

Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

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Using Ocamlacc

- Input attribute grammar is put in file $\langle\text{grammar}\rangle.\text{mly}$
- Execute $\text{ocamlacc } \langle\text{grammar}\rangle.\text{mly}$
- Produces code for parser in $\langle\text{grammar}\rangle.\text{ml}$ and interface (including type declaration for tokens) in $\langle\text{grammar}\rangle.\text{mli}$

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Parser Code

- $\langle\text{grammar}\rangle.\text{ml}$ defines one parsing function per entry point
- Parsing function takes a lexing function (lexer buffer to token) and a lexer buffer as arguments
- Returns semantic attribute of corresponding entry point

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Ocamlacc Input

- File format:

```
%{
    <header>
%}
    <declarations>
%%
    <rules>
%%
    <trailer>
```

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Ocamlacc $\langle\text{header}\rangle$

- Contains arbitrary Ocaml code
- Typically used to give types and functions needed for the semantic actions of rules and to give specialized error recovery
- May be omitted
- $\langle\text{footer}\rangle$ similar. Possibly used to call parser

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Ocamlacc $\langle\text{declarations}\rangle$

- **%token** *symbol ... symbol*
 - Declare given symbols as tokens
- **%token** $\langle\text{type}\rangle$ *symbol ... symbol*
 - Declare given symbols as token constructors, taking an argument of type $\langle\text{type}\rangle$
- **%start** *symbol ... symbol*
 - Declare given symbols as entry points; functions of same names in $\langle\text{grammar}\rangle.\text{ml}$

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Ocamlyacc <declarations>

- **%type <type> symbol ... symbol**

Specify type of attributes for given symbols.
Mandatory for start symbols

- **%left symbol ... symbol**

- **%right symbol ... symbol**

- **%nonassoc symbol ... symbol**

Associate precedence and associativity to
given symbols. Same line, same precedence;
earlier line, lower precedence (broadest
scope)

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Ocamlyacc <rules>

- **nonterminal :**

symbol ... symbol { semantic_action }

| ...

symbol ... symbol { semantic_action }

;

- Semantic actions are arbitrary Ocaml
expressions

- Must be of same type as declared (or inferred)
for **nonterminal**

- Access semantic attributes (values) of symbols
by position: \$1 for first symbol, \$2 to second ...

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Example - Base types

```
(* File: expr.ml *)
type expr =
  Term_as_Expr of term
 | Plus_Expr of (term * expr)
 | Minus_Expr of (term * expr)
and term =
  Factor_as_Term of factor
 | Mult_Term of (factor * term)
 | Div_Term of (factor * term)
and factor =
  Id_as_Factor of string
 | Parenthesized_Expr_as_Factor of expr
```

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Example - Lexer (exprlex.mll)

```
{ (*open Exprparse*) }
let numeric = ['0' - '9']
let letter =[ 'a' - 'z' 'A' - 'Z' ]
rule token = parse
  | "+" {Plus_token}
  | "-" {Minus_token}
  | "*" {Times_token}
  | "/" {Divide_token}
  | "(" {Left_parenthesis}
  | ")" {Right_parenthesis}
  | letter (letter|numeric|" ") as id {Id_token id}
  | ['t' 'n'] {token lexbuf}
  | eof {EOL}
```

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Example - Parser (exprparse.mly)

```
%{ open Expr
%}
%token <string> Id_token
%token Left_parenthesis Right_parenthesis
%token Times_token Divide_token
%token Plus_token Minus_token
%token EOL
%start main
%type <expr> main
%%
```

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Example - Parser (exprparse.mly)

```
expr:
  term
    { Term_as_Expr $1 }
  | term Plus_token expr
    { Plus_Expr ($1, $3) }
  | term Minus_token expr
    { Minus_Expr ($1, $3) }
```

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Example - Parser (exprparse.mly)

term:

```
factor
  { Factor_as_Term $1 }
| factor Times_token term
  { Mult_Term ($1, $3) }
| factor Divide_token term
  { Div_Term ($1, $3) }
```

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Example - Parser (exprparse.mly)

factor:

```
Id_token
  { Id_as_Factor $1 }
| Left_parenthesis expr Right_parenthesis
  { Parenthesized_Expr_as_Factor $2 }
```

main:

```
| expr EOL
  { $1 }
```

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Example - Using Parser

```
# #use "expr.ml";;
...
# #use "exprparse.ml";;
...
# #use "exprlex.ml";;
...
# let test s =
let lexbuf = Lexing.from_string (s^"\n") in
  main token lexbuf;;
```

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Example - Using Parser

```
# test "a + b";;
- : expr =
Plus_Expr
(Factor_as_Term (Id_as_Factor "a"),
Term_as_Expr (Factor_as_Term
(Id_as_Factor "b")))
```

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LR Parsing

- Read tokens left to right (L)
- Create a rightmost derivation (R)
- How is this possible?
- Start at the bottom (left) and work your way up
- Last step has only one non-terminal to be replaced so is right-most
- Working backwards, replace mixed strings by non-terminals
- Always proceed so that there are no non-terminals to the right of the string to be replaced

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Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

$\langle \text{Sum} \rangle \Rightarrow$

= ● (0 + 1) + 0 shift

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Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\quad \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

<Sum> =>

=	(<Sum> ●) + 0	shift
=>	(<Sum> + <Sum> ●) + 0	reduce
=>	(<Sum> + 1 ●) + 0	reduce
=	(<Sum> + ● 1) + 0	shift
=	(<Sum> ● + 1) + 0	shift
=>	(0 ● + 1) + 0	reduce
=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

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Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

<Sum> =>

=> (<Sum>)	$\bullet + 0$	reduce
=	(<Sum>) $\bullet + 0$	shift
=> (<Sum> + <Sum>)	$\bullet + 0$	reduce
=> (<Sum> + 1)	$\bullet + 0$	reduce
=	(<Sum> + 1) $\bullet + 0$	shift
=	(<Sum>) $\bullet + 1$	shift
=> (0)	$\bullet + 1$	reduce
=	(0 + 1) $\bullet + 0$	shift
=	(0 + 1) \bullet	shift

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Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

<Sum> =>

=	$<\text{Sum}> \bullet + 0$	shift
=>	$(<\text{Sum}>) \bullet + 0$	reduce
=	$(<\text{Sum}> \bullet) + 0$	shift
=>	$(<\text{Sum}> + <\text{Sum}> \bullet) + 0$	reduce
=>	$(<\text{Sum}> + 1 \bullet) + 0$	reduce
=	$(<\text{Sum}> + \bullet 1) + 0$	shift
=	$(<\text{Sum}> \bullet + 1) + 0$	shift
=>	$(0 \bullet + 1) + 0$	reduce
=	$(\bullet 0 + 1) + 0$	shift
=	$\bullet (0 + 1) + 0$	shift

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Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

<Sum> =>

=	$<\text{Sum}> + \bullet\bullet 0$	shift
=	$<\text{Sum}> \bullet + 0$	shift
=>	($<\text{Sum}>$) $\bullet + 0$	reduce
=	($<\text{Sum}>$ \bullet) $+ 0$	shift
=>	($<\text{Sum}> + <\text{Sum}>$ \bullet) $+ 0$	reduce
=>	($<\text{Sum}> + 1 \bullet$) $+ 0$	reduce
=	($<\text{Sum}> + \bullet 1$) $+ 0$	shift
=	($<\text{Sum}> \bullet + 1$) $+ 0$	shift
=>	(0 $\bullet + 1$) $+ 0$	reduce
=	($\bullet 0 + 1$) $+ 0$	shift
=	(\bullet (0 + 1)) $+ 0$	shift

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Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

<Sum> =>

=> <Sum> + 0	reduce
= <Sum> + 0	shift
= <Sum> 0	shift
=> (<Sum>) 0	reduce
= (<Sum> 0)	shift
=> (<Sum> + <Sum>) 0	reduce
=> (<Sum> + 1) 0	reduce
= (<Sum> + 1) 0	shift
= (<Sum> 1) 0	shift
=> (0 + 1) 0	reduce
= (0 + 1) 0	shift
= 0 (0 + 1)	shift

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Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

<Sum> =>

```

=> <Sum> + 0 ●          reduce
= <Sum> + ● 0           shift
= <Sum> ● + 0           shift
=> ( <Sum> ) ● + 0      reduce
= ( <Sum> ● ) + 0        shift
=> ( <Sum> + <Sum> ● ) + 0  reduce
=> ( <Sum> + 1 ● ) + 0    reduce
= ( <Sum> + ● 1 ) + 0      shift
= ( <Sum> ● + 1 ) + 0      shift
=> ( 0 ● + 1 ) + 0        reduce
= ( ● 0 + 1 ) + 0         shift
= ● ( 0 + 1 ) + 0         shift

```

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Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

```

<Sum> ● => <Sum> + <Sum> ●      reduce
      => <Sum> + 0 ●      reduce
      =  <Sum> + ● 0      shift
      =  <Sum> ● + 0      shift
      => ( <Sum> ) ● + 0      reduce
      = ( <Sum> ● ) + 0      shift
      => ( <Sum> + <Sum> ● ) + 0      reduce
      => ( <Sum> + 1 ● ) + 0      reduce
      = ( <Sum> + ● 1 ) + 0      shift
      = ( <Sum> ● + 1 ) + 0      shift
      => ( 0 ● + 1 ) + 0      reduce
      = ( ● 0 + 1 ) + 0      shift
      =  ● ( 0 + 1 ) + 0      shift
  
```

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Example

 $(\quad 0 \quad + \quad 1 \quad) \quad + \quad 0$

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Example

 $(\quad 0 \quad + \quad 1 \quad) \quad + \quad 0$

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Example

 $(\quad 0 \quad + \quad 1 \quad) \quad + \quad 0$

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Example

 $(\quad \circlearrowleft \quad 0 \quad + \quad 1 \quad) \quad + \quad 0$

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Example

 $(\quad \circlearrowleft \quad 0 \quad + \quad 1 \quad) \quad + \quad 0$

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Example

$$(\text{} \ 0 + 1) + 0$$

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Example

$$(\text{} \ 0 + \text{} \ 1) + 0$$

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$$(\text{} \ 0 + \text{} \ 1) + 0$$

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$$(\text{} \ 0 + \text{} \ 1) + 0$$

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Example

$$(\text{} \ 0 + \text{} \ 1) + 0$$

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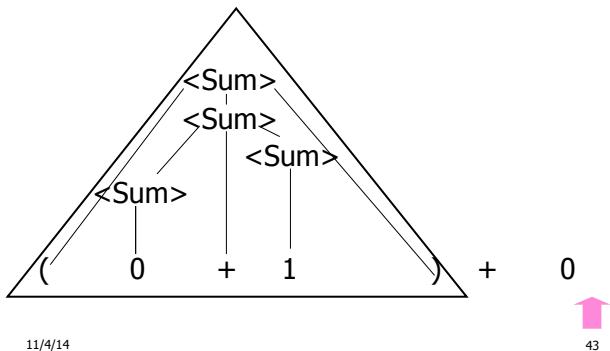
Example

$$(\text{} \ 0 + \text{} \ 1) + 0$$

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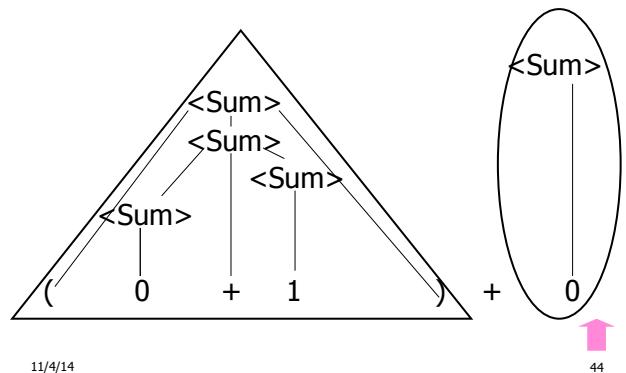
Example



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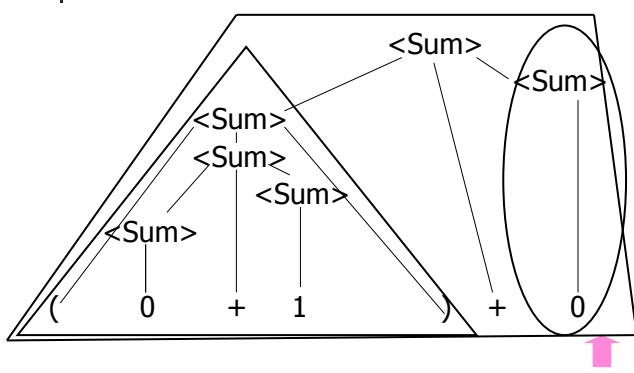
Example



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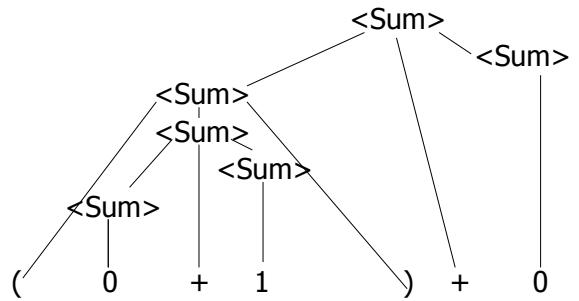
Example



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Example



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LR Parsing Tables

- Build a pair of tables, Action and Goto, from the grammar
 - This is the hardest part, we omit here
 - Rows labeled by states
 - For Action, columns labeled by terminals and “end-of-tokens” marker
 - (more generally strings of terminals of fixed length)
 - For Goto, columns labeled by non-terminals

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Action and Goto Tables

- Given a state and the next input, Action table says either
 - **shift** and go to state n , or
 - **reduce** by production k (explained in a bit)
 - **accept** or **error**
- Given a state and a non-terminal, Goto table says
 - go to state m

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LR(i) Parsing Algorithm

- Based on push-down automata
- Uses states and transitions (as recorded in Action and Goto tables)
- Uses a stack containing states, terminals and non-terminals

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LR(i) Parsing Algorithm

0. Insure token stream ends in special “end-of-tokens” symbol
1. Start in state 1 with an empty stack
2. Push **state(1)** onto stack
- 3. Look at next i tokens from token stream ($toks$) (don’t remove yet)
4. If top symbol on stack is **state(n)**, look up action in Action table at $(n, toks)$

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LR(i) Parsing Algorithm

5. If action = **shift m** ,
 - a) Remove the top token from token stream and push it onto the stack
 - b) Push **state(m)** onto stack
 - c) Go to step 3

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LR(i) Parsing Algorithm

6. If action = **reduce k** where production k is $E ::= u$
 - a) Remove $2 * \text{length}(u)$ symbols from stack (u and all the interleaved states)
 - b) If new top symbol on stack is **state(m)**, look up new state p in $\text{Goto}(m, E)$
 - c) Push E onto the stack, then push **state(p)** onto the stack
 - d) Go to step 3

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LR(i) Parsing Algorithm

7. If action = **accept**
 - Stop parsing, return success
8. If action = **error**,
 - Stop parsing, return failure

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Adding Synthesized Attributes

- Add to each **reduce** a rule for calculating the new synthesized attribute from the component attributes
- Add to each non-terminal pushed onto the stack, the attribute calculated for it
- When performing a **reduce**,
 - gather the recorded attributes from each non-terminal popped from stack
 - Compute new attribute for non-terminal pushed onto stack

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Shift-Reduce Conflicts

- **Problem:** can't decide whether the action for a state and input character should be **shift** or **reduce**
- Caused by ambiguity in grammar
- Usually caused by lack of associativity or precedence information in grammar

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Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

$\bullet 0 + 1 + 0$ -> $0 \bullet + 1 + 0$ -> $\langle \text{Sum} \rangle \bullet + 1 + 0$ -> $\langle \text{Sum} \rangle + \bullet 1 + 0$ -> $\langle \text{Sum} \rangle + 1 \bullet + 0$ -> $\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet + 0$	shift reduce shift shift reduce shift
--	--

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Example - cont

- **Problem:** shift or reduce?
- You can shift-shift-reduce-reduce or reduce-shift-shift-reduce
- Shift first - right associative
- Reduce first- left associative

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Reduce - Reduce Conflicts

- **Problem:** can't decide between two different rules to reduce by
- Again caused by ambiguity in grammar
- **Symptom:** RHS of one production suffix of another
- Requires examining grammar and rewriting it
- Harder to solve than shift-reduce errors

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Example

- $S ::= A \mid aB \quad A ::= abc \quad B ::= bc$
- $\bullet abc \quad \text{shift}$
 $a \bullet bc \quad \text{shift}$
 $ab \bullet c \quad \text{shift}$
 $abc \bullet$
- Problem: reduce by $B ::= bc$ then by $S ::= aB$, or by $A ::= abc$ then $S ::= A$?

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Semantics

- Expresses the meaning of syntax
- Static semantics
 - Meaning based only on the form of the expression without executing it
 - Usually restricted to type checking / type inference

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Dynamic semantics

- Method of describing meaning of executing a program
- Several different types:
 - Operational Semantics
 - Axiomatic Semantics
 - Denotational Semantics

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Dynamic Semantics

- Different languages better suited to different types of semantics
- Different types of semantics serve different purposes

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Operational Semantics

- Start with a simple notion of machine
- Describe how to execute (implement) programs of language on virtual machine, by describing how to execute each program statement (ie, following the *structure* of the program)
- Meaning of program is how its execution changes the state of the machine
- Useful as basis for implementations

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Axiomatic Semantics

- Also called Floyd-Hoare Logic
- Based on formal logic (first order predicate calculus)
- Axiomatic Semantics is a logical system built from *axioms* and *inference rules*
- Mainly suited to simple imperative programming languages

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Axiomatic Semantics

- Used to formally prove a property (*post-condition*) of the *state* (the values of the program variables) after the execution of program, assuming another property (*pre-condition*) of the state before execution
- Written :
 {Precondition} Program {Postcondition}
- Source of idea of *loop invariant*

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Denotational Semantics

- Construct a function \mathcal{M} assigning a mathematical meaning to each program construct
- Lambda calculus often used as the range of the meaning function
- Meaning function is compositional: meaning of construct built from meaning of parts
- Useful for proving properties of programs

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Natural Semantics

- Aka Structural Operational Semantics, aka “Big Step Semantics”
- Provide value for a program by rules and derivations, similar to type derivations
- Rule conclusions look like
 $(C, m) \Downarrow m'$
or
 $(E, m) \Downarrow v$

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Simple Imperative Programming Language

- $I \in \text{Identifiers}$
- $N \in \text{Numerals}$
- $B ::= \text{true} \mid \text{false} \mid B \& B \mid B \text{ or } B \mid \text{not } B \mid E < E \mid E = E$
- $E ::= N \mid I \mid E + E \mid E * E \mid E - E \mid - E$
- $C ::= \text{skip} \mid C; C \mid I ::= E \mid \text{if } B \text{ then } C \text{ else } C \text{ fi} \mid \text{while } B \text{ do } C \text{ od}$

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Natural Semantics of Atomic Expressions

- Identifiers: $(I, m) \Downarrow m(I)$
- Numerals are values: $(N, m) \Downarrow N$
- Booleans: $(\text{true}, m) \Downarrow \text{true}$
 $(\text{false}, m) \Downarrow \text{false}$

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Booleans:

$$\frac{(B, m) \Downarrow \text{false}}{(B \& B', m) \Downarrow \text{false}} \quad \frac{(B, m) \Downarrow \text{true} \quad (B', m) \Downarrow b}{(B \& B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(B \text{ or } B', m) \Downarrow \text{true}} \quad \frac{(B, m) \Downarrow \text{false} \quad (B', m) \Downarrow b}{(B \text{ or } B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(\text{not } B, m) \Downarrow \text{false}} \quad \frac{(B, m) \Downarrow \text{false}}{(\text{not } B, m) \Downarrow \text{true}}$$

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Relations

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \sim V = b}{(E \sim E', m) \Downarrow b}$$

- By $U \sim V = b$, we mean does (the meaning of) the relation \sim hold on the meaning of U and V
- May be specified by a mathematical expression/equation or rules matching U and V

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Arithmetic Expressions

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \text{ op } V = N}{(E \text{ op } E', m) \Downarrow N}$$

where N is the specified value for $U \text{ op } V$

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Commands

Skip: $(\text{skip}, m) \Downarrow m$

Assignment: $\frac{(E,m) \Downarrow V}{(I:=E,m) \Downarrow m[I \leftarrow V]}$

Sequencing: $\frac{(C,m) \Downarrow m' \quad (C',m') \Downarrow m''}{(C;C',m) \Downarrow m''}$

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If Then Else Command

$\frac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi, } m) \Downarrow m'}$

$\frac{(B,m) \Downarrow \text{false} \quad (C',m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi, } m) \Downarrow m'}$

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While Command

$\frac{(B,m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od, } m) \Downarrow m}$

$\frac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m' \quad (\text{while } B \text{ do } C \text{ od, } m') \Downarrow m''}{(\text{while } B \text{ do } C \text{ od, } m) \Downarrow m''}$

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Example: If Then Else Rule

$\frac{}{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?}$

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Example: If Then Else Rule

$\frac{(x > 5, \{x \rightarrow 7\}) \Downarrow ?}{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?}$

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Example: Arith Relation

$\frac{\begin{array}{c} ? > ? = ? \\ (x, \{x \rightarrow 7\}) \Downarrow ? \quad (5, \{x \rightarrow 7\}) \Downarrow ? \end{array}}{\frac{(x > 5, \{x \rightarrow 7\}) \Downarrow ?}{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?}}$

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Example: Identifier(s)

$$\begin{array}{c}
 7 > 5 = \text{true} \\
 \underline{(x,\{x->7\}) \Downarrow 7 \quad (5,\{x->7\}) \Downarrow 5} \\
 \underline{(x > 5, \{x -> 7\}) \Downarrow ?} \\
 \hline
 (\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\
 \quad \{x -> 7\}) \Downarrow ?
 \end{array}$$

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Example: Arith Relation

$$\begin{array}{c}
 7 > 5 = \text{true} \\
 \underline{(x,\{x->7\}) \Downarrow 7 \quad (5,\{x->7\}) \Downarrow 5} \\
 \underline{(x > 5, \{x -> 7\}) \Downarrow \text{true}} \\
 \hline
 (\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\
 \quad \{x -> 7\}) \Downarrow ?
 \end{array}$$

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Example: If Then Else Rule

$$\begin{array}{c}
 7 > 5 = \text{true} \\
 \underline{(x,\{x->7\}) \Downarrow 7 \quad (5,\{x->7\}) \Downarrow 5} \quad \underline{(y := 2 + 3, \{x -> 7\})} \\
 \underline{(x > 5, \{x -> 7\}) \Downarrow \text{true}} \quad \underline{\Downarrow ?} \\
 \hline
 \cdot
 \end{array}$$

(if $x > 5$ then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x -> 7\}) \Downarrow ?$

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Example: Assignment

$$\begin{array}{c}
 7 > 5 = \text{true} \\
 \underline{(x,\{x->7\}) \Downarrow 7 \quad (5,\{x->7\}) \Downarrow 5} \quad \underline{(2+3, \{x -> 7\}) \Downarrow ?} \\
 \underline{(x > 5, \{x -> 7\}) \Downarrow \text{true}} \quad \underline{\Downarrow ?} \\
 \hline
 \cdot
 \end{array}$$

(if $x > 5$ then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x -> 7\}) \Downarrow ?$

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Example: Arith Op

$$\begin{array}{c}
 ? + ? = ? \\
 \underline{(2,\{x->7\}) \Downarrow ? \quad (3,\{x->7\}) \Downarrow ?} \\
 \underline{7 > 5 = \text{true}} \quad \underline{(2+3, \{x -> 7\}) \Downarrow ?} \\
 \underline{(x,\{x->7\}) \Downarrow 7 \quad (5,\{x->7\}) \Downarrow 5} \quad \underline{(y := 2 + 3, \{x -> 7\})} \\
 \underline{(x > 5, \{x -> 7\}) \Downarrow \text{true}} \quad \underline{\Downarrow ?} \\
 \hline
 \cdot
 \end{array}$$

(if $x > 5$ then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x -> 7\}) \Downarrow ?$

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Example: Numerals

$$\begin{array}{c}
 2 + 3 = 5 \\
 \underline{(2,\{x->7\}) \Downarrow 2 \quad (3,\{x->7\}) \Downarrow 3} \\
 \underline{7 > 5 = \text{true}} \quad \underline{(2+3, \{x -> 7\}) \Downarrow ?} \\
 \underline{(x,\{x->7\}) \Downarrow 7 \quad (5,\{x->7\}) \Downarrow 5} \quad \underline{(y := 2 + 3, \{x -> 7\})} \\
 \underline{(x > 5, \{x -> 7\}) \Downarrow \text{true}} \quad \underline{\Downarrow ?} \\
 \hline
 \cdot
 \end{array}$$

(if $x > 5$ then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x -> 7\}) \Downarrow ?$

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Example: Arith Op

$$\begin{array}{c}
 2 + 3 = 5 \\
 \underline{(2,\{x>7\}) \Downarrow 2 \quad (3,\{x>7\}) \Downarrow 3} \\
 7 > 5 = \text{true} \qquad \qquad \qquad \underline{(2+3, \{x>7\}) \Downarrow 5} \\
 \underline{(x,\{x>7\}) \Downarrow 7 \quad (5,\{x>7\}) \Downarrow 5} \qquad \underline{(y := 2 + 3, \{x>7\})} \\
 \underline{(x > 5, \{x -> 7\}) \Downarrow \text{true}} \qquad \qquad \qquad \Downarrow ? \\
 \underline{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi},} \\
 \qquad \qquad \qquad \underline{\{x -> 7\}) \Downarrow ?}
 \end{array}$$

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Example: Assignment

$$\begin{array}{c}
 2 + 3 = 5 \\
 \underline{(2,\{x>7\}) \Downarrow 2 \quad (3,\{x>7\}) \Downarrow 3} \\
 7 > 5 = \text{true} \qquad \qquad \qquad \underline{(2+3, \{x>7\}) \Downarrow 5} \\
 \underline{(x,\{x>7\}) \Downarrow 7 \quad (5,\{x>7\}) \Downarrow 5} \qquad \underline{(y := 2 + 3, \{x>7\})} \\
 \underline{(x > 5, \{x -> 7\}) \Downarrow \text{true}} \qquad \qquad \qquad \Downarrow \{x -> 7, y -> 5\} \\
 \underline{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi},} \\
 \qquad \qquad \qquad \underline{\{x -> 7\}) \Downarrow ?}
 \end{array}$$

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Example: If Then Else Rule

$$\begin{array}{c}
 2 + 3 = 5 \\
 \underline{(2,\{x>7\}) \Downarrow 2 \quad (3,\{x>7\}) \Downarrow 3} \\
 7 > 5 = \text{true} \qquad \qquad \qquad \underline{(2+3, \{x>7\}) \Downarrow 5} \\
 \underline{(x,\{x>7\}) \Downarrow 7 \quad (5,\{x>7\}) \Downarrow 5} \qquad \underline{(y := 2 + 3, \{x>7\})} \\
 \underline{(x > 5, \{x -> 7\}) \Downarrow \text{true}} \qquad \qquad \qquad \Downarrow \{x -> 7, y -> 5\} \\
 \underline{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi},} \\
 \qquad \qquad \qquad \underline{\{x -> 7\}) \Downarrow \{x -> 7, y -> 5\}}
 \end{array}$$

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Let in Command

$$\frac{(E, m) \Downarrow \vee (C, m[I \leftarrow v]) \Downarrow m'}{(\text{let } I = E \text{ in } C, m) \Downarrow m'}$$

Where $m''(y) = m'(y)$ for $y \not\in I$ and $m''(I) = m(I)$ if $m(I)$ is defined, and $m''(I)$ is undefined otherwise

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Example

$$\begin{array}{c}
 \underline{(x,\{x>5\}) \Downarrow 5 \quad (3,\{x>5\}) \Downarrow 3} \\
 \underline{(x+3,\{x>5\}) \Downarrow 8} \\
 \underline{(5,\{x>17\}) \Downarrow 5 \quad (x := x + 3, \{x -> 5\}) \Downarrow \{x -> 8\}} \\
 (\text{let } x = 5 \text{ in } (x := x + 3), \{x -> 17\}) \Downarrow ?
 \end{array}$$

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Example

$$\begin{array}{c}
 \underline{(x,\{x>5\}) \Downarrow 5 \quad (3,\{x>5\}) \Downarrow 3} \\
 \underline{(x+3,\{x>5\}) \Downarrow 8} \\
 \underline{(5,\{x>17\}) \Downarrow 5 \quad (x := x + 3, \{x -> 5\}) \Downarrow \{x -> 8\}} \\
 (\text{let } x = 5 \text{ in } (x := x + 3), \{x -> 17\}) \Downarrow \{x -> 17\}
 \end{array}$$

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Comment

- Simple Imperative Programming Language introduces variables *implicitly* through assignment
- The let-in command introduces scoped variables *explicitly*
- Clash of constructs apparent in awkward semantics

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Interpretation Versus Compilation

- A **compiler** from language L1 to language L2 is a program that takes an L1 program and for each piece of code in L1 generates a piece of code in L2 of same meaning
- An **interpreter** of L1 in L2 is an L2 program that executes the meaning of a given L1 program
- Compiler would examine the body of a loop once; an interpreter would examine it every time the loop was executed

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Interpreter

- An *Interpreter* represents the operational semantics of a language L1 (source language) in the language of implementation L2 (target language)
- Built incrementally
 - Start with literals
 - Variables
 - Primitive operations
 - Evaluation of expressions
 - Evaluation of commands/declarations

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Interpreter

- Takes abstract syntax trees as input
 - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
 - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next “state”
 - To get final value, put in a loop

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Natural Semantics Example

- `compute_exp (Var(v), m) = look_up v m`
- `compute_exp (Int(n), _) = Num (n)`
- ...
- `compute_com(IfExp(b,c1,c2),m) =`
`if compute_exp (b,m) = Bool(true)`
`then compute_com (c1,m)`
`else compute_com (c2,m)`

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Natural Semantics Example

- `compute_com(While(b,c), m) =`
`if compute_exp (b,m) = Bool(false)`
`then m`
`else compute_com`
`(While(b,c), compute_com(c,m))`
- May fail to terminate - exceed stack limits
- Returns no useful information then

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