
HW 6 – Polymorphic Type Inference

CS 421 – Fall 2014

Revision 1.3

Assigned October 2, 2014

Due October 14, 2014, 23:59 pm

Extension 48 hours (20% penalty)

1 Change Log

1.3 Re-corrected the turn-in procedure.

1.2 Corrected the type annotation.

1.1 Corrected the turn-in procedure.

1.0 Initial Release.

2 Turn-In Procedure

Answer the problem below, save your work as a PDF (either scanned if handwritten or converted from a program), and hand in the PDF. Your file should be named `hw6-submission.pdf`.

3 Objectives and Background

The purpose of this HW is to test your understanding of how to use typing rules to perform polymorphic type derivations in a functional programming language (here with OCaml syntax). Another purpose of HWs is to provide you with experience answering non-programming written questions of the kind you may experience on the midterms and final.

4 Problems

(22 points) Give a complete type derivation for the following typing judgment.

```
let id = fun x -> x in if (id true) then (fun id -> id + 1) else id : int
-> int
```

As a suggestion for formatting, you may want to name subtrees of the proof and write them out separately. Note, we are asking for a type derivation not the intermediate states of a type inferencing algorithm.

Solution:

Let

- L stand for the Let-In Rule
- V stand for the Variable Rule,
- C standfor the Constant Rule,
- I stand for the If-Then-Else Rule,
- F stand for the Function Rule,
- A stand for the Application Rule and
- P for the Primitive Operation Rule.

$$\begin{array}{c}
 \frac{}{\{id: int\} \vdash id: int} V \quad \frac{}{\{id: int\} \vdash 1: int} C \\
 \hline
 \frac{}{\{id: int\} \vdash id: int} P^3 \\
 \\
 \frac{}{\{id: \forall 'a. 'a \rightarrow 'a\} \vdash id: bool \rightarrow bool} V^1 \quad \frac{}{\{id: \forall 'a. 'a \rightarrow 'a\} \vdash true: bool} C \\
 \hline
 \frac{}{\{id: \forall 'a. 'a \rightarrow 'a\} \vdash (id\ true): bool} A \quad \frac{}{\{id: int\} \vdash (id + 1): int} F \\
 \hline
 \frac{}{\{id: \forall 'a. 'a \rightarrow 'a\} \vdash (fun\ id \rightarrow id + 1): int \rightarrow int} F \quad \frac{}{\{id: \forall 'a. 'a \rightarrow 'a\} \vdash id: int \rightarrow int} V^2 \\
 \hline
 \frac{}{\{id: \forall 'a. 'a \rightarrow 'a\} \vdash if\ (id\ true)\ then\ (fun\ id \rightarrow id + 1)\ else\ id: int \rightarrow int} I \\
 \\
 \frac{}{\{x: 'a\} \vdash x: 'a} V \quad \frac{}{\{\} \vdash fun\ x \rightarrow x: 'a \rightarrow 'a} F \\
 \hline
 \frac{}{\{\} \vdash let\ id = fun\ x \rightarrow x\ in\ if\ (id\ true)\ then\ (fun\ id \rightarrow id + 1)\ else\ id: int \rightarrow int} L
 \end{array}$$

1. where we instantiate 'a with bool
2. where we instantiate 'a with int
3. where (+): int -> int -> int. I would aslo accept:

$$\frac{}{\{id: int\} \vdash (+): int \rightarrow int \rightarrow int} C \quad \frac{}{\{id: int\} \vdash id: int} V \\
 \hline
 \frac{}{\{id: int\} \vdash (+)\ id: int \rightarrow int} A \quad \frac{}{\{id: int\} \vdash 1: int} C \\
 \hline
 \frac{}{\{id: int\} \vdash (id + 1): int} A$$