Programming Languages and Compilers (CS 421)

Elsa L Gunter

2112 SC, UIUC

http://courses.engr.illinois.edu/cs421

Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Background for Unification

- Terms made from constructors and variables (for the simple first order case)
- Constructors may be applied to arguments (other terms) to make new terms
- Variables and constructors with no arguments are base cases
- Constructors applied to different number of arguments (arity) considered different
- Substitution of terms for variables

Simple Implementation Background

```
type term = Variable of string
                  | Const of (string * term list)

let rec subst var_name residue term =
    match term with Variable name ->
        if var_name = name then residue else term
      | Const (c, tys) ->
        Const (c, List.map (subst var_name residue)
                              tys);;
```

Unification Problem

Given a set of pairs of terms ("equations")

$$\{(s_1, t_1), (s_2, t_2), \ldots, (s_n, t_n)\}$$

(the *unification problem*) does there exist a substitution σ (the *unification solution*) of terms for variables such that

$$\sigma(s_i) = \sigma(t_i),$$

for all i = 1, …, n?

Uses for Unification

- Type Inference and type checking
- Pattern matching as in OCAML
  - Can use a simplified version of algorithm
- Logic Programming - Prolog
- Simple parsing

Unification Algorithm

- Let $S = \{(s_1 = t_1), (s_2 = t_2), \ldots, (s_n = t_n)\}$ be a unification problem.
- Case $S = \{ \}$: Unif(S) = Identity function (i.e., no substitution)
- Case $S = \{(s, t)\} \cup S'$: Four main steps

Unification Algorithm

- Delete: if $s = t$ (they are the same term) then $Unif(S) = Unif(S')$
- Decompose: if $s = f(q_1, \ldots, q_m)$ and $t = f(r_1, \ldots, r_m)$ (same f, same m!), then $Unif(S) = Unif(\{(q_1, r_1), \ldots, (q_m, r_m)\} \cup S')$
- Orient: if $t = x$ is a variable, and s is not a variable, $Unif(S) = Unif(\{(x = s)\} \cup S')$

Unification Algorithm

- Eliminate: if $s = x$ is a variable, and x does not occur in t (the occurs check), then
  - Let $\varphi = \{x \to t\}$
  - Let $\psi = Unif(\varphi(S'))$
  - $Unif(S) = \{x \to \psi(t)\} \circ \psi$
    - Note: $\{x \to a\} \circ \{y \to b\} = \{y \to (\{x \to a\}(b))\} \circ \{x \to a\}$ if y not in a

Tricks for Efficient Unification

- Don't return substitution, rather do it incrementally
- Make substitution be constant time
  - Requires implementation of terms to use mutable structures (or possibly lazy structures)
  - We won't discuss these

Example

- x,y,z variables, f,g constructors

- Unify {(f(x) = f(g(f(z),y))), (g(y,y) = x)} = ?

Example

- x,y,z variables, f,g constructors
- S = {(f(x) = f(g(f(z),y))), (g(y,y) = x)} is nonempty

- Unify {(f(x) = f(g(f(z),y))), (g(y,y) = x)} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (g(y,y) = x)

- Unify {(f(x) = f(g(f(z),y))), (g(y,y) = x)} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (g(y,y)) = x)
- Orient: (x = g(y,y))

- Unify {(f(x) = f(g(f(z),y))), (g(y,y) = x)} =
  Unify {(f(x) = f(g(f(z),y))), (x = g(y,y))}

by Orient

Example

- x,y,z variables, f,g constructors

- Unify {(f(x) = f(g(f(z),y))), (x = g(y,y))} = ?

Example

- x,y,z variables, f,g constructors
- {(f(x) = f(g(f(z),y))), (x = g(y,y))} is non-empty

- Unify {(f(x) = f(g(f(z),y))), (x = g(y,y))} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (x = g(y,y))
- Unify {(f(x) = f(g(f(z),y))), (x = g(y,y))} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (x = g(y,y))
- Eliminate x with substitution {x→ g(y,y)}
  - Check: x not in g(y,y)
- Unify {(f(x) = f(g(f(z),y))), (x = g(y,y))} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (x = g(y,y))
- Eliminate x with substitution {x→ g(y,y)}

- Unify {(f(x) = f(g(f(z),y))), (x = g(y,y))} =
  Unify {(f(g(y,y)) = f(g(f(z),y)))}
  o {x→ g(y,y)}

Example

- x,y,z variables, f,g constructors

- Unify {(f(g(y,y)) = f(g(f(z),y)))}
  o {x→ g(y,y)} = ?

Example

- x,y,z variables, f,g constructors
- {(f(g(y,y)) = f(g(f(z),y)))} is non-empty

- Unify {(f(g(y,y)) = f(g(f(z),y)))}
  o {x→ g(y,y)} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (f(g(y,y)) = f(g(f(z),y)))

- Unify {(f(g(y,y)) = f(g(f(z),y)))}
  o {x→ g(y,y)} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (f(g(y,y)) = f(g(f(z),y)))
- Decompose:(f(g(y,y)) = f(g(f(z),y))) becomes {(g(y,y) = g(f(z),y))}

- Unify {(f(g(y,y)) = f(g(f(z),y)))} o {x→ g(y,y)} =

Unify {(g(y,y) = g(f(z),y))} o {x→ g(y,y)}

Example

- x,y,z variables, f,g constructors
- {(g(y,y) = g(f(z),y))} is non-empty

- Unify {(g(y,y) = g(f(z),y))}
  o {x→ g(y,y)} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (g(y,y) = g(f(z),y))

- Unify {(g(y,y) = g(f(z),y))}
  o {x→ g(y,y)} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (f(g(y,y)) = f(g(f(z),y)))
- Decompose: (g(y,y)) = g(f(z),y)) becomes {(y = f(z)); (y = y)}

- Unify {(g(y,y) = g(f(z),y))} o {x→ g(y,y)} = Unify {(y = f(z)); (y = y)} o {x→ g(y,y)}

Example

- x,y,z variables, f,g constructors

- Unify {(y = f(z)); (y = y)} o {x→ g(y,y)} = ?

Example

- x,y,z variables, f,g constructors
- {(y = f(z)); (y = y)} o {x→ g(y,y)} is non-empty

- Unify {(y = f(z)); (y = y)} o {x→ g(y,y)} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (y = f(z))
- Unify {(y = f(z)); (y = y)} o {x→ g(y,y)} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (y = f(z))
- Eliminate y with {y → f(z)}
- Unify {(y = f(z)); (y = y)} o {x→ g(y,y)} =
  Unify {(f(z) = f(z))}
  o {y → f(z)} o {x→ g(y,y)}=
  Unify {(f(z) = f(z))}
  o {y → f(z); x→ g(f(z), f(z))}

Example

- x,y,z variables, f,g constructors
- Unify $\{(f(z) = f(z))\}$

  $o \{y \to f(z); x \to g(f(z), f(z))\} = ?$

Example

- x,y,z variables, f,g constructors
- $\{(f(z) = f(z))\}$ is non-empty

- Unify $\{(f(z) = f(z))\}$
  $o \{y \to f(z);\ x \to g(f(z), f(z))\} =$?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (f(z) = f(z))

- Unify {(f(z) = f(z))}
  o {y → f(z); x→ g(f(z), f(z))} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (f(z) = f(z))
- Delete
- Unify {(f(z) = f(z))}
  o {y → f(z); x→ g(f(z), f(z))} =
  Unify {} o {y → f(z); x→ g(f(z), f(z))}

Example

- x,y,z variables, f,g constructors

- Unify {} o {y → f(z); x→ g(f(z), f(z))} = ?

Example

- x,y,z variables, f,g constructors
- {} is empty
- Unify {} = identity function
- Unify {} o {y → f(z); x→ g(f(z), f(z))} =

  {y → f(z); x→ g(f(z), f(z))}

Example

- Unify {(f(x) = f(g(f(z),y))), (g(y,y) = x)} = {y → f(z); x→ g(f(z), f(z))}

f(x) = f(g(f(z), y))

→ f(g(f(z), f(z))) = f(g(f(z), f(z)))

g(y , y) = x

→ g(f(z),f(z)) = g(f(z), f(z))

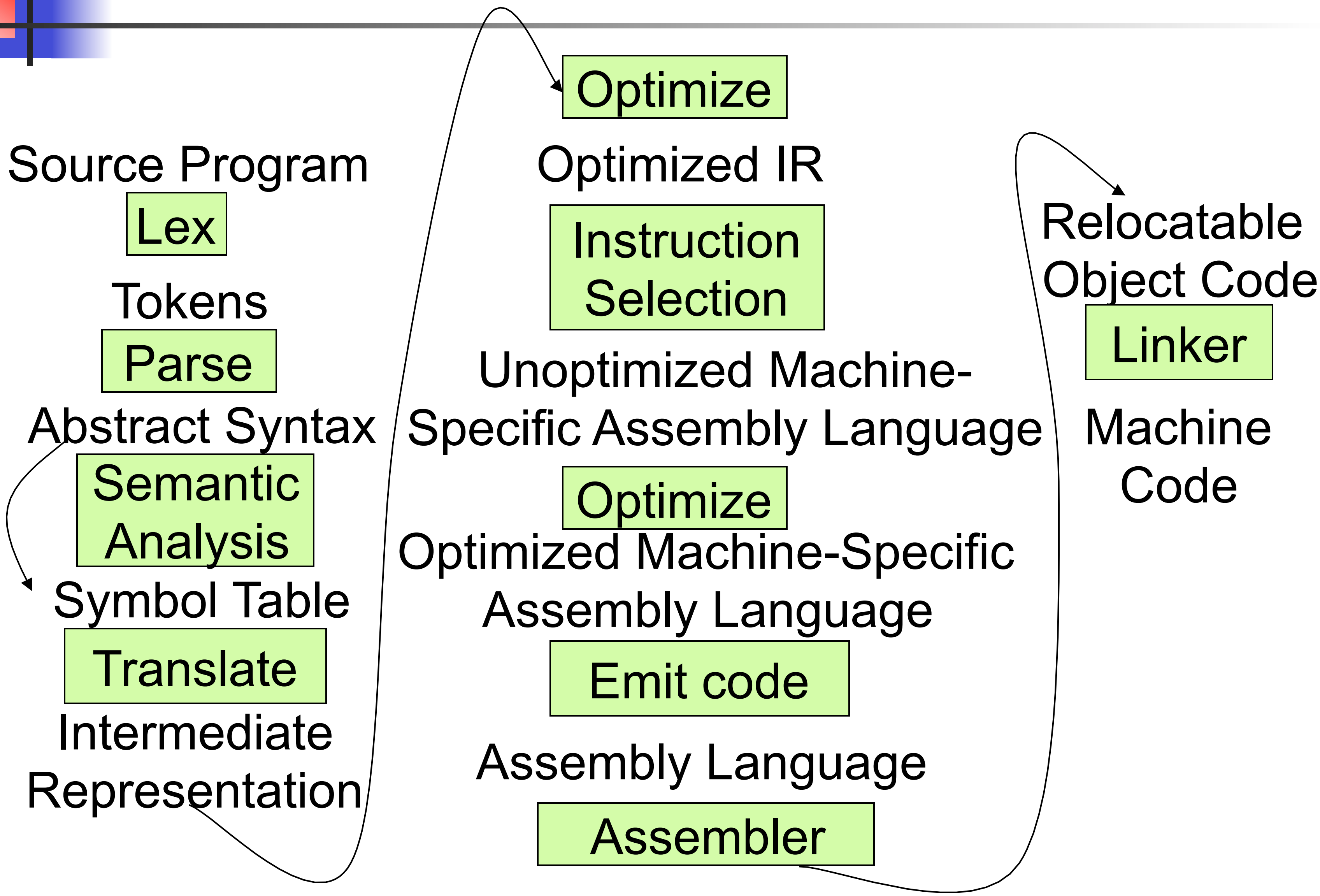
Example of Failure: Decompose

- Unify{(f(x,g(y)) = f(h(y),x))}
- Decompose: (f(x,g(y)) = f(h(y),x))
- = Unify {(x = h(y)), (g(y) = x)}
- Orient: (g(y) = x)
- = Unify {(x = h(y)), (x = g(y))}
- Eliminate: (x = h(y))
- Unify {(h(y), g(y))} o {x → h(y)}
- No rule to apply! Decompose fails!

Example of Failure: Occurs Check

- Unify{(f(x,g(x)) = f(h(x),x))}
- Decompose: (f(x,g(x)) = f(h(x),x))
- = Unify {(x = h(x)), (g(x) = x)}
- Orient: (g(y) = x)
- = Unify {(x = h(x)), (x = g(x))}
- No rules apply.

Major Phases of a Compiler



Modified from "Modern Compiler Implementation in ML", by Andrew Appel

Meta-discourse

- Language Syntax and Semantics
- Syntax
  - Regular Expressions, DFSAs and NDFSAs
  - Grammars
- Semantics
  - Natural Semantics
  - Transition Semantics

Language Syntax

- Syntax is the description of which strings of symbols are meaningful expressions in a language
- It takes more than syntax to understand a language; need meaning (semantics) too
- Syntax is the entry point

Syntax of English Language

- Pattern 1

Subject	Verb
David	*sings*
The dog	*barked*
Susan	*yawned*

- Pattern 2

Subject	Verb	Direct Object
David	*sings*	*ballads*
The professor	*wants*	*to retire*
The jury	*found*	*the defendant guilty*

Elements of Syntax

- Character set – previously always ASCII, now often 64 character sets
- Keywords – usually reserved
- Special constants – cannot be assigned to
- Identifiers – can be assigned to
- Operator symbols
- Delimiters (parenthesis, braces, brackets)
- Blanks (aka white space)

Elements of Syntax

- Expressions

  if ... then begin ... ; ... end else begin ... ; ... end

- Type expressions

  typexpr$_1$ -> *typexpr*$_2$

- Declarations (in functional languages)

  let *pattern*$_1$ = *expr*$_1$ in *expr*

- Statements (in imperative languages)

  a = b + c

- Subprograms

  let *pattern*$_1$ = let rec inner = … in *expr*

Elements of Syntax

- Modules
- Interfaces
- Classes (for object-oriented languages)

Lexing and Parsing

- Converting strings to abstract syntax trees done in two phases
  - **Lexing:** Converting string (or streams of characters) into lists (or streams) of tokens (the “words” of the language)
    - Specification Technique: Regular Expressions
  - **Parsing:** Convert a list of tokens into an abstract syntax tree
    - Specification Technique: BNF Grammars

Formal Language Descriptions

- Regular expressions, regular grammars, finite state automata
- Context-free grammars, BNF grammars, syntax diagrams
- Whole family more of grammars and automata – covered in automata theory

Grammars

- Grammars are formal descriptions of which strings over a given character set are in a particular language
- Language designers write grammar
- Language implementers use grammar to know what programs to accept
- Language users use grammar to know how to write legitimate programs