

# CS 414 – Multimedia Systems Design

## Lecture 8 – JPEG

### Compression (Part 3)

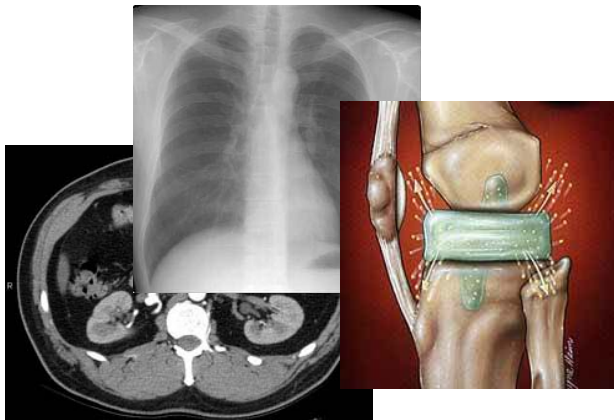
Klara Nahrstedt  
Spring 2009



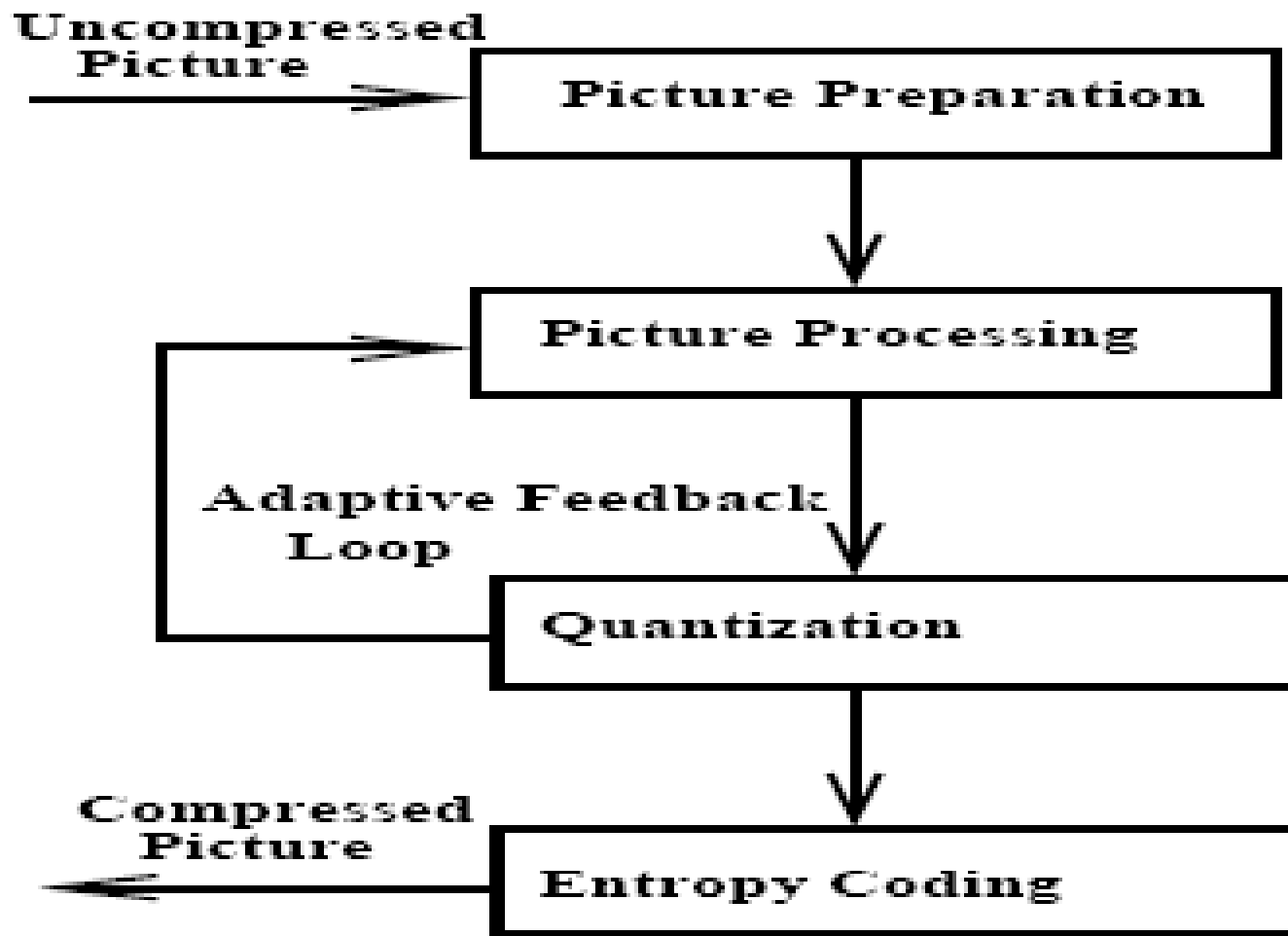
# Administrative

- MP1 is posted, deadline Monday, February 9, demonstrations 5-7pm in 0216 SC

# Ubiquitous use of digital images



# Hybrid Coding

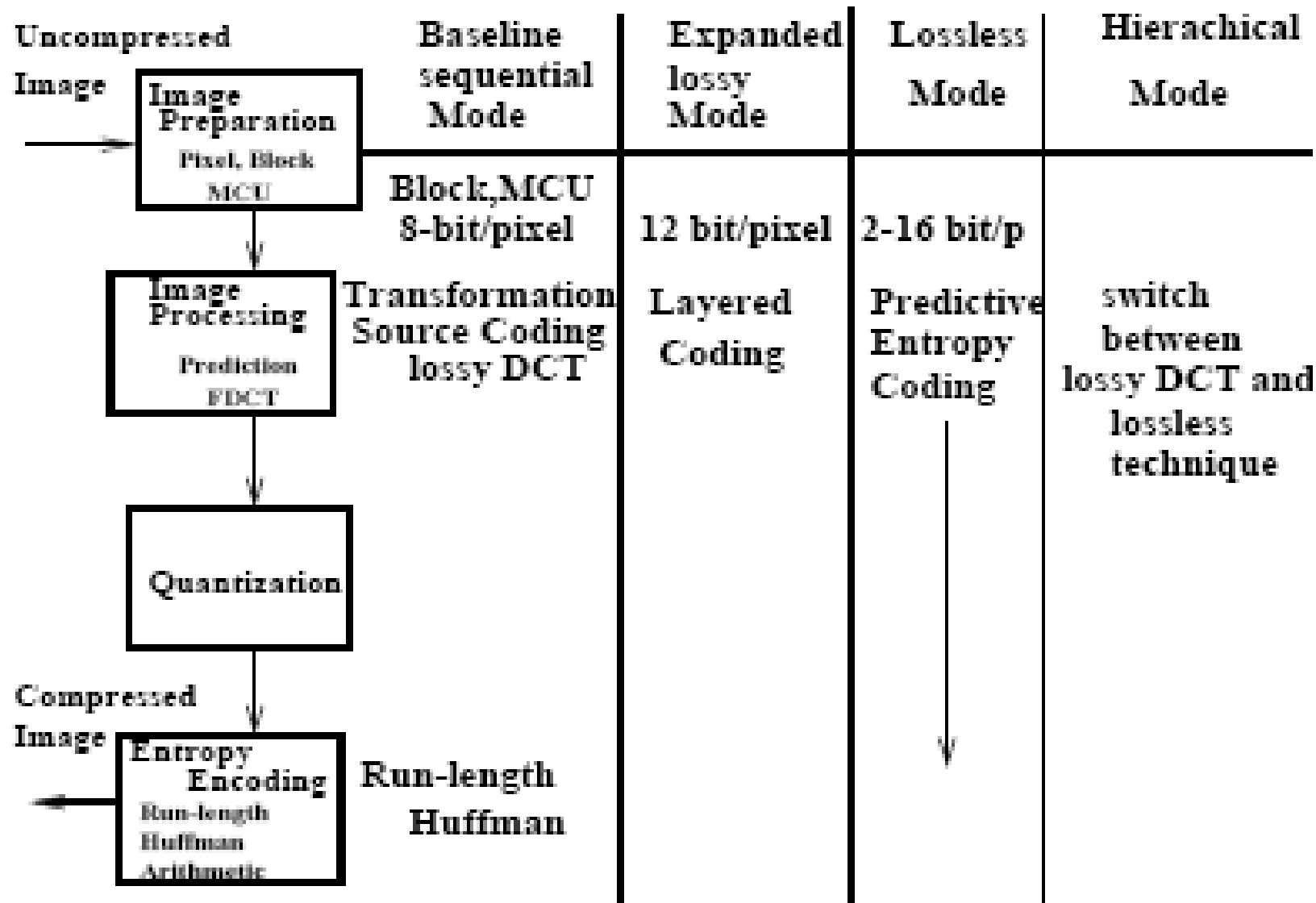




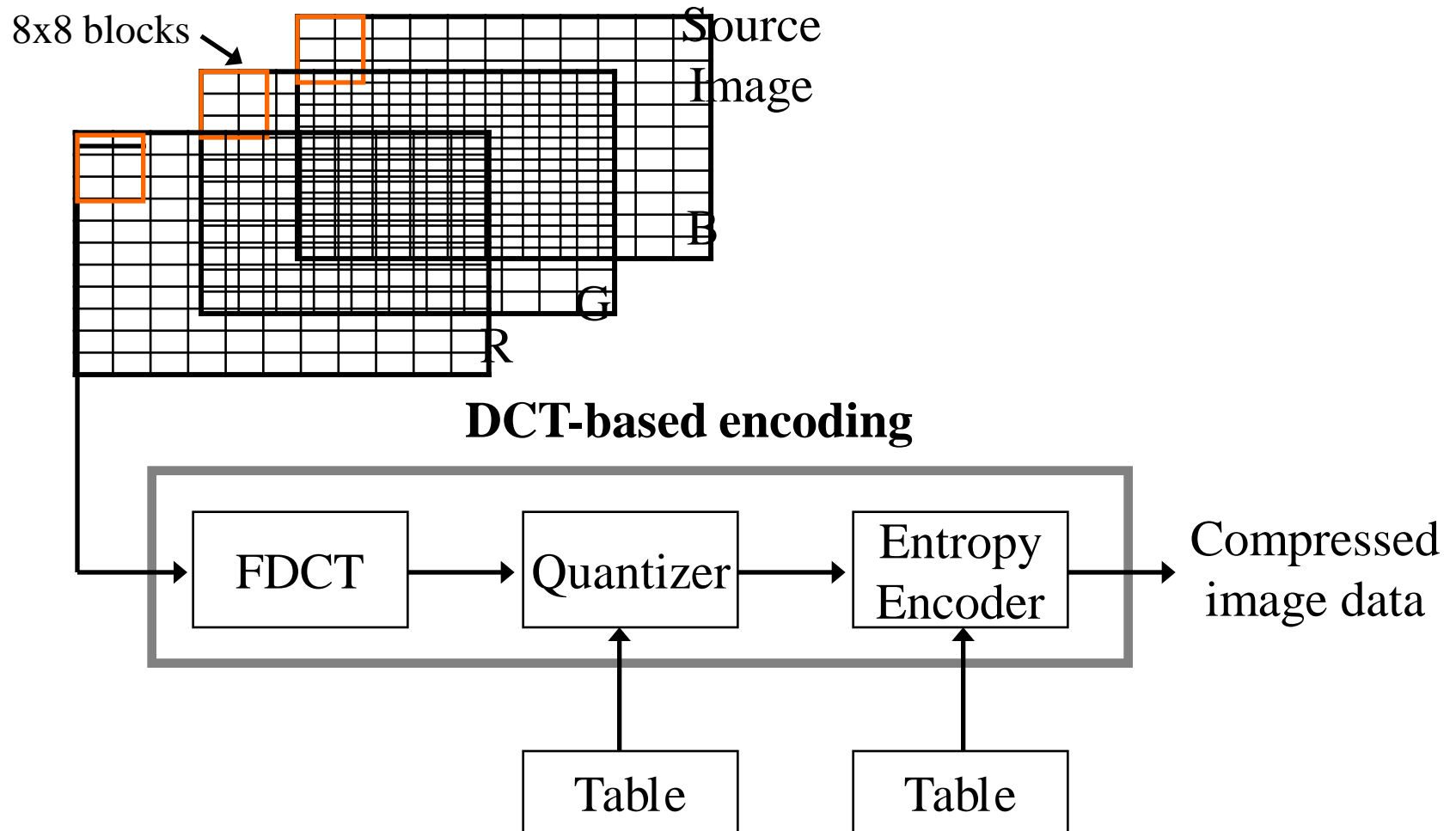
# JPEG (Joint Photographic Experts Group)

## ■ Requirements:

- ☐ Very good compression ratio and good quality image
- ☐ Independent of image size
- ☐ Applicable to any image and pixel aspect ratio
- ☐ Applicable to any complexity (with any statistical characteristics)



# JPEG Compression

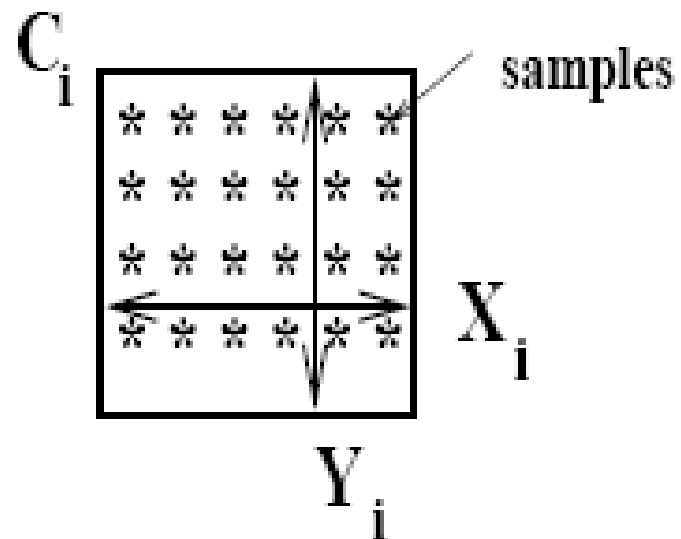
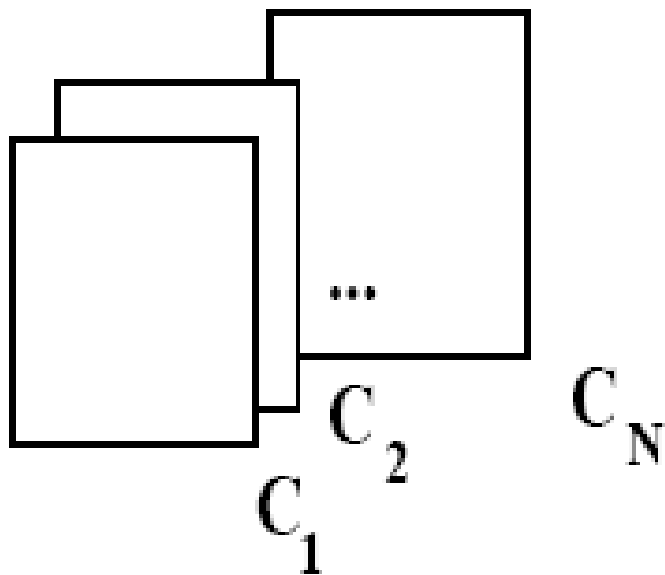


# Image Preparation

- The image preparation is **NOT BASED** on
  - 9-bit YUV encoding
  - Fixed number of lines and columns
  - Mapping of encoded chrominance
- Source image consists of components ( $C_i$ ) and to each component we assign YUV, RGB or TIQ signals.

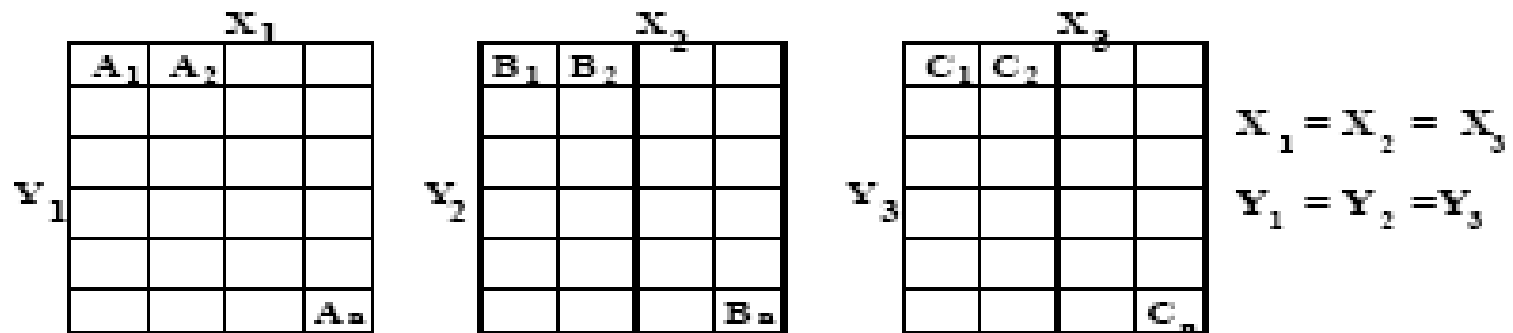


# Division of Source Image into Planes

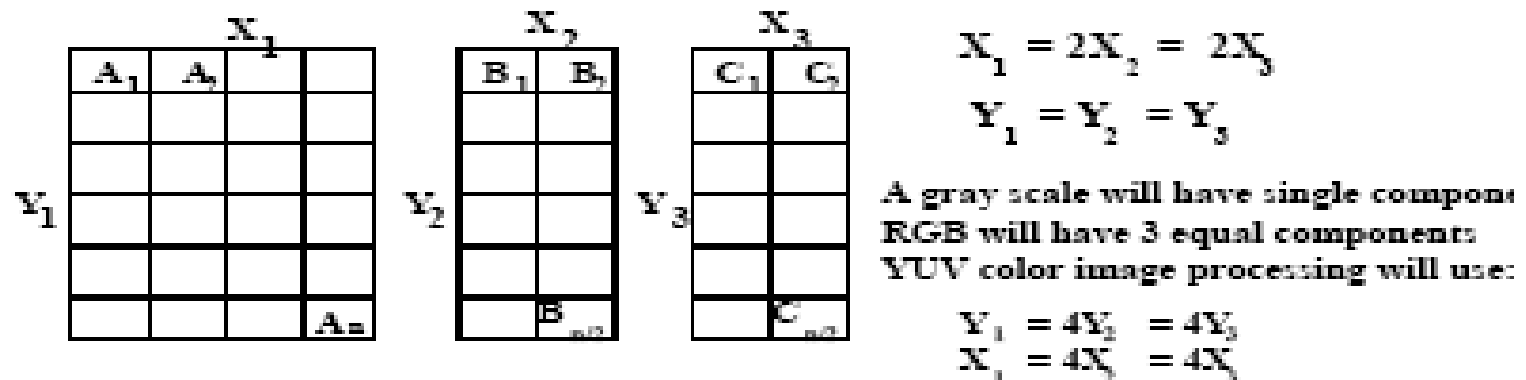


# Components and their Resolutions

## A./ Components with the same resolution



## B./ Components with different resolution



# Color Transformation (optional)

- Down-sample chrominance components
  - compress without loss of quality (color space)
  - e.g., YUV 4:2:2 or 4:1:1
- Example: 640 x 480 RGB to YUV 4:1:1
  - Y is 640x480
  - U is 160x120
  - V is 160x120

# Dimensions of Compressed Image

- $i_{th}$  color component has dimension  $(x_i, y_i)$ 
  - maximum dimension value is  $2^{16}$
  - $[X, Y]$  where  $X=\max(x_i)$  and  $Y=\max(y_i)$
- Sampling among components must be integral
  - $H_i$  and  $V_i$ ; must be within range  $[1, 4]$
  - $[H_{max}, V_{max}]$  where  $H_{max}=\max(H_i)$  and  $V_{max}=\max(V_i)$
- $x_i = X * H_i / H_{max}$
- $y_i = Y * V_i / V_{max}$

# Dimensions (Example)

Level 0:

$$C_1 \quad H_1 = 2 \quad (X=6)$$

$$V_1 = 2 \quad (Y=4)$$

	0	1	2	3	4	5
0	*	*	*	*	*	*
1	*	*	*	*	*	*
2	*	*	*	*	*	*
3	*	*	*	*	*	*

Level 1:

$$H_2 = 2 \quad (X=6)$$

$$V_2 = 1 \quad (Y=4/2)$$

$$C_2$$

*	*	*	*	*	*
*	*	*	*	*	*


Level 2:

$$H_3 = 1 \quad (X=6/2)$$

$$V_3 = 1 \quad (Y=4/2)$$

$$C_3$$

*	*	*
*	*	*



# Image Preparation (Pixel Allocation)

- Each pixel is presented by 'p' bits, value is in range of  $(0, 2^p - 1)$
- All pixels of all components within the same image are coded with the same number of bits
- Lossy modes use precision 8 or 12 bits per pixel
- Lossless mode uses precision 2 up to 12 bits per pixel

# Image Preparation - Blocks

- Images are divided into data units, called blocks – definition comes from DCT transformation since DCT operates on blocks
- Lossy mode – blocks of 8x8 pixels; lossless mode – data unit 1 pixel

# Data Unit Ordering

- *Non-interleaved*: scan from left to right, top to bottom for each color component
- *Interleaved*: compute one “unit” from each color component, then repeat
  - full color pixels after each step of decoding
  - but components may have different resolution

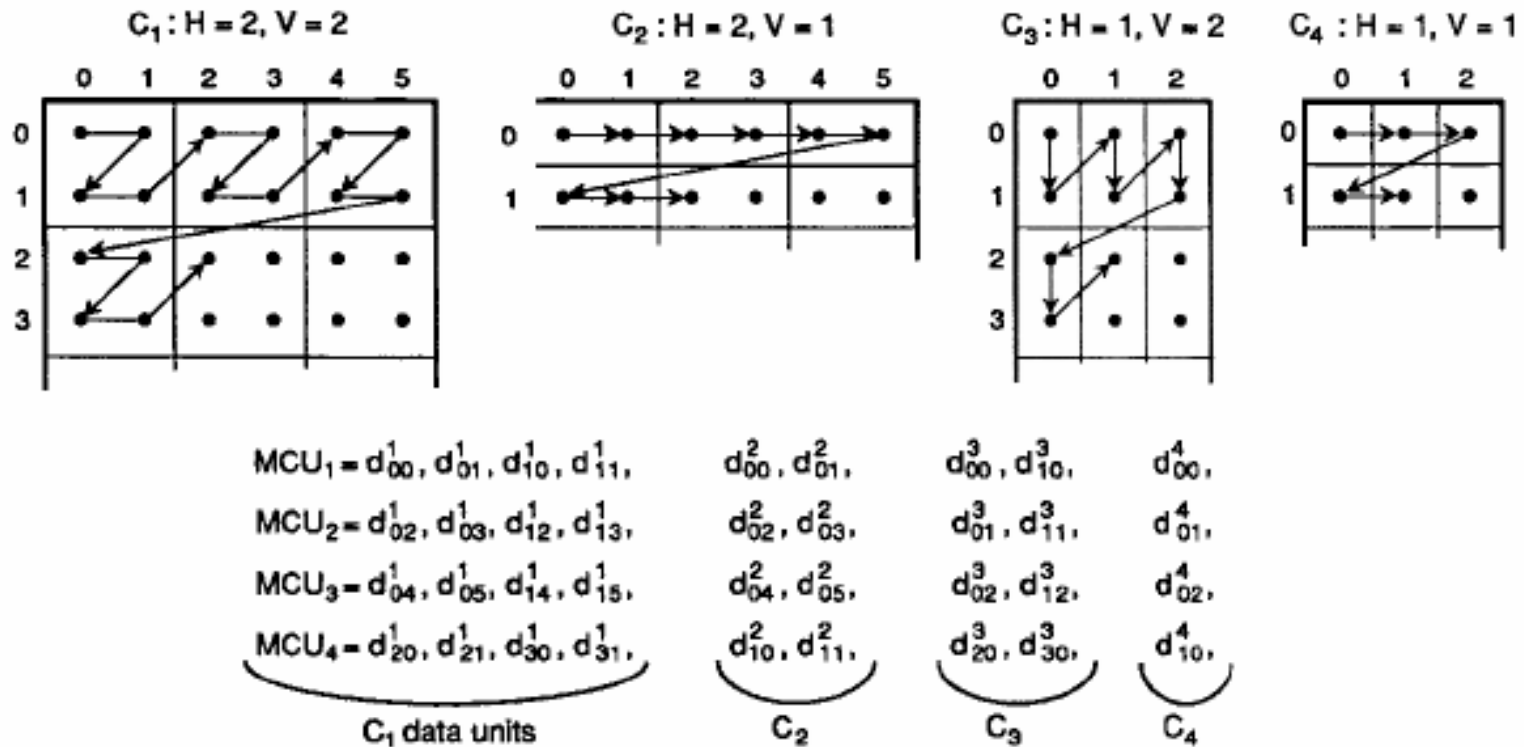




# Interleaved Data Ordering

- Interleaved data units of different components are combined into Minimum Coded Units (MCUs)
- If image has the same resolution, then MCU consists of exactly one data unit for each component
- If image has different resolution for each component, reconstruction of MCUs is more complex

# Example



# Image Processing

- Shift values  $[0, 2^P - 1]$  to  $[-2^{P-1}, 2^{P-1} - 1]$ 
  - e.g. if  $(P=8)$ , shift  $[0, 255]$  to  $[-127, 127]$
  - DCT requires range be centered around 0
- Values in  $8 \times 8$  pixel blocks are spatial values and there are 64 samples values in each block

# Forward DCT

- Convert from spatial to frequency domain
  - convert intensity function into weighted sum of periodic basis (cosine) functions
  - identify bands of spectral information that can be thrown away without loss of quality
- Intensity values in each color plane often change slowly

# Understanding DCT

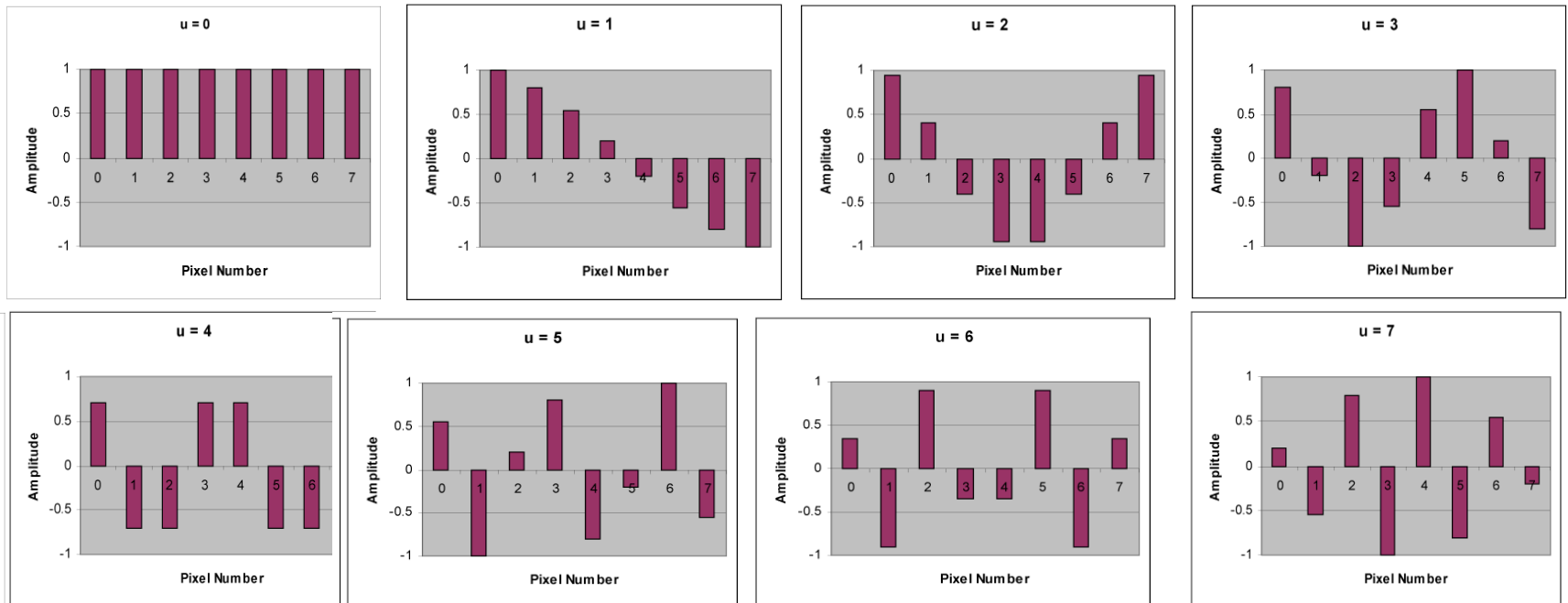
- For example, in  $\mathbb{R}^3$ , we can write  $(5, 2, 9)$  as the sum of a set of basis vectors
  - we know that  $[(1,0,0), (0,1,0), (0,0,1)]$  provides one set of basis functions in  $\mathbb{R}^3$

$$(5,2,9) = 5*(1,0,0) + 2*(0,1,0) + 9*(0,0,1)$$

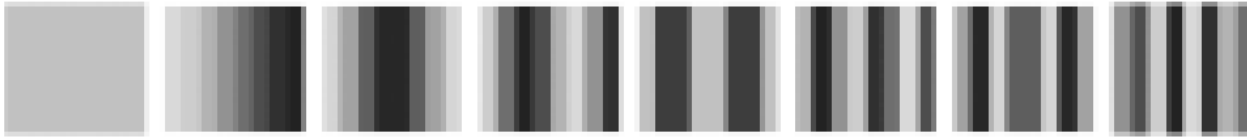
- DCT is same process in function domain

# DCT Basic Functions

- Decompose the intensity function into a weighted sum of cosine basis functions



# Alternative Visualization



# 1D Forward DCT

- Given a list of  $n$  intensity values  $I(x)$ , where  $x = 0, \dots, n-1$
- Compute the  $n$  DCT coefficients:

$$F(u) = \sqrt{\frac{2}{n}} \quad C(u) \sum_{x=0}^{n-1} I(x) \cos \frac{(2x+1)u\pi}{2n}, u = 0 \dots n-1$$

$$\text{where} \quad C(u) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for } u = 0, \\ 1 & \text{otherwise} \end{cases}$$



# 1D Inverse DCT

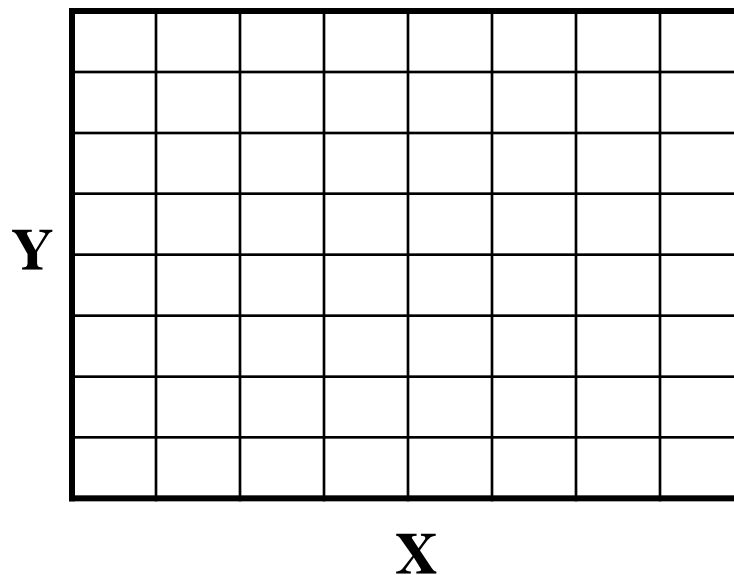
- Given a list of  $n$  DCT coefficients  $F(u)$ , where  $u = 0, \dots, n-1$
- Compute the  $n$  intensity values:

$$I(x) = \sqrt{\frac{2}{n}} \sum_{u=0}^{n-1} F(u) C(u) \cos \frac{(2x+1)u\pi}{2n}, x = 0 \dots n-1$$

$$\text{where } C(u) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for } u = 0, \\ 1 & \text{otherwise} \end{cases}$$

# Extend DCT from 1D to 2D

- Perform 1D DCT on each row of the block
- Again for each column of 1D coefficients
  - alternatively, transpose the matrix and perform DCT on the rows



# Equations for 2D DCT

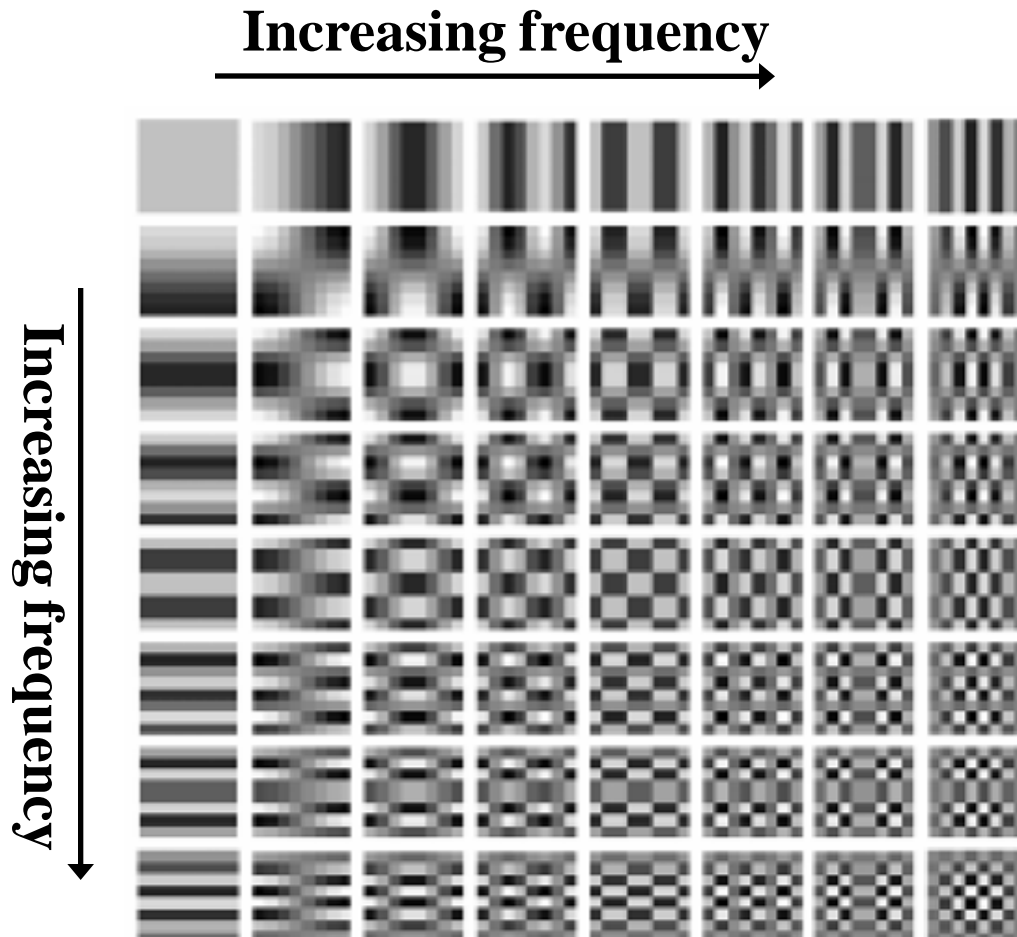
- Forward DCT:

$$F(u, v) = \frac{2}{\sqrt{nm}} C(u)C(v) \sum_{y=0}^{m-1} \sum_{x=0}^{n-1} I(x, y) * \cos\left(\frac{(2x+1)u\pi}{2n}\right) * \cos\left(\frac{(2y+1)v\pi}{2m}\right)$$

- Inverse DCT:

$$I(y, x) = \frac{2}{\sqrt{nm}} \sum_{v=0}^{m-1} \sum_{u=0}^{n-1} F(v, u) C(u)C(v) \cos\left(\frac{(2x+1)u\pi}{2n}\right) * \cos\left(\frac{(2y+1)v\pi}{2m}\right)$$

# Visualization of Basis Functions



# Coefficient Differentiation

## ■ $F(0,0)$

- includes the lowest frequency in both directions
- is called **DC coefficient**
- Determines fundamental color of the block

## ■ $F(0,1) \dots F(7,7)$

- are called **AC coefficients**
- Their frequency is non-zero in one or both directions

# Quantization

- Throw out bits
- Consider example:  $101101_2 = 45$  (6 bits)
  - We can truncate this string to 4 bits:  $1011_2 = 11$
  - We can truncate this string to 3 bits:  $101_2 = 5$  (original value 40) or  $110_2 = 6$  (original value 48)
- Uniform quantization is achieved by dividing DCT coefficients by  $N$  and round the result (e.g., above we used  $N=4$  or  $N=8$ )
- In JPEG – use quantization tables
  - $F_q(u,v) = F(u,v)/Q_{uv}$
  - Two quantization tables – one for luminance and one for two chrominance components

# De facto Quantization Table

Eye becomes less sensitive ↓

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

→ Eye becomes less sensitive

# Entropy Encoding

- Compress sequence of quantized DC and AC coefficients from quantization step
  - further increase compression, without loss
- Separate DC from AC components
  - DC components change slowly, thus will be encoded using difference encoding



# DC Encoding

- DC represents average intensity of a block
  - encode using difference encoding scheme
  - use 3x3 pattern of blocks
- Because difference tends to be near zero, can use less bits in the encoding
  - categorize difference into difference classes
  - send the index of the difference class, followed by bits representing the difference

# Difference Coding applied to DC Coefficients

PREDICTOR

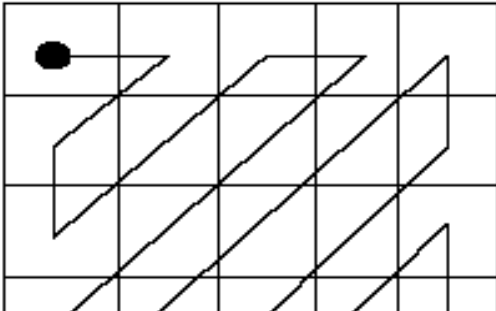
$$\text{Diff}_i = \text{DC}_i - \text{DC}_{i-1} \quad i > 0$$

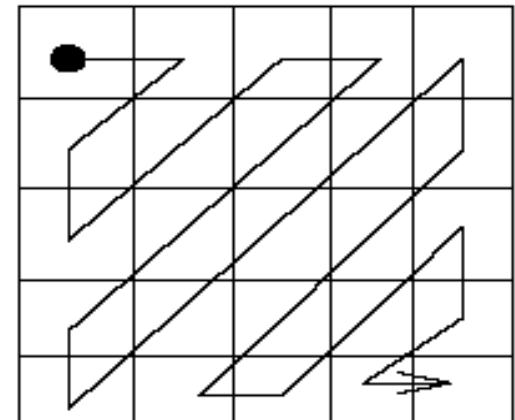
$\text{DC}_0$	$\text{DC}_1$	$\text{DC}_2$
$\text{DC}_3$	$\text{DC}_4$	$\text{DC}_5$
$\text{DC}_6$	$\text{DC}_7$	$\text{DC}_8$



$\text{DC}_0$	$\text{Diff}_1$	$\text{Diff}_2$
$\text{Diff}_3$	$\text{Diff}_4$	$\text{Diff}_5$
$\text{Diff}_6$	$\text{Diff}_7$	$\text{Diff}_8$

# AC Encoding

- Use zig-zag ordering of coefficients
    - orders frequency components from low->high
    - produce maximal series of 0s at the end
    - Ordering helps to apply efficiently entropy encoding
  - Apply Huffman coding
  - Apply RLE on AC zero values
- 
- The diagram shows a 5x5 grid representing a coefficient matrix. A black dot marks the starting point at the top-left corner (0,0). A solid line traces a zig-zag path through the grid, starting from (0,0), moving right to (4,0), then diagonally down-left to (4,4), then diagonally up-right to (0,4), then diagonally down-left to (4,3), then diagonally up-right to (0,3), then diagonally down-left to (4,2), then diagonally up-right to (0,2), then diagonally down-left to (4,1), then diagonally up-right to (0,1), then diagonally down-left to (4,0), and finally ending at (4,4). This path ensures that all coefficients are visited in a systematic order, typically used for entropy encoding.

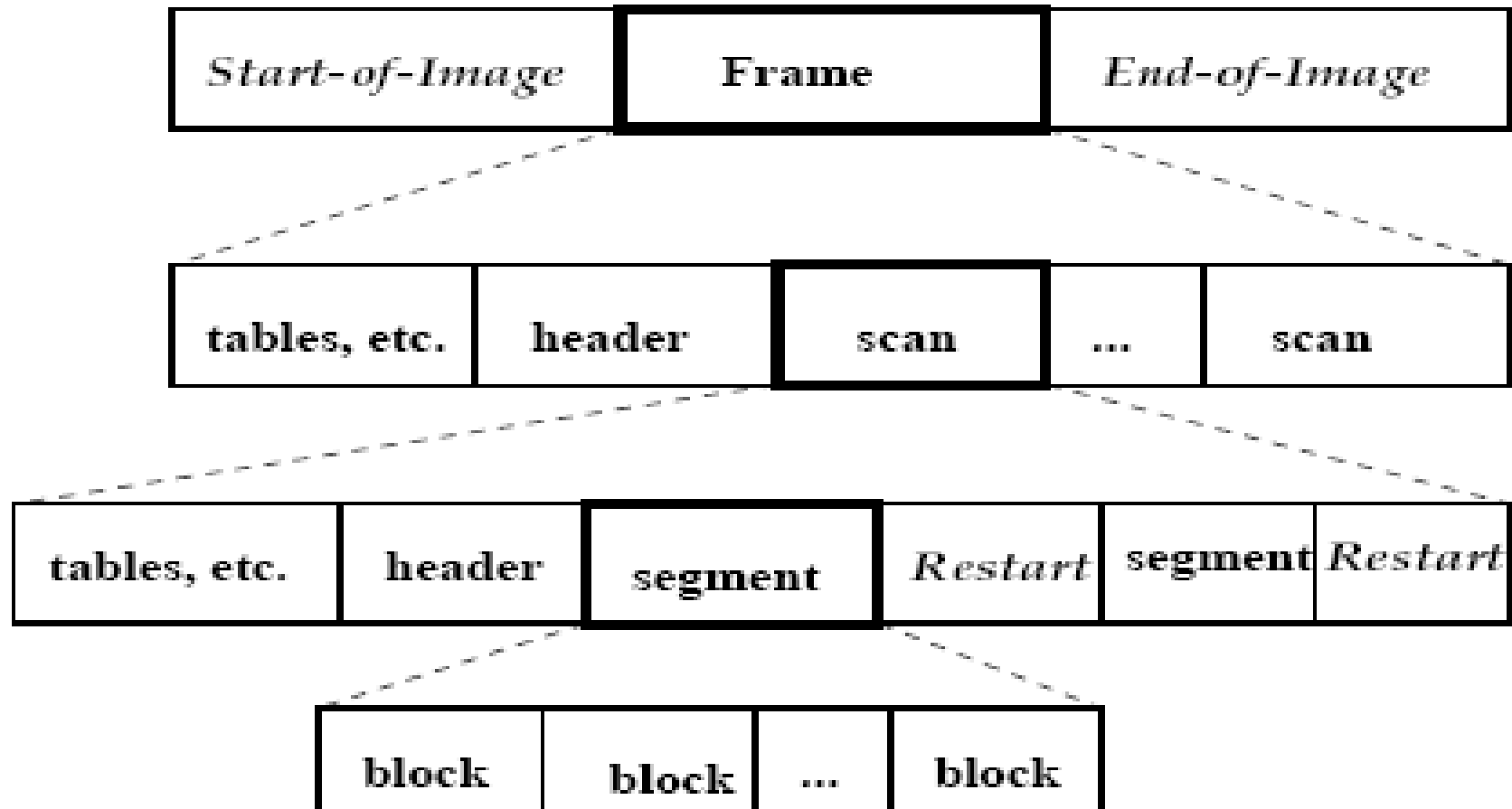




# Huffman Encoding

- Sequence of DC difference indices and values along with RLE of AC coefficients
- Apply Huffman encoding to sequence
- Attach appropriate headers
- Finally have the JPEG image!

# Interchange Format of JPEG



# Example - One Everyday Photo



**2.76M**

# Example - One Everyday Photo



**600K**



# Example - One Everyday Photo



**350K**



# Example - One Everyday Photo



**240K**

# Example - One Everyday Photo



144K

# Example - One Everyday Photo



**88K**



# Discussion

- What types of image content would JPEG work best (worst) for?
- Is image compression solved?
- What's missing from JPEG?