CS/ECE 374 A ✧ Spring 2026
Midterm 1 Problem 1 Solution

For each statement below, check "Yes" if the statement is ***always*** true and check "No" otherwise. In either case, give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

(a) There exists a DFA for the following language: All palindromes with length at most 374.

☒ Yes ☐ No

Finite languages are regular; and every regular language has an accepting DFA.

(b) For every language L, if L is not regular, then its complement $\Sigma^* \setminus L$ is regular.

☐ Yes ☒ No

$L = \{x \in \Sigma^* \mid \#(\mathtt{0},x) = \#(\mathtt{1},x)\}$ is not regular, and $\Sigma^* \setminus L = \{x \in \Sigma^* \mid \#(\mathtt{0},x) \neq \#(\mathtt{1},x)\}$ is also not regular.

(c) For every language L, if L^* is context-free then L^* is regular.

☐ Yes ☒ No

$L = \{x \in \Sigma^* \mid \#(\mathtt{0},x) = \#(\mathtt{1},x)\}$ is context-free but not regular. Since, $L^* = L$, L^* is also not regular.

(d) For every context-free language L, L^* is regular.

☐ Yes ☒ No

Same example as above.

(e) If L has a fooling set of size 374, then every DFA for L requires at least 374 states.

☒ Yes ☐ No

Every *DFA* for L requires as many states as the largest fooling set, and therefore at least 374 in this case.

(f) If F is a fooling set of a language L then F contains an infinite length string.

☐ Yes ☒ No

By definition, all strings have finite length.

(g) For language $\{\mathtt{0}^a\mathtt{1}^b \mid a = b\}$, there exists a fooling set F such that F is regular

☒ Yes ☐ No

$F = 0^*$ is a fooling set of this language, and since F has a regular expression, it is regular.

(h) For language $\{\mathtt{0}^a\mathtt{1}^b \mid a = b\}$, every fooling set is regular.

☐ Yes ☒ No

Another fooling set for this language is $F = \{\mathtt{0}^{2a}\mathtt{1}^b \mid a = b\}$ which is not regular.

(i) If L_1 has a DFA with k_1 states, and L_2 has a DFA with k_2 states, then every DFA of language $L_1 \cup L_2$ has at most $(k_1 * k_2)$ states.

☐ Yes ☒ No

Construct a DFA for $L_1 \cup L_2$ with $(k_1 \times k_2)$ states by taking the product construction, and then can add more dummy states to it that are essentially unreachable.

(j) If language L is regular then language $L' = \{xy \mid x, y \in L, |x| = |y|\}$ is regular.

☐ Yes ☒ No

$L = (01)^*00$ is regular. The corresponding L' is $\{(01)^a00(01)^a00 \mid a \geq 0\}$ has a fooling set $F = (01)^*$ of infinite size, and hence is not regular.

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Midterm 1 Problem 2 Solution

For an even length string $w \in \Sigma^*$, we define $\mathsf{Compress}(w)$ to be the function that divides w into pairs of symbols and replaces each pair ab with the result of the XOR operation $a \oplus b$.

(a) ***Prove*** that if $L \subseteq \Sigma^*$ is regular, then $\textsc{Compressed}(L) := \{\mathsf{Compress}(w) : |w| \text{ is even and } w \in L\}$ is also regular.

Solution: Let $M = (Q, s, A, \delta)$ be an arbitrary DFA for L. We construct an NFA $M' = (Q', S', A', \delta')$ for $\textsc{Compressed}(L)$ as follows.

$$\begin{aligned} Q' &= Q \\ S' &= s \\ A' &= A \end{aligned}$$

$$\begin{aligned} \delta'(q, \mathtt{0}) &= \{\delta(\delta(q, \mathtt{0}), \mathtt{0}), \delta(\delta(q, \mathtt{1}), \mathtt{1})\} \quad \forall q \in Q \\ \delta'(q, \mathtt{1}) &= \{\delta(\delta(q, \mathtt{0}), \mathtt{1}), \delta(\delta(q, \mathtt{1}), \mathtt{0})\} \quad \forall q \in Q \end{aligned}$$

At every state q, to transition on reading a in M', we read each of the two combinations of two characters in M from state q whose XOR is a. ■

Rubric: 5 points: standard language-transformation rubric (scaled)

(b) ***Prove*** that if $L \subseteq \Sigma^*$ is regular, then $\textsc{UnCompressed}(L) := \{w \in \Sigma^* : |w| \text{ is even and } \mathsf{Compress}(w) \in L\}$ is also regular.

Solution: Let $M = (Q, s, A, \delta)$ be an arbitrary DFA for L. We construct a **DFA** $M' = (Q', S', A', \delta')$ for $\textsc{UnCompressed}(L)$ as follows.

$$\begin{aligned} Q' &= Q \times \{\mathtt{0}, \mathtt{1}, \varepsilon\} \\ S' &= (s, \varepsilon) \\ A' &= \{(q, \varepsilon) \mid q \in A\} \end{aligned}$$

$$\begin{aligned} \delta'((q, \varepsilon), a) &= (q, a), \qquad \forall q \in Q, a \in \{\mathtt{0}, \mathtt{1}\} \\ \delta'((q, a), b) &= (\delta(q, a \oplus b), \varepsilon) \qquad \forall q \in Q, a, b \in \{\mathtt{0}, \mathtt{1}\} \end{aligned}$$

State (q, ε) in M' indicates we are about to read a of $(a \oplus b)$ to be read in M. (q, a) state indicates we have read a and about to read b. Once we read b at (q, a) in M', we transition in M by reading $(a \oplus b)$ from q. ■

Rubric: 5 points: standard language-transformation rubric (scaled)

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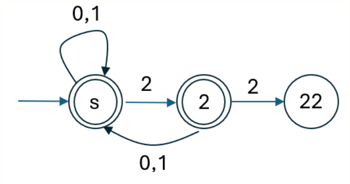
Midterm 1 Problem 3 Solution

For each of the following languages over the alphabet $\Sigma = \{0, 1, 2\}$, describe both a regular expression that matches the language and a DFA that accepts the language. You do not need to prove that your answers are correct.

(a) All strings in Σ^* that do not have 22 as a substring.

Solution:

$$(0+1)^*(2(0+1)(0+1)^*)^*(\varepsilon+2)$$



All missing transitions in the DFA go to a hidden dump state. The states are labeled as follows:

- s: The previous character is either ε or 0 or 1.
- 2: The previous character is 2
- 22: Substring 22 is encourntered, and hence is a dump state.

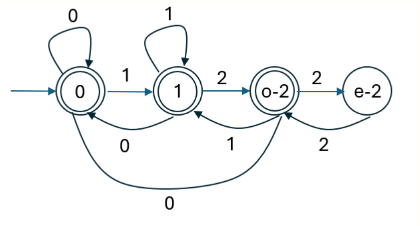
■

Rubric: 5 points = 2½ for regular expression + 2½ for DFA, standard rubrics (scaled). The state labels and explanations are not required for credit.

(b) All strings in Σ^* where each run of 2s contains the substring 22 an even number of times and is preceded by a run of 1s.

Solution: Note that a run of 2s has substring 22 even number of times if and only if the run is of odd length.

$$(0+1)^*(11^*2(22)^*(0+1)^*)^*$$



All missing transitions in the DFA go to a hidden dump state. The states are labeled with the last few symbols read:

- 0: We are either in a run of 0s or haven't read anything yet.
- 1: We are in a run of 1s.
- o-2: We are in a valid run of 2s, that which is preceeded by a run of 1s, and we have seen odd number of 2s so far within this run.
- e-2: We are in a valid run of 2s, that which is preceeded by a run of 1s, and we have seen even number of 2s so far within this run.

■

Rubric: 5 points = 2½ for regular expression + 2½ for DFA, standard rubrics (scaled). The state labels and explanations are not required for credit.

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Midterm 1 Problem 4 Solution

Prove $\#(1, w) \bmod 2 = (\mathsf{NumOdd1s}(w) - \mathsf{NumEven1s}(w)) \bmod 2$ for all strings $w \in \{0, 1\}^*$ where the functions $\mathsf{NumOdd1s}$ and $\mathsf{NumEven1s}$ are defined in the question handout.

Solution: Let w be an arbitrary string in Σ^*. Assume for all strings x shorter than w that $\#(1, x) \bmod 2 = (\mathsf{NumOdd1s}(x) - \mathsf{NumEven1s}(x)) \bmod 2$. There are three cases to consider:

- Suppose $w = \varepsilon$. Then

$$\begin{aligned}
\#(1, w) \bmod 2 &= \#(1, \varepsilon) \bmod 2 && w = \varepsilon \\
&= 0 \bmod 2 && \text{def. } \# \\
&= (0 - 0) \bmod 2 && \text{arithmetic} \\
&= (\mathsf{NumOdd1s}(\varepsilon) - \mathsf{NumEven1s}(\varepsilon)) \bmod 2 && \text{def. NO1s and NE1s} \\
&= (\mathsf{NumOdd1s}(w) - \mathsf{NumEven1s}(w)) \bmod 2 && w = \varepsilon
\end{aligned}$$

- Suppose $w = a$ for some symbol $a \in \Sigma$. Then

$$\begin{aligned}
\#(1, w) \bmod 2 &= \#(1, a) \bmod 2 && w = a \\
&= ([a = 1] + 0) \bmod 2 && \text{def. } \#\text{, arithmetic} \\
&= (\mathsf{NumOdd1s}(w) + \mathsf{NumEven1s}(w)) \bmod 2 && \text{def. NO1s and NE1s} \\
&= (\mathsf{NumOdd1s}(w) - \mathsf{NumEven1s}(w)) \bmod 2 && \text{modular arithmetic}
\end{aligned}$$

- Suppose $w = abx$ for some string x and symbols $a, b \in \Sigma$. Then

$$\begin{aligned}
\#(1, w) \bmod 2 &= \#(1, abx) \bmod 2 && w = abx \\
&= ([a = 1] + [b = 1] + \#(1, x)) \bmod 2 && \text{def. } \#\text{, arithmetic} \\
&= ([a = 1] + [b = 1] + (\#(1, x) \bmod 2)) \bmod 2 && \text{modular arithmetic} \\
&= ([a = 1] + [b = 1] + \\
&\qquad ((\mathsf{NumOdd1s}(x) - \mathsf{NumEven1s}(x)) \bmod 2)) \bmod 2 && \textbf{ind. hyp.} \\
&= ([a = 1] + \mathsf{NumOdd1s}(x) + \\
&\qquad [b = 1] - \mathsf{NumEven1s}(x)) \bmod 2 && \text{modular arithmetic} \\
&= (([a = 1] + \mathsf{NumOdd1s}(x)) - \\
&\qquad [b = 1] - \mathsf{NumEven1s}(x)) \bmod 2 && \text{modular arithmetic} \\
&= (\mathsf{NumOdd1s}(abx) - \mathsf{NumEven1s}(abx)) \bmod 2 && \text{def. NO1s and NE1s} \\
&= (\mathsf{NumOdd1s}(w) - \mathsf{NumEven1s}(w)) \bmod 2 && w = abx
\end{aligned}$$

In all three cases, we conclude that. $\#(1, w) \bmod 2 = (\mathsf{NumOdd1s}(w) - \mathsf{NumEven1s}(w)) \bmod 2$. ■

Rubric: 10 points: standard induction rubric. This solution is more detailed than necessary for full credit.

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Midterm 1 Problem 5 Solution

Let $L \subseteq \{\mathtt{0}, \mathtt{1}\}^*$ be a set of strings in which 0s and 1s appear the same number of times, and substrings 01 and 10 also appear the same number of times.

(a) Prove that L is not a regular language.

Solution: Consider the set $F = \mathtt{0}^*$.

Let x and y be distinct strings in F.

Then $x = \mathtt{0}^i$ and $y = \mathtt{0}^j$ for some integers $i \neq j$.

Let $z = \mathtt{1}^{i+1}\mathtt{0}$.

- $xz = \mathtt{0}^i\mathtt{1}^{i+1}\mathtt{0} \in L$, because 0 and 1 each appear exactly $i+1$ times, and 01 and 10 each appear exactly once.
- $yz = \mathtt{0}^j\mathtt{1}^{i+1}\mathtt{0} \notin L$, because 0 appears $j+1$ times, 1 appears $i+1$ times, and $i \neq j$.

We conclude that F is a fooling set for L.

Because F is infinite, L cannot be regular. ■

Rubric: 5 points: Standard fooling set rubric. This is not the only correct solution.

(b) Describe a context-free grammar for L.

Solution:

$$\begin{aligned} S &\to \varepsilon \mid \mathtt{0}A\mathtt{1}A\mathtt{1}A\mathtt{0} \mid \mathtt{1}A\mathtt{0}A\mathtt{0}A\mathtt{1} \\ A &\to \varepsilon \mid \mathtt{0}A\mathtt{1}A \mid \mathtt{1}A\mathtt{0}A && \{w \mid \#(\mathtt{0}, w)) = \#(\mathtt{1}, w)\} \end{aligned}$$

L is also the set of strings in which 0s and 1s appear the same number of times and (are either empty or begin and end with the same symbol). ■

Solution:

$$\begin{aligned} S &\to \varepsilon \mid \mathtt{0}A\mathtt{1}A\mathtt{1}A\mathtt{0} \mid \mathtt{1}A\mathtt{0}A\mathtt{0}A\mathtt{1} \\ A &\to \varepsilon \mid AA \mid \mathtt{0}A\mathtt{1} \mid \mathtt{1}A\mathtt{0} && \{w \mid \#(\mathtt{0}, w)) = \#(\mathtt{1}, w)\} \end{aligned}$$

■

Rubric: 5 points: Standard CFG rubric. These are not the only correct solutions.