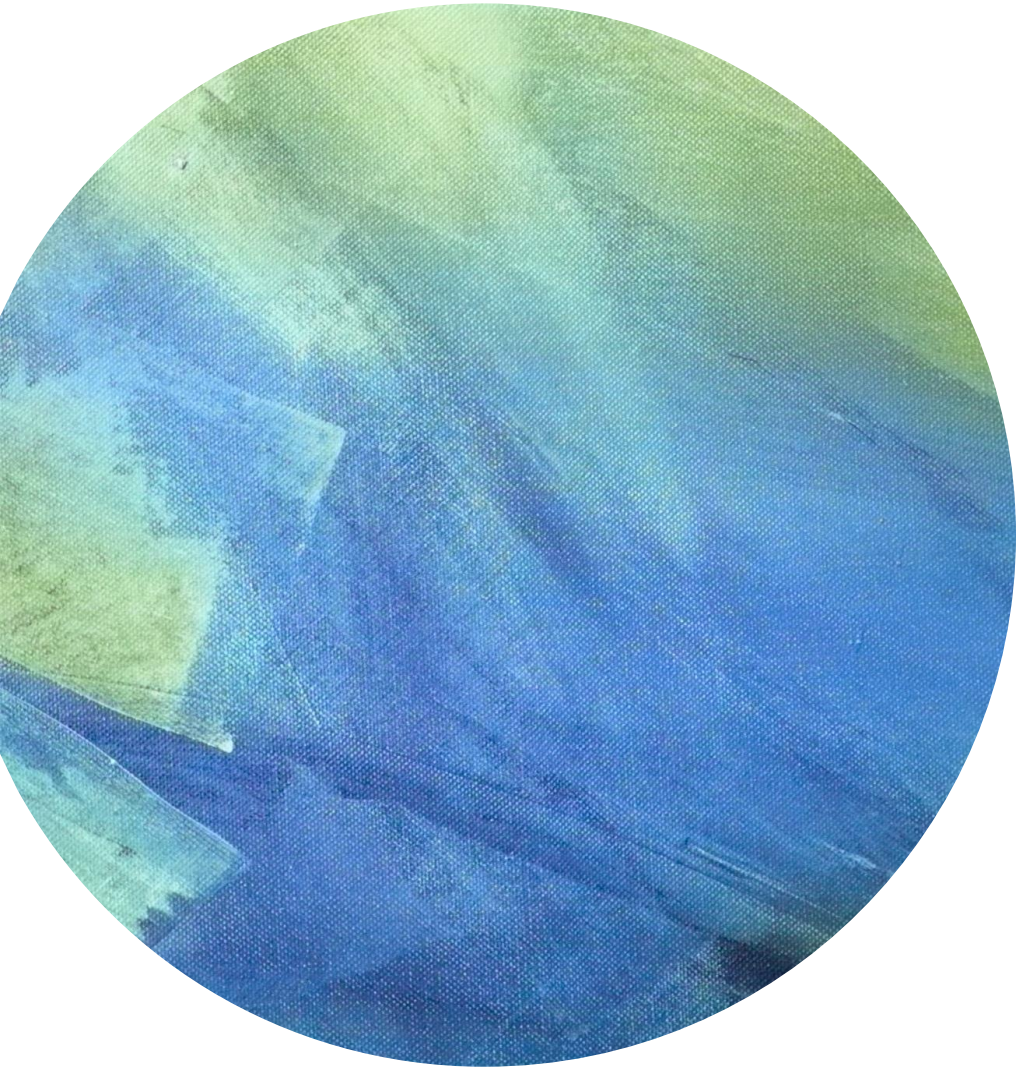
Lecture 19



Outline

Wrap-up commitments

NIZKs from pairings

Pedersen Commitments

Pedersen Commitments

- Unconditionally hiding
  - Given a commitment c, every value x is equally likely to be the value committed in c.
  - For example, given x,r, and any x’, there exists r’ such that $g^x h^r = g^{x'} h^{r'}$, in fact $r = (x-x')a^{-1} + r \bmod q$.

Pedersen Commitments

- Computationally binding
  - Suppose committer sent $g^x h^r \bmod p$ for some (x, r)
  - Now it finds $x' \neq x$ and r' such that $c = g^{x'} h^{r'}$.
  - This means that the sender ``knows'' $\log_g(h) = (x'-x)\cdot(r-r')^{-1}$.
  - This means: assuming DL is hard, the sender cannot open the commitment to a different value.

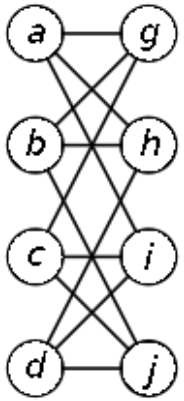
Application: Coin Tossing

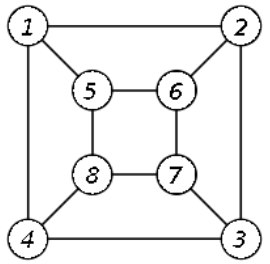
- Alice and Bob want to decide on something by tossing a coin over a phone. How to do this securely?
- Solution: Alice commits to a random bit $b_A \leftarrow \{0, 1\}$, and sends Com $(b_A; r)$ to Bob
- Bob selects a random bit $b_B \leftarrow \{0, 1\}$ and sends it to Alice
- Alice decommits b_A
- Alice and Bob output b_A xor b_B

Zero-Knowledge

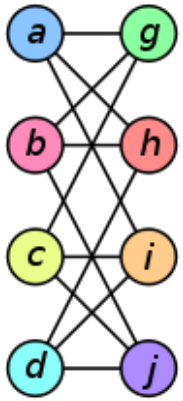
Problems in NP

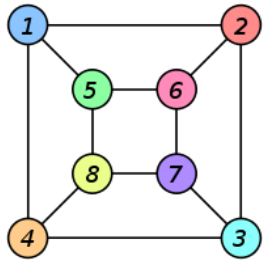
Graph Isomorphism





Graph Isomorphism

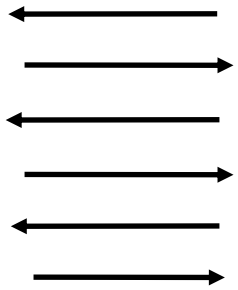




Real World

Prover

Verifier



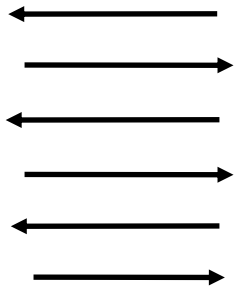
NP Statement **x**

Witness that **x** is true

Real World

Prover

Verifier



Didn't learn witness

NP Statement **x**

Witness that **x** is true

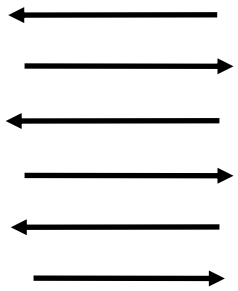
Outputs view

Real World

Prover

NP Statement **x**

Witness that **x** is true



Verifier

Didn't learn witness

Outputs view

Ideal World (Proof)

Simulator

NP Statement **x**

No witness

Real World

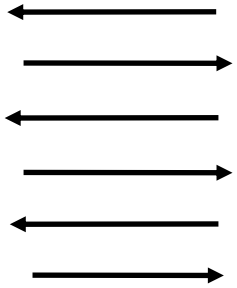
Prover

NP Statement **x**

Witness that **x** is true

Verifier

Didn't learn witness



Outputs view

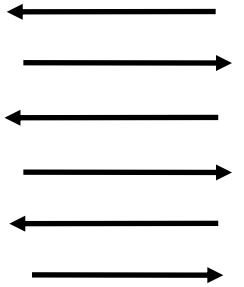
Ideal World (Proof)

Simulator

NP Statement **x**

No witness

Verifier



Real World

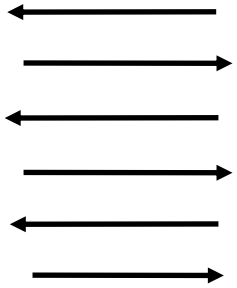
Prover

NP Statement **x**

Witness that **x** is true

Verifier

Didn't learn witness



Outputs view

Ideal World (Proof)

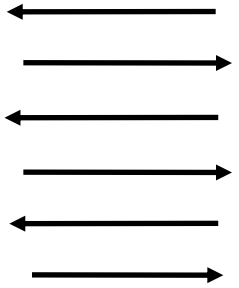
Simulator

NP Statement **x**

No witness

Verifier

Didn't learn witness



Outputs similar view

Real World

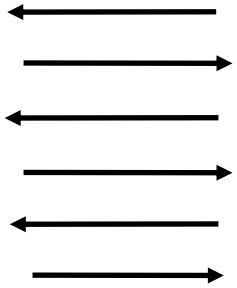
Prover

NP Statement **x**

Witness that **x** is true

Verifier

Didn't learn witness



Outputs view

Ideal World (Proof)

Simulator

NP Statement **x**

No witness

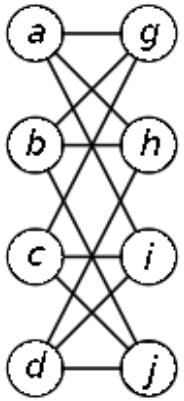
Verifier

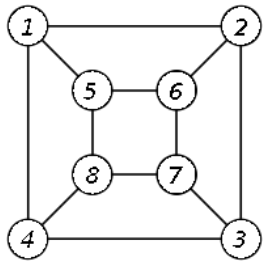
Didn't learn witness

Outputs similar view

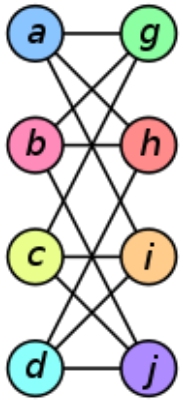
Cannot distinguish the two

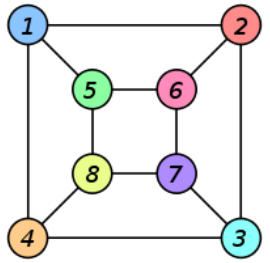
Graph Isomorphism



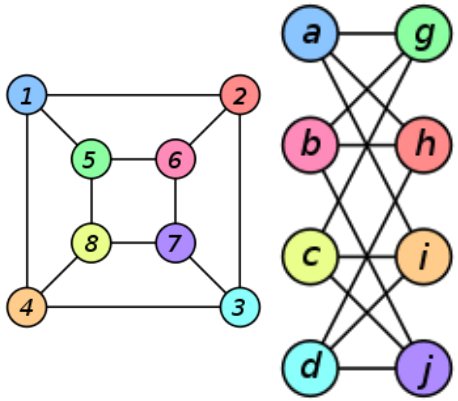


Graph Isomorphism



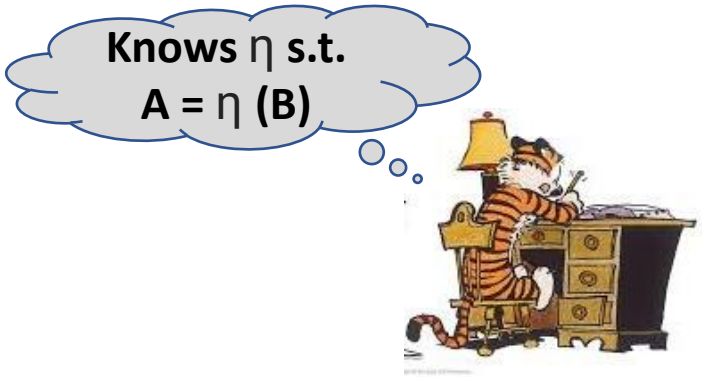


Graph Isomorphism



Prover

X = (A, B)

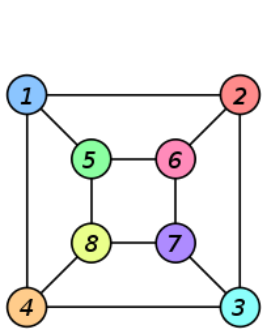


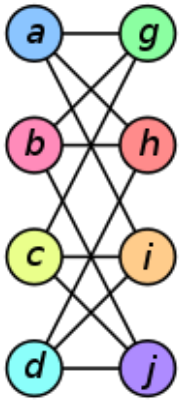
Verifier

Graph Isomorphism

Prover

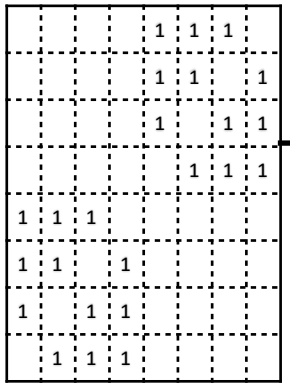
X = (A, B)



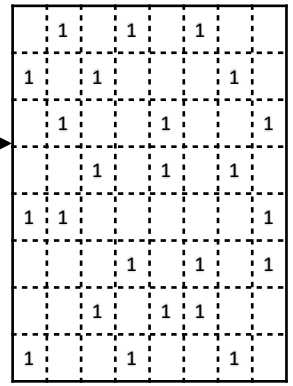


Knows η s.t.
A = η (B)

A

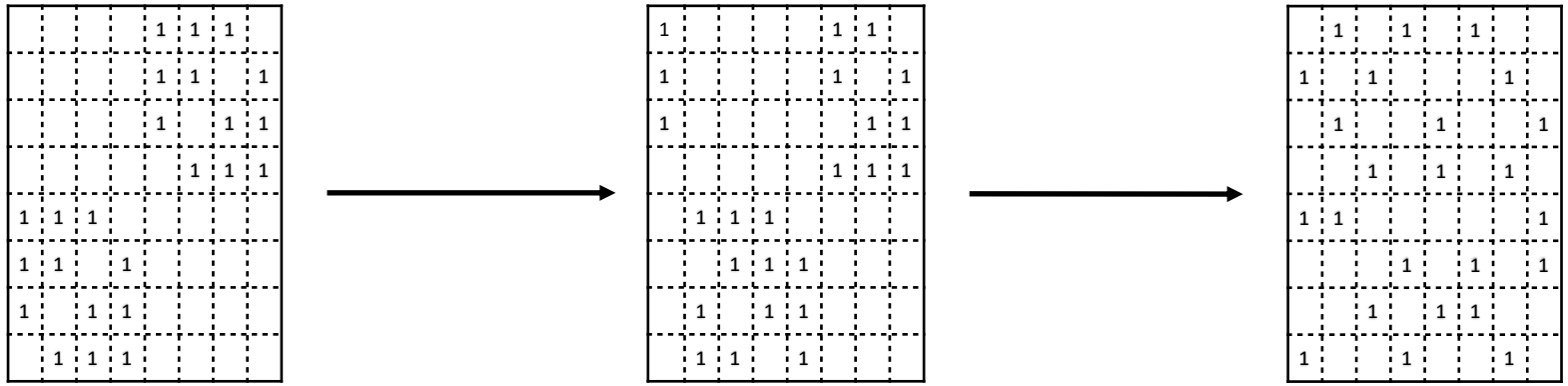


G = φ(A)



Verifier

Permuting the Graph

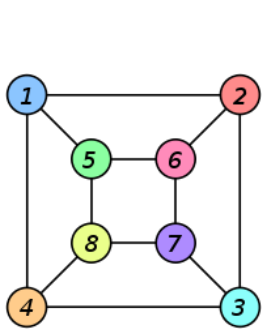


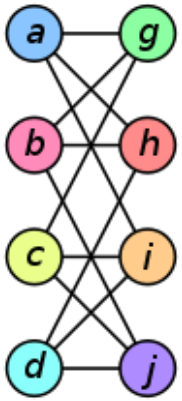
$G = \varphi(A)$

Graph Isomorphism

Prover

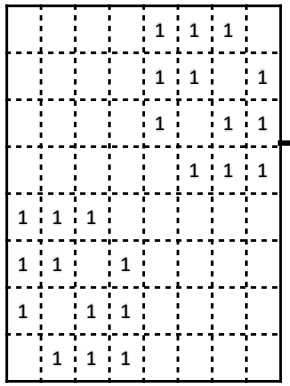
X = (A, B)



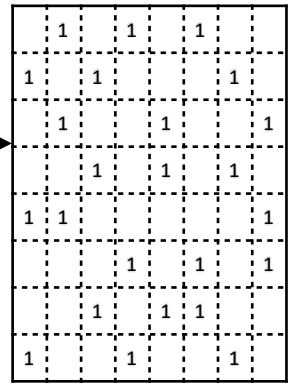


Knows η s.t.
A = η (B)

A



G = φ(A)



G

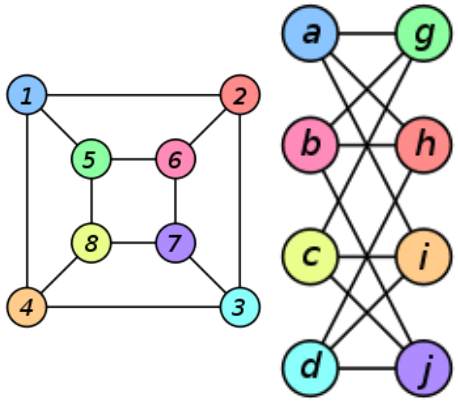
c = A or B

Verifier

Graph Isomorphism

Simulator

X = (A, B)



Verifier

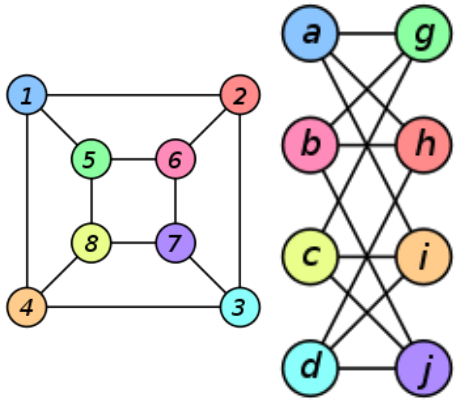
Knows c in advance

G

Graph Isomorphism

Simulator

X = (A, B)



Verifier

Knows c in advance

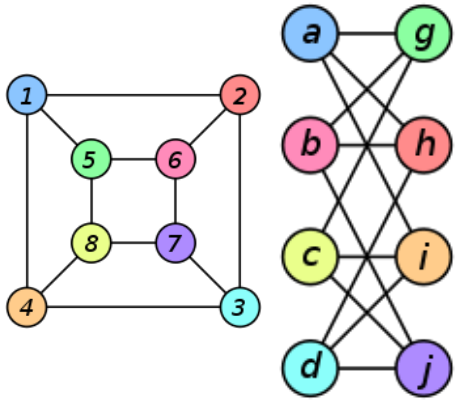
G = φ(c)

G

Graph Isomorphism

Simulator

X = (A, B)



Verifier

Knows c in advance

G = φ(c)

G

c = A or B

φ