Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Semantics

- Expresses the meaning of syntax
- Static semantics
  - Meaning based only on the form of the expression without executing it
  - Usually restricted to type checking / type inference

Dynamic semantics

- Method of describing meaning of executing a program
- Several different types:
  - Operational Semantics
  - Axiomatic Semantics
  - Denotational Semantics

Dynamic Semantics

- Different languages better suited to different types of semantics
- Different types of semantics serve different purposes

Operational Semantics

- Start with a simple notion of machine
- Describe how to execute (implement) programs of language on virtual machine, by describing how to execute each program statement (ie, following the *structure* of the program)
- Meaning of program is how its execution changes the state of the machine
- Useful as basis for implementations

Axiomatic Semantics

- Also called Floyd-Hoare Logic
- Based on formal logic (first order predicate calculus)
- Axiomatic Semantics is a logical system built from *axioms* and *inference rules*
- Mainly suited to simple imperative programming languages

Axiomatic Semantics

- Used to formally prove a property (*post-condition*) of the *state* (the values of the program variables) after the execution of program, assuming another property (*pre-condition*) of the state before execution
- Written :

  {Precondition} Program {Postcondition}
- Source of idea of *loop invariant*

Denotational Semantics

- Construct a function $\mathcal{M}$ assigning a mathematical meaning to each program construct
- Lambda calculus often used as the range of the meaning function
- Meaning function is compositional: meaning of construct built from meaning of parts
- Useful for proving properties of programs

Natural Semantics

- Aka Structural Operational Semantics, aka "Big Step Semantics"
- Provide value for a program by rules and derivations, similar to type derivations
- Rule conclusions look like

$$(C, m) \Downarrow m'$$

or

$$(E, m) \Downarrow v$$

Simple Imperative Programming Language

- $I \in Identifiers$
- $N \in Numerals$
- B ::= true | false | B & B | B or B | not B | $E < E$ | $E = E$
- $E ::= N \mid I \mid E + E \mid E * E \mid E - E \mid - E$
- C::= skip | C;C | I ::= E | if B then C else C fi | while B do C od

Natural Semantics of Atomic Expressions

- Identifiers: $(I,m) \Downarrow m(I)$
- Numerals are values: $(N,m) \Downarrow N$
- Booleans: $(\text{true},m) \Downarrow \text{true}$

  $(\text{false}\ ,m) \Downarrow \text{false}$

Booleans:

$$\frac{(B, m) \Downarrow \text{false}}{(B \,\&\, B', m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{true} \quad (B', m) \Downarrow b}{(B \,\&\, B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(B \text{ or } B', m) \Downarrow \text{true}} \qquad \frac{(B, m) \Downarrow \text{false} \quad (B', m) \Downarrow b}{(B \text{ or } B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(\text{not } B, m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{false}}{(\text{not } B, m) \Downarrow \text{true}}$$

Relations

$$\frac{(E, m) \Downarrow U \qquad (E', m) \Downarrow V \quad U \sim V = b}{(E \sim E', m) \Downarrow b}$$

- By $U \sim V = b$, we mean does (the meaning of) the relation ~ hold on the meaning of U and V
- May be specified by a mathematical expression/equation or rules matching U and V

Arithmetic Expressions

$$\frac{(E, m) \Downarrow U \qquad (E', m) \Downarrow V \quad U \; op \; V = N}{(E \; op \; E', m) \Downarrow N}$$

where N is the specified value for $U \; op \; V$

Commands

Skip: $(\text{skip}, m) \Downarrow m$

Assignment:
$$\frac{(E,m) \Downarrow V}{(I::=E,m) \Downarrow m[I <\text{--}\ V\,]}$$

Sequencing:
$$\frac{(C,m) \Downarrow m' \quad (C',m') \Downarrow m''}{(C;C',\ m) \Downarrow m''}$$

If Then Else Command

$$\frac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

$$\frac{(B,m) \Downarrow \text{false} \quad (C',m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

While Command

$$\frac{(B,m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m}$$

$$\frac{(B,m)\Downarrow\text{true} \quad (C,m)\Downarrow m' \quad (\text{while } B \text{ do } C \text{ od}, m')\Downarrow m''}{(\text{while } B \text{ do C od}, m) \Downarrow m''}$$

Example: If Then Else Rule

$$\frac{}{\text{(if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi,} \\ \{x \text{ -> } 7\}) \Downarrow \ ?}$$

Example: If Then Else Rule

$$\frac{\overline{\qquad\qquad\qquad\qquad} \\ (x > 5, \{x \to 7\}) \Downarrow ?}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \\ \{x \to 7\}) \Downarrow\ ?}$$

Example: Arith Relation

$$\frac{\dfrac{\begin{array}{c} ? > ? = ? \\ (x,\{x\text{->}7\})\Downarrow ? \quad (5,\{x\text{->}7\})\Downarrow ? \end{array}}{(x > 5, \{x \text{ -> } 7\})\Downarrow ?}}{\begin{array}{c}(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\\ \{x \text{ -> } 7\}) \Downarrow ?\end{array}}$$

Example: Identifier(s)

$7 > 5 = \text{true}$

$$\dfrac{\dfrac{(x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow ?}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\ \{x \text{ -> } 7\}) \Downarrow\ ?}$$

Example: Arith Relation

$$\cfrac{\cfrac{7 > 5 = \text{true}}{(x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow \text{true}}$$

$$\overline{\text{(if } x > 5 \text{ then } y\text{:= } 2 + 3 \text{ else } y\text{:=}3 + 4 \text{ fi}, \quad \{x \text{ -> } 7\}) \Downarrow ?}$$

Example: If Then Else Rule

$$\dfrac{\dfrac{\begin{array}{c}7 > 5 = \text{true} \\ (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5\end{array}}{(x > 5, \{x \text{->} 7\})\Downarrow \text{true}} \qquad \dfrac{}{\begin{array}{c}(y:= 2 + 3, \{x\text{->} 7\} \\ \Downarrow ? \qquad .\end{array}}}{\begin{array}{c}(\text{if } x > 5 \text{ then } y:= 2 + 3 \text{ else } y:=3 + 4 \text{ fi}, \\ \{x \text{->} 7\}) \Downarrow ?\end{array}}$$

Example: Assignment

7 > 5 = true

(x,{x->7})⇓7 (5,{x->7})⇓5

(x > 5, {x -> 7})⇓true

(2+3, {x->7})⇓?

(y:= 2 + 3, {x-> 7}) ⇓ ?

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi, {x -> 7}) ⇓ ?

Example: Arith Op

$$\frac{\dfrac{\begin{array}{c}7 > 5 = \text{true}\\ (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5\end{array}}{(x > 5, \{x \text{ -> } 7\})\Downarrow \text{true}} \qquad \dfrac{\dfrac{\begin{array}{c}? + ? = ?\\ (2,\{x\text{->}7\})\Downarrow ? \quad (3,\{x\text{->}7\})\Downarrow ?\end{array}}{(2+3, \{x\text{->}7\})\Downarrow ?}}{(y:= 2 + 3, \{x\text{->} 7\}) \Downarrow ?}}{(\text{if } x > 5 \text{ then } y:= 2 + 3 \text{ else } y:=3 + 4 \text{ fi}, \{x \text{ -> } 7\}) \Downarrow ?}$$

Example: Numerals

$$
\dfrac{
\dfrac{\overset{7 > 5 = \text{true}}{(x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}}{(x > 5,\ \{x \text{->}\ 7\})\Downarrow \text{true}}
\qquad
\dfrac{\dfrac{\overset{2 + 3 = 5}{(2,\{x\text{->}7\})\Downarrow 2 \quad (3,\{x\text{->}7\})\Downarrow 3}}{(2+3,\ \{x\text{->}7\})\Downarrow ?}}{(y{:=}\ 2 + 3,\ \{x\text{->}\ 7\}) \Downarrow ?}
}{
(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\ \{x \text{->}\ 7\}) \Downarrow ?
}
$$

Example: Arith Op

$$\dfrac{\dfrac{7 > 5 = \mathrm{true} \qquad (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x \text{->} 7\})\Downarrow \mathrm{true}} \qquad \dfrac{\dfrac{2 + 3 = 5 \qquad (2,\{x\text{->}7\})\Downarrow 2 \quad (3,\{x\text{->}7\})\Downarrow 3}{(2+3,\ \{x\text{->}7\})\Downarrow 5}}{(y{:=}\ 2 + 3,\ \{x\text{->}\ 7\}) \Downarrow ?}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\ \{x \text{->} 7\}) \Downarrow\ ?}$$

Example: Assignment

$$\dfrac{\dfrac{7 > 5 = \text{true} \qquad (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5, \{x \text{->} 7\})\Downarrow \text{true}} \qquad \dfrac{\dfrac{2 + 3 = 5 \qquad (2,\{x\text{->}7\})\Downarrow 2 \quad (3,\{x\text{->}7\}) \Downarrow 3}{(2+3, \{x\text{->}7\})\Downarrow 5}}{(y:= 2 + 3, \{x\text{->} 7\}) \Downarrow \{x\text{->}7, y\text{->}5\}}}{(\text{if } x > 5 \text{ then } y:= 2 + 3 \text{ else } y:=3 + 4 \text{ fi}, \{x \text{->} 7\}) \Downarrow ?}$$

Example: If Then Else Rule

$$
\frac{
\dfrac{7 > 5 = \text{true} \qquad (x,\{x\text{->}7\}) \Downarrow 7 \quad (5,\{x\text{->}7\}) \Downarrow 5}{(x > 5,\ \{x \text{->} 7\}) \Downarrow \text{true}}
\qquad
\dfrac{\dfrac{2 + 3 = 5 \qquad (2,\{x\text{->}7\}) \Downarrow 2 \quad (3,\{x\text{->}7\}) \Downarrow 3}{(2+3,\ \{x\text{->}7\}) \Downarrow 5}}{(y := 2 + 3,\ \{x\text{->}\ 7\}) \Downarrow \{x\text{->}7,\ y\text{->}5\}}
}{
(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi},\ \{x \text{->} 7\}) \Downarrow \{x\text{->}7,\ y\text{->}5\}
}
$$

Let in Command

$$\frac{(E,m) \Downarrow v \quad (C, m[I\text{<-}v]) \Downarrow m'}{(\text{let } I = E \text{ in } C,\ m) \Downarrow m''}$$

Where $m''(y) = m'(y)$ for $y \neq I$ and $m''(I) = m(I)$ if $m(I)$ is defined, and $m''(I)$ is undefined otherwise

Example

$$\frac{(5,\{x\text{->}17\}) \Downarrow 5 \qquad \dfrac{\dfrac{(x,\{x\text{->}5\}) \Downarrow 5 \quad (3,\{x\text{->}5\}) \Downarrow 3}{(x+3,\{x\text{->}5\}) \Downarrow 8}}{(x:=x+3,\{x\text{->}5\}) \Downarrow \{x\text{->}8\}}}{(\text{let } x = 5 \text{ in } (x:=x+3),\ \{x \text{ -> } 17\}) \Downarrow\ ?}$$

Example

$$\dfrac{(5,\{x\text{->}17\}) \Downarrow 5 \qquad \dfrac{\dfrac{(x,\{x\text{->}5\}) \Downarrow 5 \quad (3,\{x\text{->}5\}) \Downarrow 3}{(x+3,\{x\text{->}5\}) \Downarrow 8}}{(x:=x+3,\{x\text{->}5\}) \Downarrow \{x\text{->}8\}}}{(\text{let } x = 5 \text{ in } (x:=x+3), \{x \text{ -> } 17\}) \Downarrow \{x\text{->}17\}}$$

Comment

- Simple Imperative Programming Language introduces variables *implicitly* through assignment
- The let-in command introduces scoped variables *explictly*
- Clash of constructs apparent in awkward semantics

Interpretation Versus Compilation

- A **compiler** from language L1 to language L2 is a program that takes an L1 program and for each piece of code in L1 generates a piece of code in L2 of same meaning
- An **interpreter** of L1 in L2 is an L2 program that executes the meaning of a given L1 program
- Compiler would examine the body of a loop once; an interpreter would examine it every time the loop was executed

Interpreter

- An *Interpreter* represents the operational semantics of a language L1 (source language) in the language of implementation L2 (target language)
- Built incrementally
  - Start with literals
  - Variables
  - Primitive operations
  - Evaluation of expressions
  - Evaluation of commands/declarations

Interpreter

- Takes abstract syntax trees as input
  - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
  - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next "state"
  - To get final value, put in a loop

Natural Semantics Example

- compute_exp (Var(v), m) = look_up v m
- compute_exp (Int(n), _) = Num (n)
- ...
- compute_com(IfExp(b,c1,c2),m) =

  if compute_exp (b,m) = Bool(true)

  then compute_com (c1,m)

  else compute_com (c2,m)

Natural Semantics Example

- compute_com(While(b,c), m) =
  if compute_exp (b,m) = Bool(false)
  then m
  else compute_com
  (While(b,c), compute_com(c,m))

- May fail to terminate - exceed stack limits
- Returns no useful information then