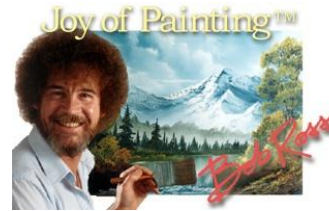


Visible Surface Determination

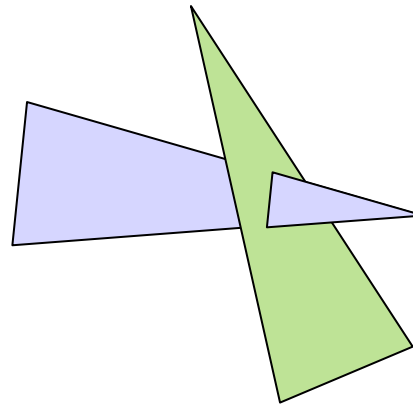
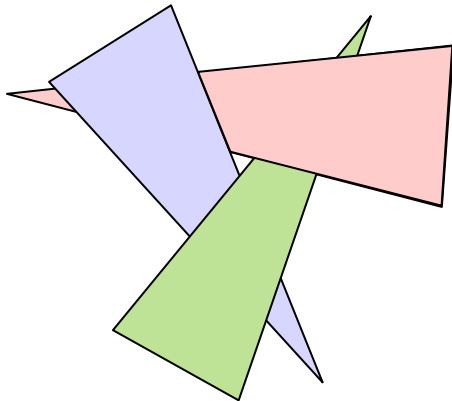
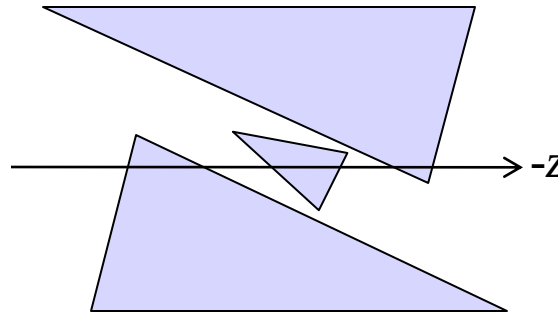
CS418 Computer Graphics

John C. Hart

Painter's Algorithm

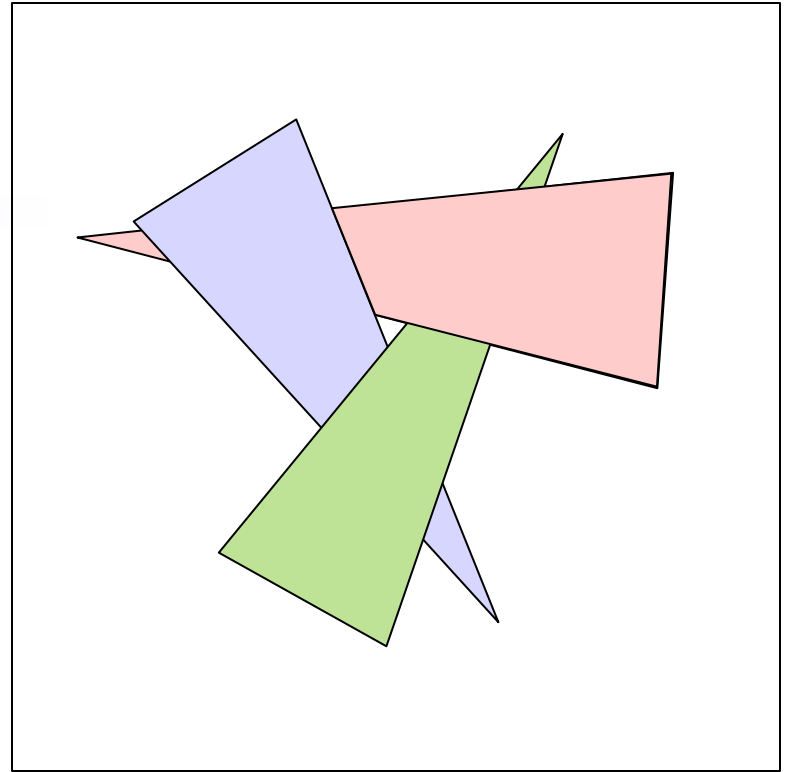


- Display polygons in back-to-front order
- Sort polygons by z-value
 - Which vertex?
 - $O(n \log n)$
- Problems...



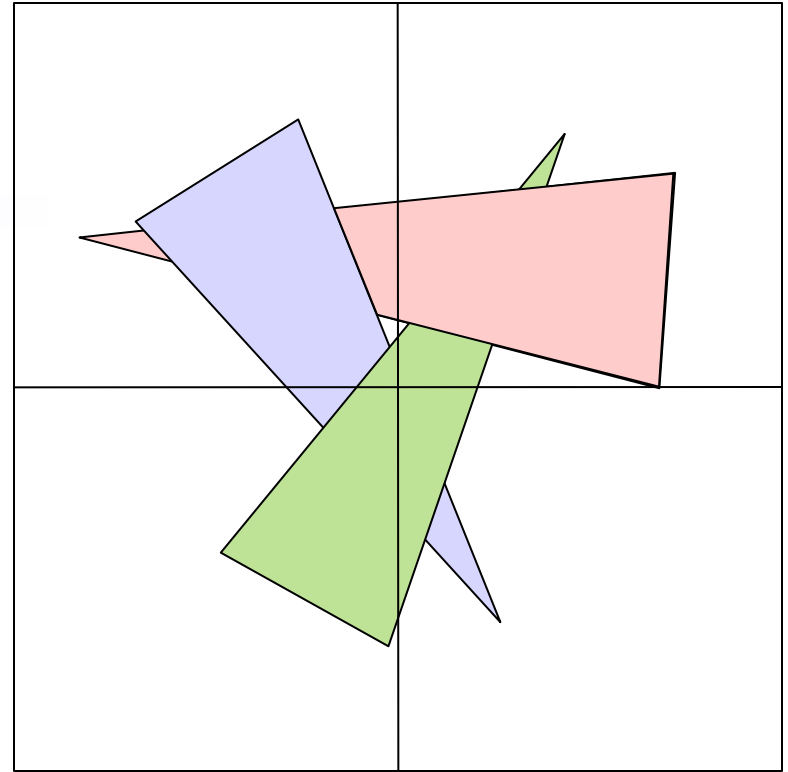
Quadtree Algorithm

- Sort polygons
- Subdivide screen until each region contains one or zero edges
- Invented by John Warnock in 1969



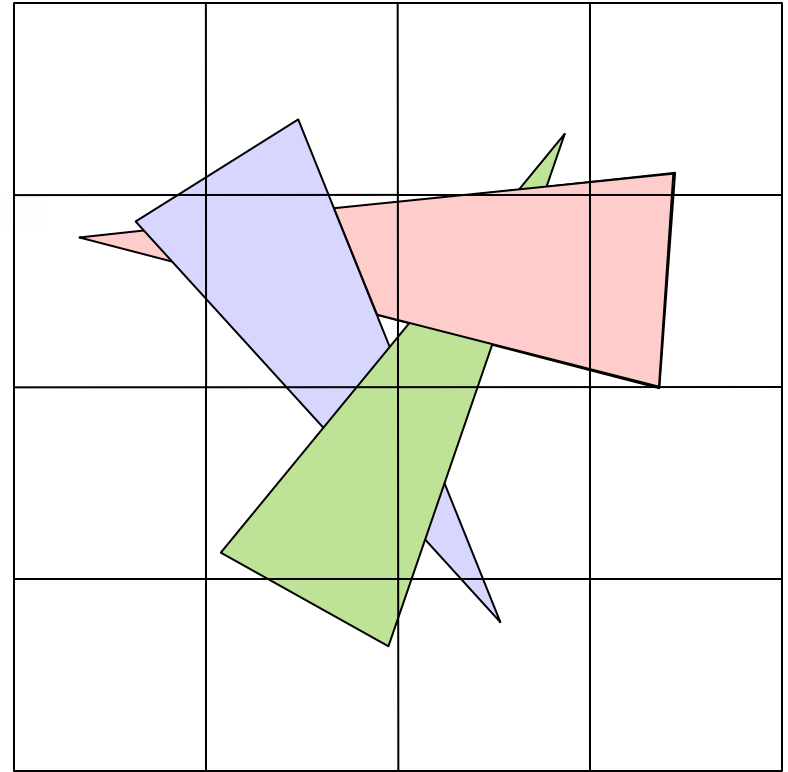
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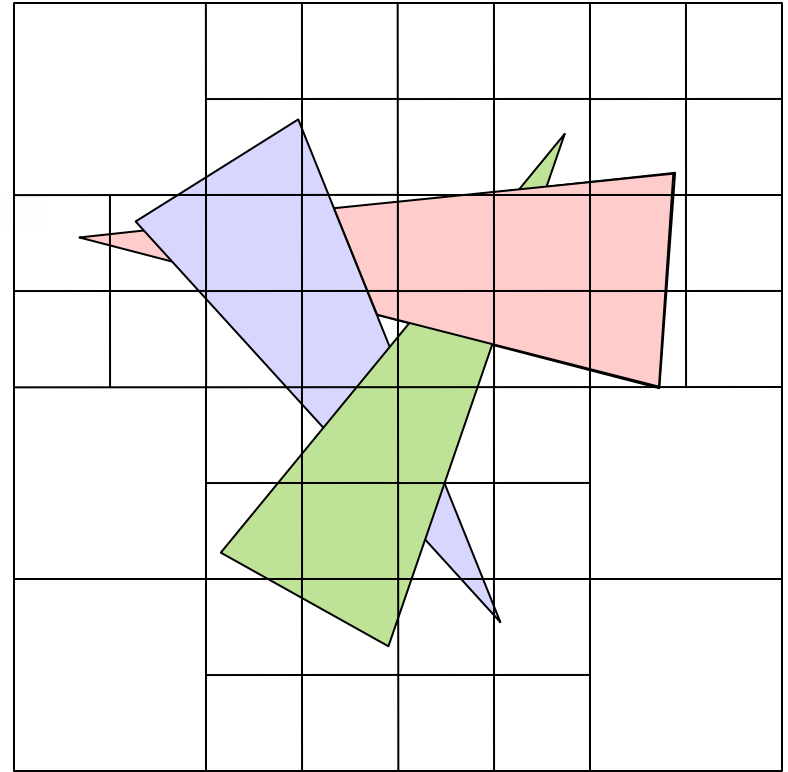
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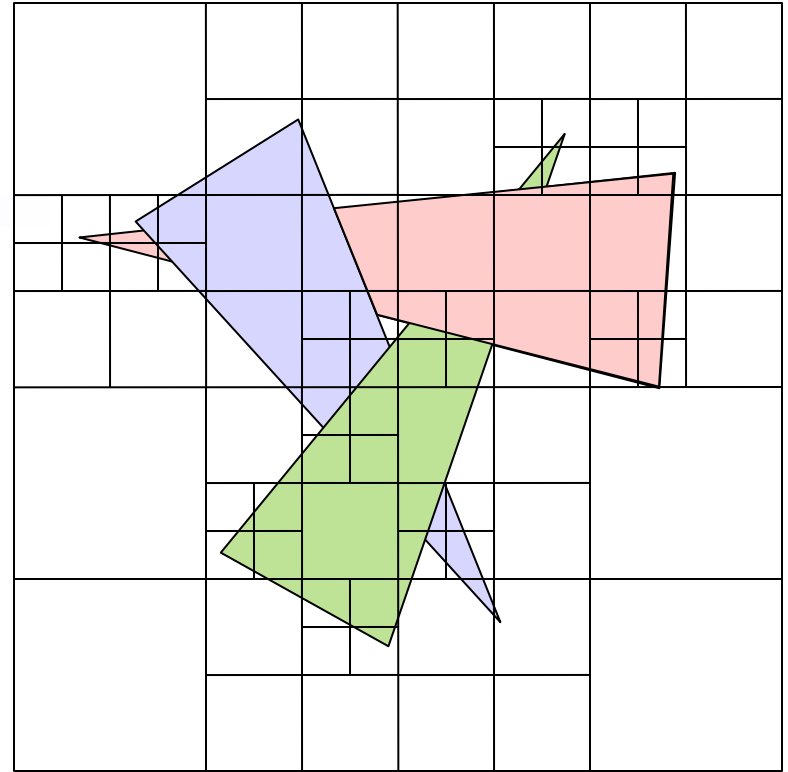
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Quadtree Algorithm

- Sort polygons
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- Don't need to sort triangles, just find for each pixel the closest triangle
- Z-buffer: one fixed or floating point value per pixel
- Algorithm:

If $z > \text{zbuffer}(x,y)$ then

$$\text{zbuffer}(x,y) = z$$


Z-Buffer

Key Observation: Each pixel displays color of only one triangle, ignores everything behind it

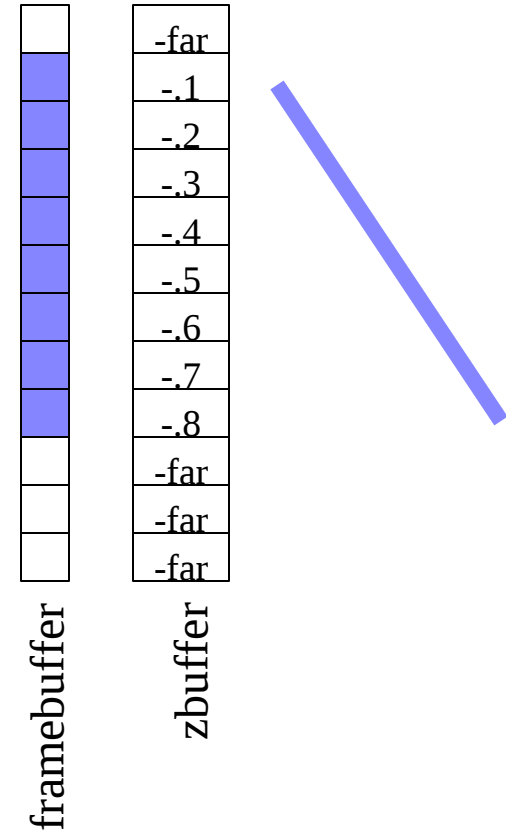
- Don't need to sort triangles, just find for each pixel the closest triangle
- Z-buffer: one fixed or floating point value per pixel
- Algorithm:

For each rasterized fragment (x,y)

If $z > \text{zbuffer}(x,y)$ then

$\text{framebuffer}(x,y) = \text{fragment color}$

$\text{zbuffer}(x,y) = z$



Z-Buffer

Key Observation: Each pixel displays color of only one triangle, ignores everything behind it

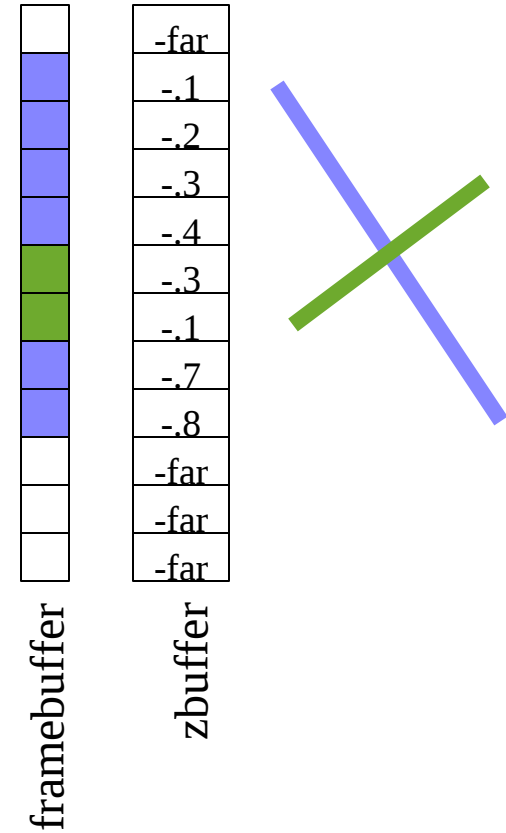
- Don't need to sort triangles, just find for each pixel the closest triangle
- Z-buffer: one fixed or floating point value per pixel
- Algorithm:

For each rasterized fragment (x,y)

If $z > \text{zbuffer}(x,y)$ then

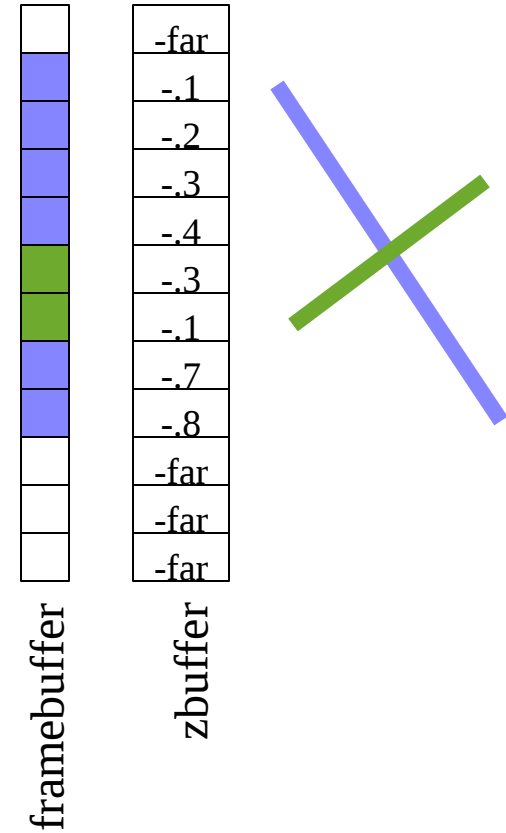
$\text{framebuffer}(x,y) = \text{fragment color}$

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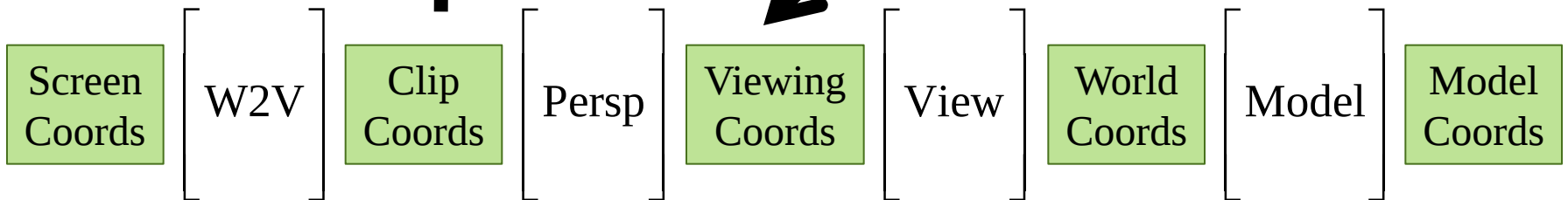
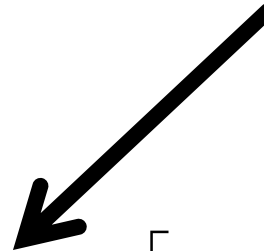
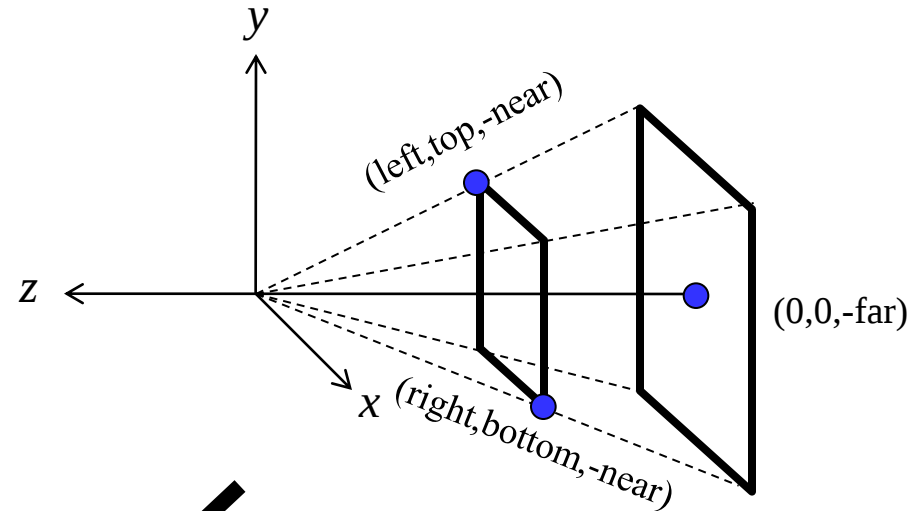
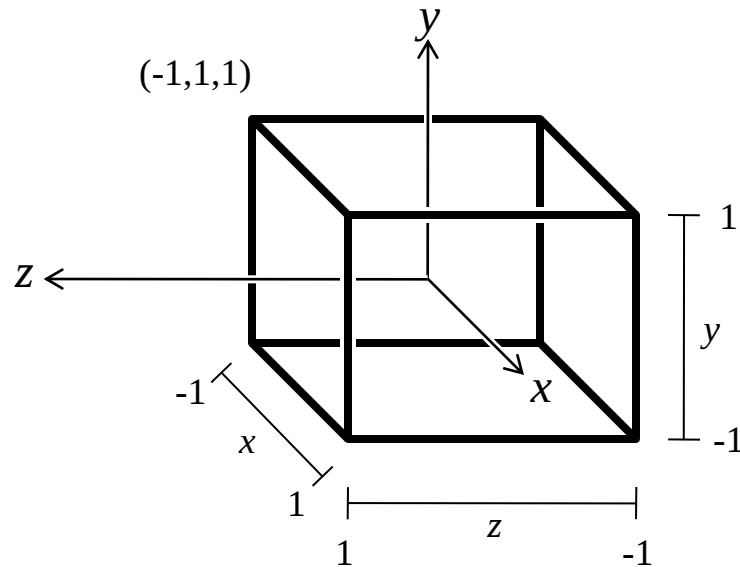
Z-Buffer

- Get fragment z-values by interpolating z-values at vertices during rasterization
- Perspective projection destroys z-values, setting them all to $-d$
- Need a perspective distortion that preserves at least the ordering of z-values

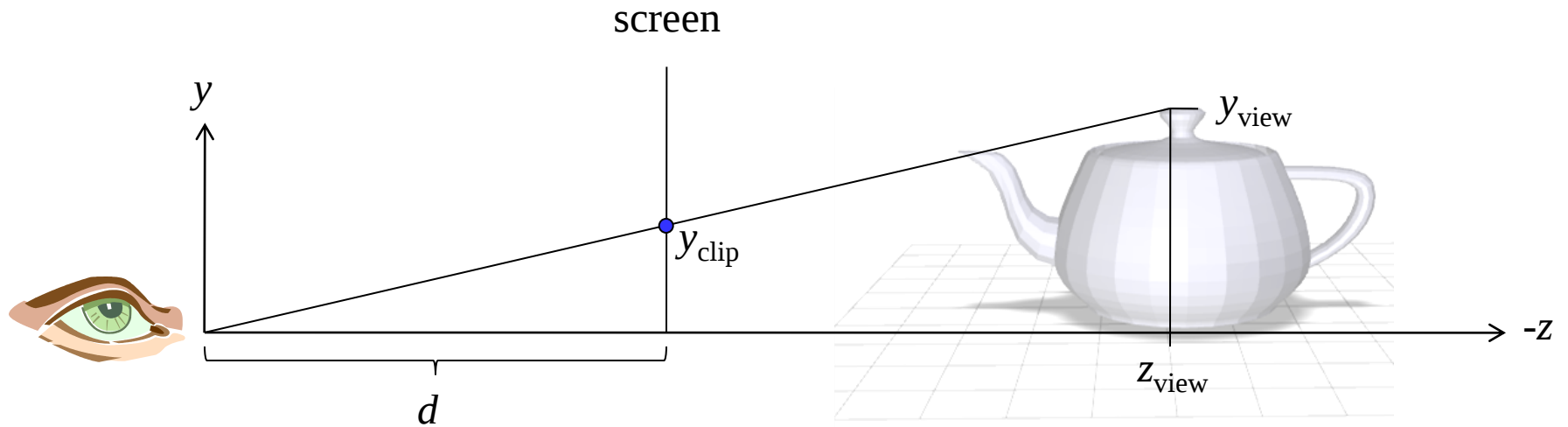


Normalized View Volume

```
glFrustum(left, right, bottom, top, near, far)
```



Perspective Projection



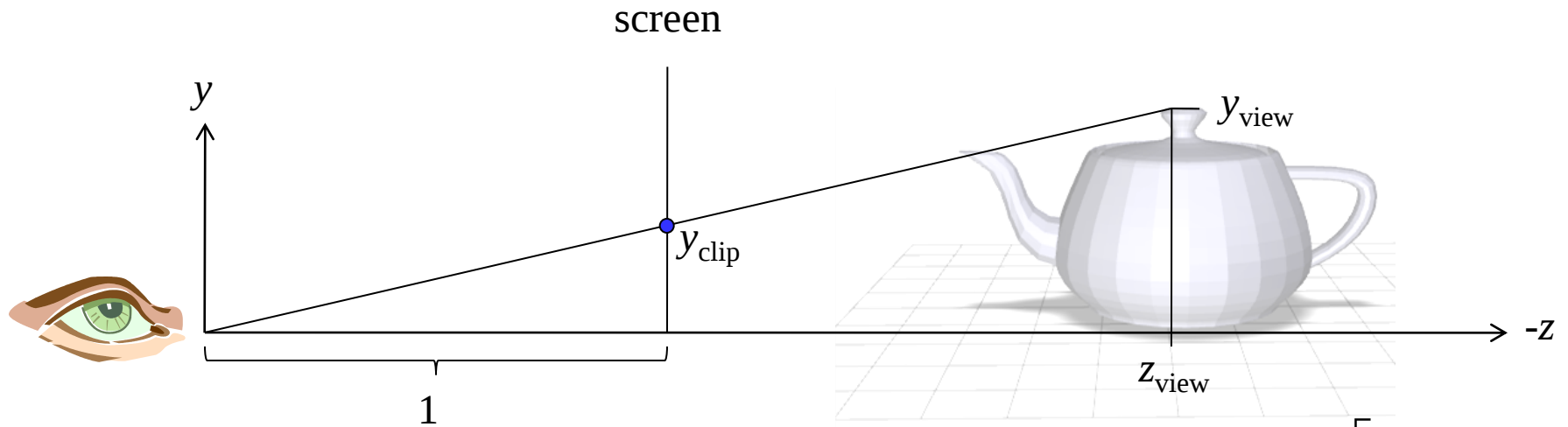
$$\frac{y_{\text{clip}}}{d} = \frac{y_{\text{view}}}{-z_{\text{view}}}$$

$$y_{\text{clip}} = \frac{y_{\text{view}}}{-z_{\text{view}} / d}$$

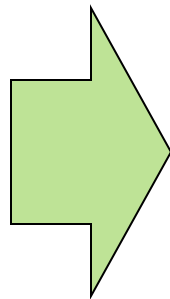
➔

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & -1/d & 0 \end{bmatrix} \begin{bmatrix} x_{\text{view}} \\ y_{\text{view}} \\ z_{\text{view}} \\ 1 \end{bmatrix} = \begin{bmatrix} x_{\text{view}} \\ y_{\text{view}} \\ z_{\text{view}} \\ -z_{\text{view}} / d \end{bmatrix} \equiv \begin{bmatrix} \frac{x_{\text{view}}}{-z_{\text{view}} / d} \\ \frac{y_{\text{view}}}{-z_{\text{view}} / d} \\ -d \\ 1 \end{bmatrix}$$

Perspective Distortion

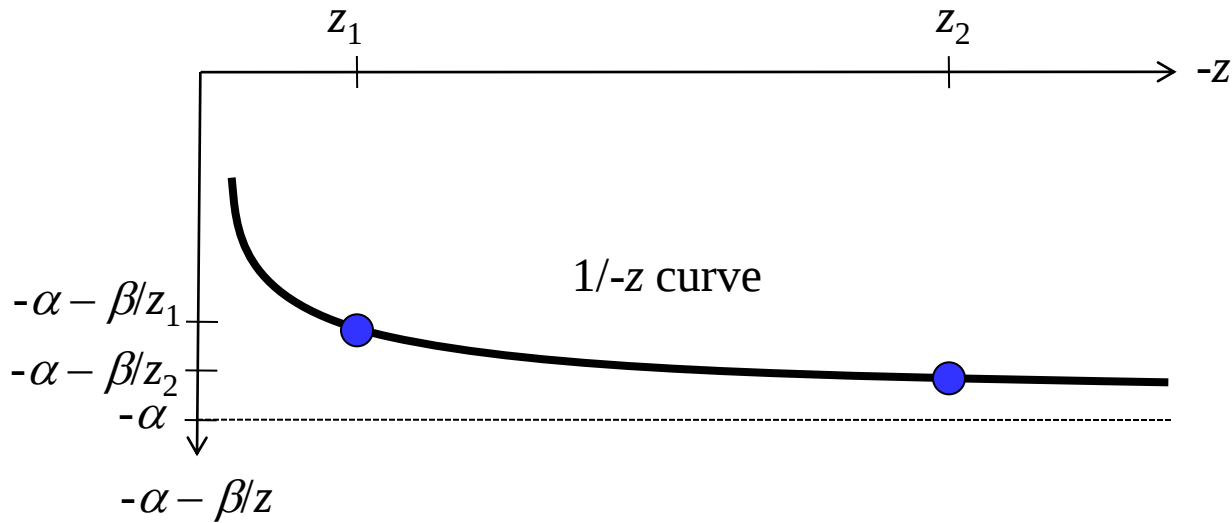


$$y_{clip} = \frac{y_{view}}{-z_{view}}$$



$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \alpha & \beta \\ & & -1 & 0 \end{bmatrix} \begin{bmatrix} x_{view} \\ y_{view} \\ z_{view} \\ 1 \end{bmatrix} = \begin{bmatrix} x_{view} \\ y_{view} \\ \alpha z_{view} + \beta \\ -z_{view} \end{bmatrix} \equiv \begin{bmatrix} \frac{x_{view}}{-z_{view}} \\ \frac{y_{view}}{-z_{view}} \\ -\alpha - \frac{\beta}{z_{view}} \\ 1 \end{bmatrix}$$

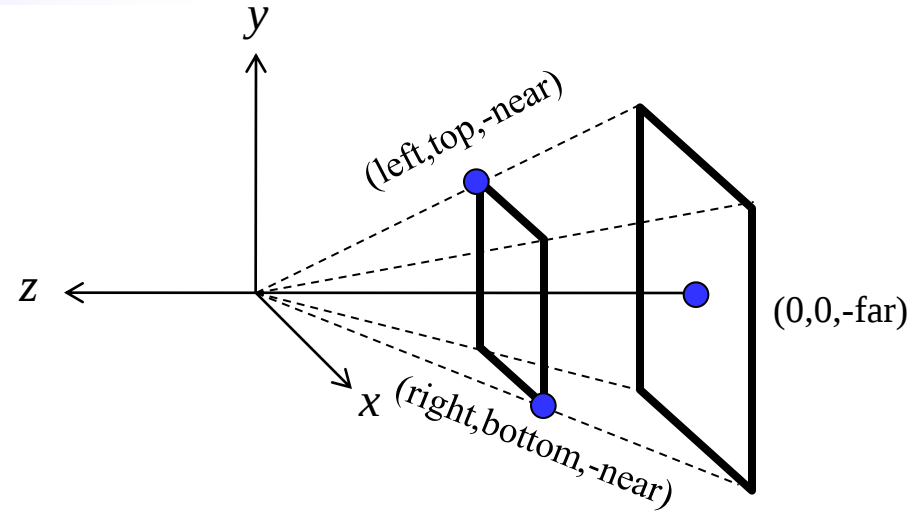
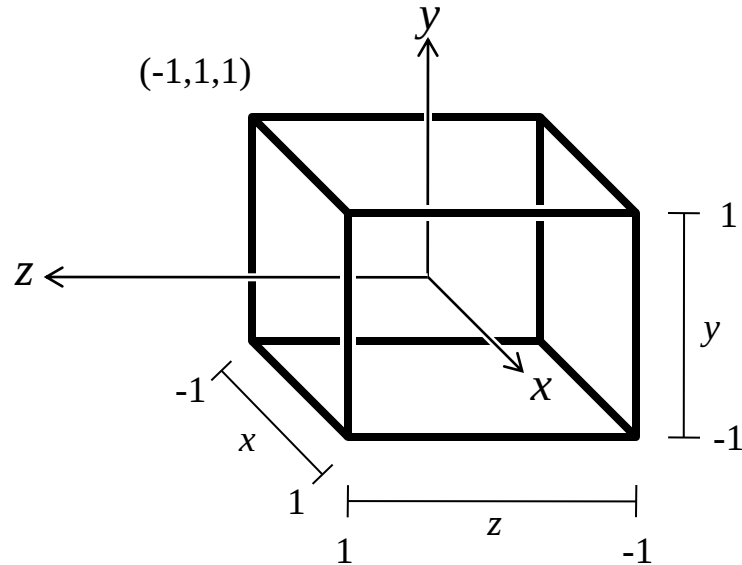
Distorted z-Values



if $z_1 > z_2$ then
 $- \alpha - \beta/z_1 > - \alpha - \beta/z_2$

$$y_{\text{clip}} = \frac{y_{\text{view}}}{-z_{\text{view}}} \quad \rightarrow \quad \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \alpha & \beta \\ & & -1 & 0 \end{bmatrix} \begin{bmatrix} x_{\text{view}} \\ y_{\text{view}} \\ z_{\text{view}} \\ 1 \end{bmatrix} = \begin{bmatrix} x_{\text{view}} \\ y_{\text{view}} \\ \alpha z_{\text{view}} + \beta \\ -z_{\text{view}} \end{bmatrix} \equiv \begin{bmatrix} \frac{x_{\text{view}}}{-z_{\text{view}}} \\ \frac{y_{\text{view}}}{-z_{\text{view}}} \\ - \alpha - \frac{\beta}{z_{\text{view}}} \\ 1 \end{bmatrix}$$

Normalized Perspective Distortion



$$\begin{bmatrix} \frac{-2 \times \text{near}}{\text{right} - \text{left}} & \frac{\text{right} + \text{left}}{\text{right} - \text{left}} & \\ \frac{-2 \times \text{near}}{\text{top} - \text{bottom}} & \frac{\text{top} + \text{bottom}}{\text{top} - \text{bottom}} & \\ -\frac{\text{far} + \text{near}}{\text{far} - \text{near}} & \frac{2 \times \text{far} \times \text{near}}{\text{far} - \text{near}} & \\ -1 & 0 & \end{bmatrix}$$

Hierarchical Z-Buffer

- Invented by Ned Green in 1994
- Creates a MIP-map of the z-buffer
 - z-value equal to farthest z-value of its four children
- Before rasterizing a triangle...
 - Check z-value of its nearest vertex against z-value of the smallest quadtree cell containing the triangle
 - If z-test fails, then the entire triangle is hidden and need not be rasterized
- Works best when displaying front-to-back



framebuffer

-0.2	-0.3	-0.5	-far
-0.3			
-0.4	-0.5		
-0.5			
-0.6	-0.7	-far	
-0.7			
-0.8	-far		
-far			

hierarchical zbuffer

