Programming Languages and Compilers (CS 421)

Elsa L Gunter
2112 SC, UIUC
http://courses.engr.illinois.edu/cs421

Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Natural Semantics

- Aka Structural Operational Semantics, aka "Big Step Semantics"
- Provide value for a program by rules and derivations, similar to type derivations
- Rule conclusions look like

$$(C, m) \Downarrow m'$$

or

$$(E, m) \Downarrow v$$

Simple Imperative Programming Language

- $I \in$ *Identifiers*
- $N \in$ *Numerals*
- B ::= true | false | B & B | B or B | not B | $E < E$ | $E = E$
- $E ::= N \mid I \mid E + E \mid E * E \mid E - E \mid - E$
- C::= skip | $C;C$ | I ::= E | if B then C else C fi | while B do C od

Natural Semantics of Atomic Expressions

- Identifiers: $(I,m) \Downarrow m(I)$
- Numerals are values: $(N,m) \Downarrow N$
- Booleans: $(\text{true},m) \Downarrow \text{true}$
  $(\text{false} ,m) \Downarrow \text{false}$

Booleans:

$$\frac{(B, m) \Downarrow \text{false}}{(B \,\&\, B', m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{true} \quad (B', m) \Downarrow b}{(B \,\&\, B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(B \text{ or } B', m) \Downarrow \text{true}} \qquad \frac{(B, m) \Downarrow \text{false} \quad (B', m) \Downarrow b}{(B \text{ or } B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(\text{not } B, m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{false}}{(\text{not } B, m) \Downarrow \text{true}}$$

Relations

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \sim V = b}{(E \sim E', m) \Downarrow b}$$

- By $U \sim V = b$, we mean does (the meaning of) the relation $\sim$ hold on the meaning of U and V
- May be specified by a mathematical expression/equation or rules matching U and V

Arithmetic Expressions

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \; op \; V = N}{(E \; op \; E', m) \Downarrow N}$$

where N is the specified value for $U \; op \; V$

Commands

Skip: (skip, m) $\Downarrow m$

Assignment: $$\frac{(E,m) \Downarrow V}{(I::=E,m) \Downarrow m[I \text{ <-- } V]}$$

Sequencing: $$\frac{(C,m) \Downarrow m' \quad (C',m') \Downarrow m''}{(C;C', m) \Downarrow m''}$$

If Then Else Command

$$\frac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

$$\frac{(B,m) \Downarrow \text{false} \quad (C',m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

While Command

$$\frac{(B,m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m}$$

$$\frac{(B,m)\Downarrow\text{true} \; (C,m)\Downarrow m' \quad (\text{while } B \text{ do } C \text{ od}, m')\Downarrow m''}{(\text{while } B \text{ do C od}, m) \Downarrow m''}$$

Example: If Then Else Rule

$$\frac{}{(\text{if x > 5 then y:= 2 + 3 else y:=3 + 4 fi, \{x -> 7\}}) \Downarrow ?}$$

Example: If Then Else Rule

$$\frac{\frac{}{(\text{x > 5, \{x -> 7\}})\Downarrow ?}}{(\text{if x > 5 then y:= 2 + 3 else y:=3 + 4 fi, \{x -> 7\}}) \Downarrow ?}$$

Example: Arith Relation

? > ? = ?
(x,{x->7})⇓? (5,{x->7})⇓?
(x > 5, {x -> 7})⇓?
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: Identifier(s)

7 > 5 = true
(x,{x->7})⇓7 (5,{x->7})⇓5
(x > 5, {x -> 7})⇓?
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: Arith Relation

7 > 5 = true
(x,{x->7})⇓7 (5,{x->7})⇓5
(x > 5, {x -> 7})⇓true
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: If Then Else Rule

7 > 5 = true
(x,{x->7})⇓7 (5,{x->7})⇓5 (y:= 2 + 3, {x-> 7}
(x > 5, {x -> 7})⇓true ⇓ ? .
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: Assignment

(2+3, {x->7})⇓?
7 > 5 = true
(x,{x->7})⇓7 (5,{x->7})⇓5 (y:= 2 + 3, {x-> 7}
(x > 5, {x -> 7})⇓true ⇓ ?
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: Arith Op

? + ? = ?
(2,{x->7})⇓? (3,{x->7}) ⇓?
7 > 5 = true (2+3, {x->7})⇓?
(x,{x->7})⇓7 (5,{x->7})⇓5 (y:= 2 + 3, {x-> 7}
(x > 5, {x -> 7})⇓true ⇓? .
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: Numerals

2 + 3 = 5
(2,{x->7})⇓2   (3,{x->7}) ⇓3
7 > 5 = true   (2+3, {x->7})⇓?
(x,{x->7})⇓7   (5,{x->7})⇓5   (y:= 2 + 3, {x-> 7}
(x > 5, {x -> 7})⇓true   ⇓ ?
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: Arith Op

2 + 3 = 5
(2,{x->7})⇓2   (3,{x->7}) ⇓3
7 > 5 = true   (2+3, {x->7})⇓5
(x,{x->7})⇓7   (5,{x->7})⇓5   (y:= 2 + 3, {x-> 7}
(x > 5, {x -> 7})⇓true   ⇓?
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: Assignment

2 + 3 = 5
(2,{x->7})⇓2   (3,{x->7}) ⇓3
7 > 5 = true   (2+3, {x->7})⇓5
(x,{x->7})⇓7   (5,{x->7})⇓5   (y:= 2 + 3, {x-> 7}
(x > 5, {x -> 7})⇓true   ⇓ {x->7, y->5}
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓?

Example: If Then Else Rule

2 + 3 = 5
(2,{x->7})⇓2   (3,{x->7}) ⇓3
7 > 5 = true   (2+3, {x->7})⇓5
(x,{x->7})⇓7   (5,{x->7})⇓5   (y:= 2 + 3, {x-> 7}
(x > 5, {x -> 7})⇓true   ⇓ {x->7, y->5}
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ {x->7, y->5}

Let in Command

(*E*,*m*) ⇓v (*C*,*m*[*I*<-*v*]) ⇓ *m'*
(let *I* = *E* in *C*, *m*) ⇓ *m''*

Where *m''* (*y*) = *m'* (*y*) for *y*≠ *I* and
m'' (*I*) = *m* (*I*) if *m*(*I*) is defined,
and *m''* (*I*) is undefined otherwise

Example

(x,{x->5}) ⇓ 5   (3,{x->5}) ⇓ 3
(x+3,{x->5}) ⇓ 8
(5,{x->17}) ⇓ 5   (x:=x+3,{x->5}) ⇓ {x->8}
(let x = 5 in (x:=x+3), {x -> 17}) ⇓ ?

Example

(x,{x->5}) ⇓ 5   (3,{x->5}) ⇓ 3
———————————————————
(x+3,{x->5}) ⇓ 8
(5,{x->17}) ⇓ 5   (x:=x+3,{x->5}) ⇓ {x->8}
———————————————————
(let x = 5 in (x:=x+3), {x -> 17}) ⇓ {x->17}

Comment

- Simple Imperative Programming Language introduces variables *implicitly* through assignment
- The let-in command introduces scoped variables *explictly*
- Clash of constructs apparent in awkward semantics

Interpretation Versus Compilation

- A **compiler** from language L1 to language L2 is a program that takes an L1 program and for each piece of code in L1 generates a piece of code in L2 of same meaning
- An **interpreter** of L1 in L2 is an L2 program that executes the meaning of a given L1 program
- Compiler would examine the body of a loop once; an interpreter would examine it every time the loop was executed

Interpreter

- An *Interpreter* represents the operational semantics of a language L1 (source language) in the language of implementation L2 (target language)
- Built incrementally
  - Start with literals
  - Variables
  - Primitive operations
  - Evaluation of expressions
  - Evaluation of commands/declarations

Interpreter

- Takes abstract syntax trees as input
  - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
  - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next "state"
  - To get final value, put in a loop

Natural Semantics Example

- compute_exp (Var(v), m) = look_up v m
- compute_exp (Int(n), _) = Num (n)
- …
- compute_com(IfExp(b,c1,c2),m) =
  if compute_exp (b,m) = Bool(true)
  then compute_com (c1,m)
  else compute_com (c2,m)

Natural Semantics Example

- compute_com(While(b,c), m) =
  if compute_exp (b,m) = Bool(false)
  then m
  else compute_com
  (While(b,c), compute_com(c,m))

- May fail to terminate - exceed stack limits
- Returns no useful information then

Transition Semantics

- Form of operational semantics
- Describes how each program construct transforms machine state by *transitions*
- Rules look like
  (C, *m*) --> (C', *m'*) or (C,*m*) --> *m'*
- *C, C'* is code remaining to be executed
- m, m' represent the state/store/memory/ environment
  - Partial mapping from identifiers to values
  - Sometimes *m* (or *C*) not needed
- Indicates exactly one step of computation

Expressions and Values

- *C, C'* used for commands; *E, E' for* expressions; *U,V* for values
- Special class of expressions designated as *values*
  - Eg 2, 3 are values, but 2+3 is only an expression
- Memory only holds values
  - Other possibilities exist

Evaluation Semantics

- Transitions successfully stops when E/*C* is a value/memory
- Evaluation fails if no transition possible, but not at value/memory
- Value/memory is the final *meaning* of original expression/command (in the given state)
- Coarse semantics: final value / memory
- More fine grained: whole transition sequence

Simple Imperative Programming Language

- *I* ∈ *Identifiers*
- *N* ∈ *Numerals*
- *B* ::= true | false | *B* & *B* | *B* or *B* | not *B* | *E* < *E* | *E* = *E*
- *E::= N | I | E + E | E * E | E - E | - E*
- C::= skip | *C;C* | *I* ::= *E*
  | if *B* then *C* else *C* fi | while *B* do *C* od

Transitions for Expressions

- Numerals are values
- Boolean values = {true, false}
- Identifiers: (*I*,*m*) --> (*m*(*I*), *m*)

Boolean Operations:

- Operators: (short-circuit)

(false & *B*, *m*) --> (false,*m*)

(true & *B*, *m*) --> (*B*,*m*)

$$\frac{(B, m) \dashrightarrow (B'', m)}{(B \,\&\, B', m) \dashrightarrow (B'' \,\&\, B', m)}$$

(true or *B*, *m*) --> (true,*m*)

(false or *B*, *m*) --> (*B*,*m*)

$$\frac{(B, m) \dashrightarrow (B'', m)}{(B \text{ or } B', m) \dashrightarrow (B'' \text{ or } B', m)}$$

(not true, m) --> (false,*m*)

(not false, m) --> (true,*m*)

$$\frac{(B, m) \dashrightarrow (B', m)}{(\text{not } B, m) \dashrightarrow (\text{not } B', m)}$$

Relations

$$\frac{(E, m) \dashrightarrow (E'', m)}{(E \sim E', m) \dashrightarrow (E'' \sim E', m)}$$

$$\frac{(E, m) \dashrightarrow (E', m)}{(V \sim E, m) \dashrightarrow (V \sim E', m)}$$

(*U* ~ *V*, *m*) --> (true,*m*) or (false,*m*) depending on whether *U* ~ *V* holds or not

Arithmetic Expressions

$$\frac{(E, m) \dashrightarrow (E'', m)}{(E \text{ op } E', m) \dashrightarrow (E'' \text{ op } E', m)}$$

$$\frac{(E, m) \dashrightarrow (E', m)}{(V \text{ op } E, m) \dashrightarrow (V \text{ op } E', m)}$$

(*U op V*, *m*) --> (*N*,*m*) where *N* is the specified value for *U op V*

Commands - in English

- skip means done evaluating
- When evaluating an assignment, evaluate the expression first
- If the expression being assigned is already a value, update the memory with the new value for the identifier
- When evaluating a sequence, work on the first command in the sequence first
- If the first command evaluates to a new memory (ie completes), evaluate remainder with new memory

Commands

(skip, *m*) --> *m*

$$\frac{(E, m) \dashrightarrow (E', m)}{(I ::= E, m) \dashrightarrow (I ::= E', m)}$$

(*I*::=*V*,*m*) --> *m*[*I* <-- *V*]

$$\frac{(C, m) \dashrightarrow (C'', m')}{(C;C', m) \dashrightarrow (C'';C', m')} \qquad \frac{(C, m) \dashrightarrow m'}{(C;C', m) \dashrightarrow (C', m')}$$

If Then Else Command - in English

- If the boolean guard in an if_then_else is true, then evaluate the first branch
- If it is false, evaluate the second branch
- If the boolean guard is not a value, then start by evaluating it first.

If Then Else Command

(if true then *C* else *C'* fi, *m*) --> (*C*, *m*)

(if false then *C* else *C'* fi, *m*) --> (*C'*, *m*)

(*B*,*m*) --> (*B'*,*m*)
(if *B* then *C* else *C'* fi, *m*)
--> (if *B'* then *C* else *C'* fi, *m*)

While Command

(while *B* do *C* od, *m*) -->
(if *B* then *C*; while *B* do *C* od else skip fi, m)

In English: Expand a While into a test of the boolean guard, with the true case being to do the body and then try the while loop again, and the false case being to stop.

Example Evaluation

- First step:

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7})
--> ?

Example Evaluation

- First step:

(x > 5, {x -> 7}) --> ?
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7})
--> ?

Example Evaluation

- First step:

(x,{x -> 7}) --> (7, {x -> 7})
(x > 5, {x -> 7}) --> ?
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7})
--> ?

Example Evaluation

- First step:

(x,{x -> 7}) --> (7, {x -> 7})
(x > 5, {x -> 7}) --> (7 > 5, {x -> 7})
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7})
--> ?

Example Evaluation

- First step:

(x,{x -> 7}) --> (7, {x -> 7})
____
(x > 5, {x -> 7}) --> (7 > 5, {x -> 7})
____
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7})
--> (if 7 > 5 then y:=2 + 3 else y:=3 + 4 fi,
{x -> 7})

Example Evaluation

- Second Step:

(7 > 5, {x -> 7}) --> (true, {x -> 7})
____
(if 7 > 5 then y:=2 + 3 else y:=3 + 4 fi,
{x -> 7})
--> (if true then y:=2 + 3 else y:=3 + 4 fi,
{x -> 7})

- Third Step:

(if true then y:=2 + 3 else y:=3 + 4 fi, {x -> 7})
-->(y:=2+3, {x->7})

Example Evaluation

- Fourth Step:

(2+3, {x-> 7}) --> (5, {x -> 7})
____
(y:=2+3, {x->7}) --> (y:=5, {x->7})

- Fifth Step:

(y:=5, {x->7}) --> {y -> 5, x -> 7}

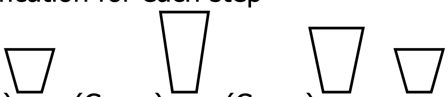
Example Evaluation

- Bottom Line:

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7})
--> (if 7 > 5 then y:=2 + 3 else y:=3 + 4 fi,
{x -> 7})
-->(if true then y:=2 + 3 else y:=3 + 4 fi,
{x -> 7})
-->(y:=2+3, {x->7})
--> (y:=5, {x->7}) --> {y -> 5, x -> 7}

Transition Semantics Evaluation

- A sequence of steps with trees of justification for each step



(C_1,m_1) --> (C_2,m_2) --> (C_3,m_3) --> … --> m

- Let -->* be the transitive closure of -->
- Ie, the smallest transitive relation containing -->

Adding Local Declarations

- Add to expressions:
- *E* ::= … | let *I* = *E* in *E'* | fun *I* -> *E* | *E E'*
- fun *I* -> *E* is a value
- Could handle local binding using state, but have assumption that evaluating expressions doesn't alter the environment
- We will use substitution here instead
- **Notation**: *E* [*E'* / *I*] means replace all free occurrence of *I* by *E'* in *E*

Call-by-value (Eager Evaluation)

(let I = V in E, m) --> (E[V/ I],m)

$$\frac{(E, m) \dashrightarrow (E'', m)}{(\text{let } I = E \text{ in } E', m) \dashrightarrow (\text{let } I = E'' \text{ in } E')}$$

((fun I -> E) V, m) --> (E[V / I],m)

$$\frac{(E', m) \dashrightarrow (E'', m)}{((\text{fun } I \text{ -> } E)\ E', m) \dashrightarrow ((\text{fun } I \text{ -> } E)\ E'', m)}$$

Call-by-name (Lazy Evaluation)

- (let I = E in E', m) --> (E' [E / I],m)
- ((fun I -> E') E, m) --> (E' [E / I],m)
- Question: Does it make a difference?
- It can depending on the language

Church-Rosser Property

- Church-Rosser Property: If E-->* E_1 and E-->* E_2, if there exists a value V such that E_1 -->* V, then E_2 -->* V
- Also called **confluence** or **diamond property**
- Example: E= 2 + 3 + 4

  E_1 = 5 + 4 E_2= 2 + 7

  V =9

Does It always Hold?

- No. Languages with side-effects tend not be Church-Rosser with the combination of call-by-name and call-by-value
- Alonzo Church and Barkley Rosser proved in 1936 the λ-calculus does have it
- Benefit of Church-Rosser: can check equality of terms by evaluating them (Given evaluation strategy might not terminate, though)

Transition Semantics for λ-Calculus

- Application (version 1)

  (λ x . E) E' --> E[E'/x]
- Application (version 2)

  (λ x . E) V --> E[V/x]

$$\frac{E' \dashrightarrow E''}{(\lambda\ x\ .\ E)\ E' \dashrightarrow (\lambda\ x\ .\ E)\ E''}$$