,** but ) **all_pos (Snoc(Snoc(Null,~1))** and **all_pos (Snoc(Snoc(Null, 3),0))** should both return **false.**
**Solution:**
    let rec all_pos s =
       (match s with Null -> true
         | Snoc(seq, x) -> if x <= 0 then false else all_pos seq);;