

## A Polymorphic Typing Rules

Polymorphic constant signatures:

$$\begin{aligned}
 \text{sig}(n) &= \text{int} & n \text{ an integer constant} & & \text{sig}(\oplus) &= \text{int} \rightarrow \text{int} \rightarrow \text{int} & \text{for } \oplus \in \{+, -, *, \% \dots\} \\
 \text{sig}(\text{true}) &= \text{bool} & & & \text{sig}(\text{false}) &= \text{bool} \\
 \text{sig}(\sim) &= \forall \alpha. \alpha \rightarrow \alpha \rightarrow \text{bool} & \text{for } \sim \in \{<, >, =, \leq, \geq\} \\
 \text{sig}([\ ]) &= \forall \alpha. \alpha \text{list} & & & \text{sig}((::)) &= \forall \alpha. \alpha \rightarrow \alpha \text{list} \rightarrow \alpha \text{list} \\
 \text{sig}((, )) &= \forall \alpha \beta. \alpha \rightarrow \beta \rightarrow \alpha * \beta
 \end{aligned}$$

Constants:

$$\frac{}{\Gamma \vdash c : \tau'} \text{CONST} \quad \text{where } c \text{ is a constant listed above, } \text{sig}(c) = \forall \alpha_1 \dots \alpha_n. \tau \text{ and} \\
 \text{there exist } \sigma_1, \dots, \sigma_n \text{ such that } \tau' = \tau[\sigma_1/\alpha_1; \dots; \sigma_n/\alpha_n]$$

Variables:

$$\frac{}{\Gamma \vdash x : \tau'} \text{VAR} \quad \text{where } \forall \alpha_1 \dots \alpha_n. \tau = \Gamma(x) \text{ and} \\
 \text{there exist } \sigma_1, \dots, \sigma_n \text{ such that } \tau' = \tau[\sigma_1/\alpha_1; \dots; \sigma_n/\alpha_n]$$

Connectives:

$$\frac{\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : \text{bool}}{\Gamma \vdash e_1 \ \&\& \ e_2 : \text{bool}} \text{CONN} \quad \frac{\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : \text{bool}}{\Gamma \vdash e_1 \ || \ e_2 : \text{bool}} \text{CONN}$$

If\_then\_else rule:

$$\frac{\Gamma \vdash e_c : \text{bool} \quad \Gamma \vdash e_t : \tau \quad \Gamma \vdash e_e : \tau}{\Gamma \vdash \text{if } e_c \ \text{then } e_t \ \text{else } e_e : \tau} \text{IF}$$

Application rule:

$$\frac{\Gamma \vdash e_1 : \tau_1 \rightarrow \tau_2 \quad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1 \ e_2 : \tau_2} \text{APP}$$

Function rule:

$$\frac{[x : \tau_1] + \Gamma \vdash e : \tau_2}{\Gamma \vdash \text{fun } x \rightarrow e : \tau_1 \rightarrow \tau_2} \text{FUN}$$

Let rule:

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad [x : \text{Gen}(\tau_1, \Gamma)] + \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash \text{let } x = e_1 \ \text{in } e_2 : \tau_2} \text{LET}$$

Let Rec rule:

$$\frac{[x : \tau_1] + \Gamma \vdash e_1 : \tau_1 \quad [x : \text{Gen}(\tau_1, \Gamma)] + \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash \text{let rec } x = e_1 \ \text{in } e_2 : \tau_2} \text{REC}$$