HW 7 – Unification
CS 421 – Fall 2014
Revision 1.0

Assigned Tuesday, October 14, 2014
Due Tuesday, October 21, 2014, 23:59 pm
Extension 48 hours (20% penalty)

1 Change Log

1.0 Initial Release.

2 Turn-In Procedure

Answer the problems below, then using your favorite tool(s), save your work as a PDF (either scanned if handwritten or converted from a program), and commit the PDF in your your repository in the directory `assignments/hw7`. Your file should be named `hw7-submission.pdf`. The command to commit this file is:

```
svn commit -m "Turning in hw7." hw7-submission.pdf
```

3 Objectives and Background

The purpose of this HW is to test your understanding of

- How to unify a system of equations

Another purpose of HW7 is to provide you with experience answering non-programming written questions of the kind you may experience on the second midterm and final.

4 Problems

1. (24 points) Give a most general unifier for the following set of equations (unification problem). The uppercase letters A, B, C, D and E denote variables of unification. The lowercase letters f and p are term constructors of arity (*i.e.* take two arguments), and l is a term constructor of arity 1 (*i.e.* takes one argument). Show all your work by listing the operations performed in each step of the unification and the result of that step. Your final result should be a single simultaneous substitution, not a composition of separate substitutions.

$$\mathsf{Unify}\{(p(A, C) = p(B, D)); (l(C) = E); (l(f(n, A)) = E); (l(f(A, B)) = E)\}$$

Solution:

$\mathsf{Unify}\{(p(A,C) = p(B,D)); (l(C) = E); (l(f(n,A)) = E); (l(f(A,B)) = E)\}$

$= \mathsf{Unify}\{(A = B); (C = D); (l(C) = E); (l(f(n,A)) = E); (l(f(A,B)) = E)\}$ by Decompose

$= \mathsf{Unify}\{(C = D); (l(C) = E); (l(f(n,B)) = E); (l(f(B,B)) = E)\}o\{A \mapsto B\}$ by Eliminate

$= \mathsf{Unify}\{(l(D) = E); (l(f(n,B)) = E); (l(f(B,B)) = E)\}o\{C \mapsto D\}o\{A \mapsto B\}$ by Eliminate

$= \mathsf{Unify}\{(l(D) = E); (l(f(n,B)) = E); (l(f(B,B)) = E)\}o\{C \mapsto D; A \mapsto B\}$

$= \mathsf{Unify}\{(E = l(D)); (l(f(n,B)) = E); (l(f(B,B)) = E)\}o\{C \mapsto D; A \mapsto B\}$ by Orient

$= \mathsf{Unify}\{(l(f(n,B)) = l(D)); (l(f(B,B)) = l(D))\}o\{E \mapsto l(D)\}o\{C \mapsto D; A \mapsto B\}$ by Eliminate

$= \mathsf{Unify}\{(l(f(n,B)) = l(D)); (l(f(B,B)) = l(D))\}o\{E \mapsto l(D) \ C \mapsto D; A \mapsto B\}$

$= \mathsf{Unify}\{(f(n,B) = D); (l(f(B,B)) = l(D))\}o\{E \mapsto l(D) \ C \mapsto D; A \mapsto B\}$ by Decompose

$= \mathsf{Unify}\{(D = f(n,B)); (l(f(B,B)) = l(D))\}o\{E \mapsto l(D) \ C \mapsto D; A \mapsto B\}$ by Orient

$= \mathsf{Unify}\{(l(f(B,B)) = l(f(n,B)))\}o\{D \mapsto f(n,B)\}o\{E \mapsto l(D) \ C \mapsto D; A \mapsto B\}$ by Eliminate

$= \mathsf{Unify}\{(l(f(B,B)) = l(f(n,B)))\}o\{D \mapsto f(n,B); E \mapsto l(f(n,B)); C \mapsto f(n,B); A \mapsto B\}$

$= \mathsf{Unify}\{(f(B,B) = f(n,B))\}o\{D \mapsto f(n,B); E \mapsto l(f(n,B)); C \mapsto f(n,B); A \mapsto B\}$ by Decompose

$= \mathsf{Unify}\{(B = n); (B = B)\}o\{D \mapsto f(n,B); E \mapsto l(f(n,B)); C \mapsto f(n,B); A \mapsto B\}$ by Decompose

$= \mathsf{Unify}\{(n = n)\}o\{B \mapsto n\}o\{D \mapsto f(n,B); E \mapsto l(f(n,B)); C \mapsto f(n,B); A \mapsto B\}$ by Eliminate

$= \mathsf{Unify}\{(n = n)\}o\{B \mapsto n; D \mapsto f(n,n); E \mapsto l(f(n,n)); C \mapsto f(n,n); A \mapsto n\}$

$= \mathsf{Unify}\{ \ \}o\{B \mapsto n; D \mapsto f(n,n); E \mapsto l(f(n,n)); C \mapsto f(n,n); A \mapsto n\}$ by Delete*

$= \{B \mapsto n; D \mapsto f(n,n); E \mapsto l(f(n,n)); C \mapsto f(n,n); A \mapsto n\}$ by Base Case

Note (*): This is also true by Decompose, since n has no arguments.