Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Unification Algorithm

- Let $S = \{(s_1, t_1), (s_2, t_2), \ldots, (s_n, t_n)\}$ be a unification problem.

- Case $S = \{ \}$: Unif(S) = Identity function (i.e., no substitution)

- Case $S = \{(s, t)\} \cup S'$: Four main steps

Unification Algorithm

- Delete: if s = t (they are the same term) then Unif(S) = Unif(S')
- Decompose: if $s = f(q_1, \ldots, q_m)$ and $t = f(r_1, \ldots, r_m)$ (same f, same m!), then $\mathrm{Unif}(S) = \mathrm{Unif}(\{(q_1, r_1), \ldots, (q_m, r_m)\} \cup S')$
- Orient: if t = x is a variable, and s is not a variable, $\mathrm{Unif}(S) = \mathrm{Unif}(\{(x,s)\} \cup S')$

Unification Algorithm

- Eliminate: if s = x is a variable, and x does not occur in t (the occurs check), then
  - Let $\varphi = x \mid\!\rightarrow t$
  - Let $\psi = \mathrm{Unif}(\varphi(S'))$
  - $\mathrm{Unif}(S) = \{x \mid\!\rightarrow \psi(t)\} \circ \psi$
    - Note: $\{x \mid\!\rightarrow a\} \circ \{y \mid\!\rightarrow b\} = \{y \mid\!\rightarrow (\{x \mid\!\rightarrow a\}(b))\} \circ \{x \mid\!\rightarrow a\}$ if y not in a

Tricks for Efficient Unification

- Don't return substitution, rather do it incrementally
- Make substitution be constant time
  - Requires implementation of terms to use mutable structures (or possibly lazy structures)
  - We won't discuss these

Example

- x,y,z variables, f,g constructors

- S = {(f(x), f(g(y,z))), (g(y,f(y)), x)}

Example

- x,y,z variables, f,g constructors
- S is nonempty

- S = {(f(x), f(g(y,z))), (g(y,f(y)), x)}

Example

- x,y,z variables, f,g constructors
- Pick a pair: (g(y,f(y)), x)

- S = {(f(x), f(g(y,z))), (g(y,f(y)), x)}

Example

- x,y,z variables, f,g constructors
- Pick a pair: (g(y,f(y))), x)
- Orient: (x, g(y,f(y)))
- S = {(f(x), f(g(y,z))), (g(y,f(y)), x)}
- -> {(f(x), f(g(y,z))), (x, g(y,f(y)))}

Example

- x,y,z variables, f,g constructors
- S -> {(f(x), f(g(y,z))), (x, g(y,f(y)))}

Example

- x,y,z variables, f,g constructors
- Pick a pair: (f(x), f(g(y,z)))

- S -> {(f(x), f(g(y,z))), (x, g(y,f(y)))}

Example

- x,y,z variables, f,g constructors
- Pick a pair: (f(x), f(g(y,z)))
- Decompose: (x, g(y,z))
- S -> {(f(x), f(g(y,z))), (x, g(y,f(y)))}
- -> {(x, g(y,z)), (x, g(y,f(y)))}

Example

- x,y,z variables, f,g constructors
- Pick a pair: (x, g(y,f(y)))
- Substitute: {x |-> g(y,f(y))}
- S -> {(x, g(y,z)), (x, g(y,f(y)))}
- -> {(g(y,f(y)), g(y,z))}

- With {x |-> g(y,f(y))}

Example

- x,y,z variables, f,g constructors
- Pick a pair: (g(y,f(y)), g(y,z))

- S -> {(g(y,f(y)), g(y,z))}

With $\{x \mapsto g(y,f(y))\}$

Example

- x,y,z variables, f,g constructors
- Pick a pair: (g(y,f(y)), g(y,z))
- Decompose: (y, y) and (f(y), z)
- S -> {(g(y,f(y)), g(y,z))}
- -> {(y, y), (f(y), z)}

With {x |→ g(y,f(y))}

Example

- x,y,z variables, f,g constructors
- Pick a pair: (y, y)

- S -> {(y, y), (f(y), z)}

With $\{x \mid\rightarrow g(y,f(y))\}$

Example

- x,y,z variables, f,g constructors
- Pick a pair: (y, y)
- Delete
- S -> {(y, y), (f(y), z)}
- -> {(f(y), z)}

With $\{x \mid\!\rightarrow g(y,f(y))\}$

Example

- x,y,z variables, f,g constructors
- Pick a pair: (f(y), z)

- S -> {(f(y), z)}

With $\{x \mapsto g(y,f(y))\}$

Example

- x,y,z variables, f,g constructors
- Pick a pair: (f(y), z)
- Orient: (z, f(y))
- S -> {(f(y), z)}
- -> {(z, f(y))}

With $\{x \mapsto g(y,f(y))\}$

Example

- x,y,z variables, f,g constructors
- Pick a pair: (z, f(y))

- S -> {(z, f(y))}

With $\{x \mapsto g(y,f(y))\}$

Example

- x,y,z variables, f,g constructors
- Pick a pair: (z, f(y))
- Eliminate: {z|-> f(y)}
- S -> {(z, f(y))}
- -> { }

With $\{x \mapsto \{z \mapsto f(y)\}\ (g(y,f(y)))\ \}$

$\circ\ \{z \mapsto f(y)\}$

Example

- x,y,z variables, f,g constructors
- Pick a pair: (z, f(y))
- Eliminate: {z|-> f(y)}
- S -> {(z, f(y))}
- -> { }

With {x |→ g(y,f(y))} o {(z |→ f(y))}

Example

$S = \{(f(x), f(g(y,z))), (g(y,f(y)),x)\}$

Solved by $\{x \mapsto g(y,f(y))\} \circ \{(z \mapsto f(y))\}$

$$f(\underbrace{g(y,f(y))}_{x}) = f(g(y,\underbrace{f(y)}_{z}))$$

and

$$g(y,f(y)) = \underbrace{g(y,f(y))}_{x}$$

Example of Failure: Decompose

- $S = \{(f(x,g(y)), f(h(y),x))\}$
- Decompose: $(f(x,g(y)), f(h(y),x))$
- $S \to \{(x,h(y)), (g(y),x)\}$
- Orient: $(g(y),x)$
- $S \to \{(x,h(y)), (x,g(y))\}$
- Eliminate: $(x,h(y))$
- $S \to \{(h(y), g(y))\}$ with $\{x \mapsto h(y)\}$
- No rule to apply! Decompose fails!

Example of Failure: Occurs Check

- S = {(f(x,g(x)), f(h(x),x))}
- Decompose: (f(x,g(x)), f(h(x),x))
- S -> {(x,h(x)), (g(x),x)}
- Orient: (g(y),x)
- S -> {(x,h(x)), (x,g(x))}
- No rules apply.

Where We Are Going

- We want to turn strings (code) into computer instructions
- Done in phases
- Turn strings into abstract syntax trees (parse)
- Translate abstract syntax trees into executable instructions (interpret or compile)

Major Phases of a Compiler

Source Program

Lex

Tokens

Parse

Abstract Syntax

Semantic Analysis

Symbol Table

Translate

Intermediate Representation

Optimize

Optimized IR

Instruction Selection

Unoptimized Machine-Specific Assembly Language

Optimize

Optimized Machine-Specific Assembly Language

Emit code

Assembly Language

Assembler

Relocatable Object Code

Linker

Machine Code

Modified from “Modern Compiler Implementation in ML”, by Andrew Appel

Meta-discourse

- Language Syntax and Semantics
- Syntax
  - Regular Expressions, DFSAs and NDFSAs
  - Grammars
- Semantics
  - Natural Semantics
  - Transition Semantics

Language Syntax

- Syntax is the description of which strings of symbols are meaningful expressions in a language
- It takes more than syntax to understand a language; need meaning (semantics) too
- Syntax is the entry point

Syntax of English Language

- Pattern 1

Subject	Verb
David	*sings*
The dog	*barked*
Susan	*yawned*

- Pattern 2

Subject	Verb	Direct Object
David	*sings*	*ballads*
The professor	*wants*	*to retire*
The jury	*found*	*the defendant guilty*

Elements of Syntax

- Character set – previously always ASCII, now often 64 character sets
- Keywords – usually reserved
- Special constants – cannot be assigned to
- Identifiers – can be assigned to
- Operator symbols
- Delimiters (parenthesis, braces, brackets)
- Blanks (aka white space)

Elements of Syntax

- Expressions

  if ... then begin ... ; ... end else begin ... ; ... end

- Type expressions

  $typexpr_1$ -> $typexpr_2$

- Declarations (in functional languages)

  let $pattern_1$ = $expr_1$ in $expr$

- Statements (in imperative languages)

  a = b + c

- Subprograms

  let $pattern_1$ = let rec inner = … in $expr$

Elements of Syntax

- Modules
- Interfaces
- Classes (for object-oriented languages)

Lexing and Parsing

- Converting strings to abstract syntax trees done in two phases
  - **Lexing:** Converting string (or streams of characters) into lists (or streams) of tokens (the "words" of the language)
    - Specification Technique: Regular Expressions
  - **Parsing:** Convert a list of tokens into an abstract syntax tree
    - Specification Technique: BNF Grammars

Formal Language Descriptions

- Regular expressions, regular grammars, finite state automata
- Context-free grammars, BNF grammars, syntax diagrams
- Whole family more of grammars and automata – covered in automata theory

Grammars

- Grammars are formal descriptions of which strings over a given character set are in a particular language
- Language designers write grammar
- Language implementers use grammar to know what programs to accept
- Language users use grammar to know how to write legitimate programs

Regular Expressions - Review

- Start with a given character set – **a, b, c**…
- Each character is a regular expression
  - It represents the set of one string containing just that character

Regular Expressions

- If **x** and **y** are regular expressions, then **xy** is a regular expression
  - It represents the set of all strings made from first a string described by **x** then a string described by **y**

If x={a,ab} and y={c,d} then xy ={ac,ad,abc,abd}.

- If **x** and **y** are regular expressions, then **x**∨**y** is a regular expression
  - It represents the set of strings described by either **x** or **y**

  If x={a,ab} and y={c,d} then x ∨ y={a,ab,c,d}

Regular Expressions

- If **x** is a regular expression, then so is **(x)**
  - It represents the same thing as **x**
- If **x** is a regular expression, then so is **x***
  - It represents strings made from concatenating zero or more strings from **x**

  If x = {a,ab}

  then x* ={"",a,ab,aa,aab,abab,aaa,aaab,…}
- **ε**
  - It represents {""}, set containing the empty string

Example Regular Expressions

- **(0**v**1)*1**
  - The set of all strings of **0**'s and **1**'s ending in 1, **{1, 01, 11,…}**
- **a*b(a*)**
  - The set of all strings of a's and b's with exactly one b
- **((01)** v**(10))***
  - You tell me
- Regular expressions (equivalently, regular grammars) important for lexing, breaking strings into recognized words

Example: Lexing

- Regular expressions good for describing lexemes (words) in a programming language
  - Identifier = (a ∨ b ∨ … ∨ z ∨ A ∨ B ∨ … ∨ Z) (a ∨ b ∨ … ∨ z ∨ A ∨ B ∨ … ∨ Z ∨ 0 ∨ 1 ∨ … ∨ 9)*
  - Digit = (0 ∨ 1 ∨ … ∨ 9)
  - Number = 0 ∨ (1 ∨ … ∨ 9)(0 ∨ … ∨ 9)* ∨ ~ (1 ∨ … ∨ 9)(0 ∨ … ∨ 9)*
  - Keywords: if = if, while = while,…

Implementing Regular Expressions

- Regular expressions reasonable way to generate strings in language
- Not so good for recognizing when a string is in language
- Problems with Regular Expressions
  - which option to choose,
  - how many repetitions to make
- Answer: finite state automata
- Should have covered this in CS373

Lexing

- Different syntactic categories of “words”: tokens

Example:

- Convert sequence of characters into sequence of strings, integers, and floating point numbers.
- "asd 123 jkl 3.14" will become:

  [String "asd"; Int 123; String "jkl"; Float 3.14]

Lex, ocamllex

- Could write the reg exp, then translate to DFA by hand
  - A lot of work
- Better: Write program to take reg exp as input and automatically generates automata
- Lex is such a program
- ocamllex version for ocaml

How to do it

- To use regular expressions to parse our input we need:
  - Some way to identify the input string — call it a lexing buffer
  - Set of regular expressions,
  - Corresponding set of actions to take when they are matched.

How to do it

- The lexer will take the regular expressions and generate a state machine.
- The state machine will take our lexing buffer and apply the transitions...
- If we reach an accepting state from which we can go no further, the machine will perform the appropriate action.

Mechanics

- Put table of reg exp and corresponding actions (written in ocaml) into a file <*filename*>.mll
- Call

  ocamllex <*filename*>.mll

- Produces Ocaml code for a lexical analyzer in file <*filename*>.ml

Sample Input

```
rule main = parse
 ['0'-'9']+ { print_string "Int\n"}
 | ['0'-'9']+'.'['0'-'9']+ { print_string "Float\n"}
 | ['a'-'z']+ { print_string "String\n"}
 | _ { main lexbuf }
 {
 let newlexbuf = (Lexing.from_channel stdin) in
 print_string "Ready to lex.\n";
 main newlexbuf
}
```

General Input

```
{ header }
let ident = regexp ...
rule entrypoint [arg1... argn] = parse
      regexp { action }
    | ...
    | regexp { action }
and entrypoint [arg1... argn] =
  parse ...and ...
{ trailer }
```

Ocamllex Input

- *header* and *trailer* contain arbitrary ocaml code put at top an bottom of *<filename>*.ml

- let *ident* = *regexp* ... Introduces *ident* for use in later regular expressions

Ocamllex Input

- *<filename>*.ml contains one lexing function per *entrypoint*
  - Name of function is name given for *entrypoint*
  - Each entry point becomes an Ocaml function that takes *n*+1 arguments, the extra implicit last argument being of type Lexing.lexbuf
- *arg1*... a*rgn* are for use in *action*

Ocamllex Regular Expression

- Single quoted characters for letters: 'a'
- _: (underscore) matches any letter
- Eof: special "end_of_file" marker
- Concatenation same as usual
- "*string*": concatenation of sequence of characters
- $e_1 \mid e_2$: choice - what was $e_1 \vee e_2$

Ocamllex Regular Expression

- $[c_1 - c_2]$: choice of any character between first and second inclusive, as determined by character codes
- $[\^{}c_1 - c_2]$: choice of any character NOT in set
- $e*$: same as before
- $e+$: same as $e\ e*$
- $e?$: option - was $e_1 \vee \varepsilon$

Ocamllex Regular Expression

- e_1 # e_2: the characters in e_1 but not in e_2; e_1 and e_2 must describe just sets of characters
- *ident*: abbreviation for earlier reg exp in let *ident* = *regexp*
- e_1 as *id*: binds the result of e_1 to *id* to be used in the associated *action*

Ocamllex Manual

- More details can be found at

http://caml.inria.fr/pub/docs/manual-ocaml/manual026.html

Example : test.mll

```
{ type result = Int of int | Float of float |
  String of string }
let digit = ['0'-'9']
let digits = digit +
let lower_case = ['a'-'z']
let upper_case = ['A'-'Z']
let letter = upper_case | lower_case
let letters = letter +
```

Example : test.mll

```
rule main = parse
   (digits)'.'digits as f  { Float (float_of_string f) }
 | digits as n              { Int (int_of_string n) }
 | letters as s             { String s}
 | _ { main lexbuf }
 { let newlexbuf = (Lexing.from_channel stdin) in
 print_string "Ready to lex.";
 print_newline ();
 main newlexbuf  }
```

Example

```
# #use "test.ml";;
…
val main : Lexing.lexbuf -> result = <fun>
val __ocaml_lex_main_rec : Lexing.lexbuf -> int ->
   result = <fun>
Ready to lex.
hi there 234 5.2
- : result = String "hi"
```

What happened to the rest?!?

Example

```
# let b = Lexing.from_channel stdin;;
# main b;;
hi 673 there
- : result = String "hi"
# main b;;
- : result = Int 673
# main b;;
- : result = String "there"
```

Problem

- How to get lexer to look at more than the first token at one time?
- Answer: *action* has to tell it to -- recursive calls
- Side Benefit: can add “state” into lexing
- Note: already used this with the _ case

Example

```
rule main = parse
  (digits) '.' digits as f { Float
  (float_of_string f) :: main lexbuf}
 | digits as n          { Int (int_of_string n) ::
  main lexbuf }
 | letters as s         { String s :: main
  lexbuf}
 | eof                   { [] }
 | _                      { main lexbuf }
```

Example Results

Ready to lex.

hi there 234 5.2

- : result list = [String "hi"; String "there"; Int 234; Float 5.2]

#

Used Ctrl-d to send the end-of-file signal

Dealing with comments

First Attempt

```
let open_comment = "(*"
let close_comment = "*)"
rule main = parse
   (digits) '.' digits as f { Float (float_of_string
   f) :: main lexbuf}
  | digits as n          { Int (int_of_string n) ::
   main lexbuf }
  | letters as s         { String s :: main lexbuf}
```

Dealing with comments

```
 | open_comment          { comment  lexbuf}
 | eof                 { [] }
 | _ { main lexbuf }
and comment = parse
   close_comment       { main lexbuf }
 | _                  { comment lexbuf }
```

Dealing with nested comments

```
rule main = parse …
 | open_comment          { comment 1 lexbuf}
 | eof                   { [] }
 | _ { main lexbuf }
and comment depth = parse
   open_comment          { comment (depth+1)
   lexbuf }
 | close_comment         { if depth = 1
                           then main lexbuf
                           else comment (depth - 1) lexbuf }
 | _                     { comment depth lexbuf }
```

Dealing with nested comments

```
rule main = parse
  (digits) '.' digits as f { Float (float_of_string f) :: main lexbuf}
 | digits as n            { Int (int_of_string n) :: main lexbuf }
 | letters as s           { String s :: main lexbuf}
 | open_comment           { (comment 1 lexbuf}
 | eof                    { [] }
 | _ { main lexbuf }
```

Dealing with nested comments

```
and comment depth = parse
   open_comment        { comment (depth+1) lexbuf }
 | close_comment       { if depth = 1
                         then main lexbuf
                         else comment (depth - 1) lexbuf }
 | _                   { comment depth lexbuf }
```