You have 120 minutes to answer five questions.

Write your answers in the separate answer booklet.

Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check "Yes" if the statement is always true and check "No" otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

   (a) Every integer in the empty set is prime.

   (b) The language $\{0^m1^n \mid m+n \leq 374\}$ is regular.

   (c) The language $\{0^m1^n \mid m-n \leq 374\}$ is regular.

   (d) For all languages L, the language L^* is regular.

   (e) For all languages L, the language L^* is infinite.

   (f) For all languages $L \subseteq \Sigma^*$, if L can be represented by a regular expression, then $\Sigma^* \setminus L$ is recognized by a DFA.

   (g) For all languages L and L', if $L \cap L' = \varnothing$ and L' is not regular, then L is regular.

   exists

   (h) Every regular language is recognized by a DFA with at least 374 accepting states.

   (i) Every regular language is recognized by an NFA with at most 374 accepting states.

   (j) Every context-free language has an infinite fooling set.

2. The *parity* of a bit-string w is 0 if w has an even number of 1s, and 1 if w has an odd number of 1s. For example:

$$parity(\varepsilon) = 0 \qquad parity(0010100) = 0 \qquad parity(00101110100) = 1$$

   (a) Give a *self-contained*, formal, recursive definition of the *parity* function. (In particular, do ***not*** refer to # or other functions defined in class.)

   (b) ***Prove*** that for every regular language L, the language $\textsc{OddParity}(L) := \{w \in L \mid parity(w) = 1\}$ is also regular.

   (c) ***Prove*** that for every regular language L, the language $\textsc{AddParity}(L) := \{parity(w) \cdot w \mid w \in L\}$ is also regular.

give DFA, reg. ex, or NFA

3. For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either ~~***prove***~~ that the language is regular or ***prove*** that the language is not regular. Both of these languages contain the string 00110100000110100.

   (a) $\{0^n w 0^n \mid w \in \Sigma^+ \text{ and } n > 0\}$

   (b) $\{w 0^n w \mid w \in \Sigma^+ \text{ and } n > 0\}$

   [Hint: Exactly one of these two languages is regular.]

4. For any string $w \in \{0, 1\}^*$, let $\mathsf{take2skip2}(w)$ denote the subsequence of w containing symbols at positions $1, 2, 5, 6, 9, 10, \ldots 4i+1, 4i+2, \ldots$. In other words, $\mathsf{take2skip2}(w)$ takes the first two symbols of w, skips the next two, takes the next two, skips the next two, and so on.For example:

$$\mathsf{take2skip2}(\underline{1}) = 1$$
$$\mathsf{take2skip2}(\underline{01}0) = 01$$
$$\mathsf{take2skip2}(\underline{01}00\underline{11}11\underline{00}01\underline{1}) = 0111001$$

   Let L be an arbitrary regular language over $\{0, 1\}$.

   (a) ***Prove*** that the language $\{w \in \Sigma^* \mid \mathsf{take2skip2}(w) \in L\}$ is regular.

   (b) ***Prove*** that the language $\{\mathsf{take2skip2}(w) \mid w \in L\}$ is regular.

5. For each of the following languages L over the alphabet $\{0, 1\}$, describe a DFA that accepts L **and** give a regular expression that represents L. You do ***not*** need to prove that your answers are correct.

   (a) All strings in which every run of 1s has even length and every run of 0s has odd length. (Recall that a *run* is a maximal substring in which all symbols are equal.)

   (b) All strings in 0^*10^* whose length is a multiple of 3.

Midterm 1 : Mon Feb 23 7pm - 9pm
Conflict : Tue Feb 24 TBA

CS/ECE 374 A ✧ Spring 2026

Midterm 1 Practice 1

February 19, 2026

Name:	Emily Fox
NetID:	ekfox

- ***Don't panic!***
- You have 120 minutes to answer five questions. The questions are described in more detail in a separate handout.
- If you brought anything except your writing implements, your **hand-written** double-sided 8½" × 11" cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
- Please clearly print your name and your NetID in the boxes above.
- Please also print your name at the top of every page of the answer booklet, except this cover page. We want to make sure that if a staple falls out, we can reassemble your answer booklet. (It doesn't happen often, but it does happen.)
- Proofs or other justifications are required for full credit if and only if we explicitly ask for them, using the word ***prove*** or ***justify*** in bold italics.
- **Do not write outside the black boxes on each page**. These indicate the area of the page that our scanners will actually scan. If the scanner can't see your work, we can't grade it.
- If you run out of space for an answer, please use the overflow/scratch pages at the back of the answer booklet, but **please clearly indicate where we should look**. If we can't find your work, we can't grade it.
- **Only work that is written into the stapled answer booklet will be graded**. In particular, you are welcome to detach scratch pages from the answer booklet, but any work on those detached pages will not be graded. Please let us know if you detach a page accidentally. We will provide additional scratch paper on request.
- Please return ***all*** paper with your answer booklet: your question sheet, your cheat sheet, and all scratch paper. **Please put all loose paper *inside* your answer booklet.**

Is between
between is and between
or between between and is
or between and and or
or between or and between
or between between and and
or between or and and
or between or and or?

CS/ECE 374 A ✧ Spring 2026 **Midterm 1 Practice 1 Problem 1**	Name: Emily Fox

For each statement below, check "Yes" if the statement is always true and check "No" otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

(a) Every integer in the empty set is prime.

[X] Yes [ ] No — Universally quantified over $\emptyset$.

(b) The language $\{0^m 1^n \mid m + n \leq 374\}$ is regular.

[X] Yes [ ] No — finite

(c) The language $\{0^m 1^n \mid m - n \leq 374\}$ is regular.

[ ] Yes [X] No — $F = (0^{375})^*$

(d) For all languages L, the language L^* is regular.

[ ] Yes [X] No — $L = \{0^n 1^n \mid n \geq 0\}$

(e) For all languages L, the language L^* is infinite.

[ ] Yes [X] No — $L = \emptyset$ ~~or $L = \{\varepsilon\}$~~

(f) For all languages $L \subseteq \Sigma^*$, if L can be represented by a regular expression, then $\Sigma^* \setminus L$ is recognized by a DFA.

[X] Yes [ ] No — regularity closed under complement

(g) For all languages L and L', if $L \cap L' = \emptyset$ and L' is not regular, then L is regular.

[ ] Yes [X] No — $L' = \{0^n 1^n \mid n \geq 1\}$ + $L = \{1^n 0^n \mid n \geq 1\}$

(h) Every regular language is recognized by a DFA with at least 374 accepting states.

[X] Yes [ ] No — add 374 accepting states to any DFA

(i) Every regular language is recognized by an NFA with at most 374 accepting states.

[X] Yes [ ] No — Add edges to single new accepting state.

(j) Every context-free language has an infinite fooling set.

[ ] Yes [X] No — $L = \emptyset$

CS/ECE 374 A ✧ Spring 2026 Midterm 1 Practice 1 Problem 2	Name: Emily Fox

The *parity* of a bit-string w is 0 if w has an even number of 1s, and 1 if w has an odd number of 1s.

(a) Give a *self-contained*, formal, recursive definition of the *parity* function. (In particular, do **not** refer to # or other functions defined in class.)

$L \cap \{w \mid parity(w) = 1\}$

(b) ***Prove*** that for every regular language L, the language $\textsc{OddParity}(L) := \{w \in L \mid parity(w) = 1\}$ is also regular.

or or

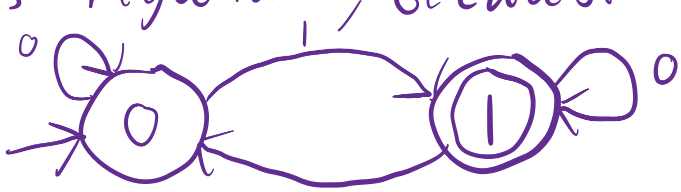
(c) ***Prove*** that for every regular language L, the language $\textsc{AddParity}(L) := \{parity(w) \cdot w \mid w \in L\}$ is also regular.

Be told ~~Guess~~ final parity + simulate while computing it.

(a) $$parity(w) = \begin{cases} 0 & \text{if } w = \varepsilon \\ (1 - parity(x)) & \text{if } w = 1x \\ parity(x) & \text{if } w = 0x \end{cases}$$

(b) $Odd := \{w \in \{0,1\}^* \mid parity(w) = 1\}$

Odd is regular, because this DFA accepts it.



Therefore $OddParity(L) = L \cap Odd$ is regular.

(c) $EvenParity(L) := \{w \in L \mid parity(w) = 0\}$.

$EvenParity(L) = L \setminus OddParity(L)$ is regular.

So, $AddParity(L) = \{0\} \bullet EvenParity(L) \cup \{1\} \bullet OddParity(L)$

is regular.

CS/ECE 374 A ✧ Spring 2026 Midterm 1 Practice 1 Problem 3	Name: Emily Fox

For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either ***prove*** that the language is regular or ***prove*** that the language is not regular. Both of these languages contain the string 00110100000110100.

(a) $\{0^n w 0^n \mid w \in \Sigma^+ \text{ and } n > 0\}$ ☺

(b) $\{w 0^n w \mid w \in \Sigma^+ \text{ and } n > 0\}$

(a) $0(0+1)(0+1)^*0$ regular

(b) not regular

Let $F = 1 1^* 0$.

Let $x, y \in F$ be distinct.

$x = 1^i 0$ & $1^j 0$ for $i > 0$, $j > 0$, $i \neq j$

Let $z = 1^i$

$xz = 1^i 0 1^i \in L$, $yz = 1^j 0 1^i \notin L$.

F is an infinite fooling set of the language.

CS/ECE 374 A ✧ Spring 2026 Midterm 1 Practice 1 Problem 4	Name: Emily Fox

For any string $w \in \{0, 1\}^*$, let take2skip2(w) denote the subsequence of w containing symbols at positions $1, 2, 5, 6, 9, 10, \ldots 4i+1, 4i+2, \ldots$. In other words, take2skip2(w) takes the first two symbols of w, skips the next two, takes the next two, skips the next two, and so on. Let L be an arbitrary regular language over $\{0, 1\}$.

(a) ***Prove*** that the language $\{w \in \Sigma^* \mid \text{take2skip2}(w) \in L\}$ is regular.

(b) ***Prove*** that the language $\{\text{take2skip2}(w) \mid w \in L\}$ is regular.

(a) Let $M = (Q, \delta, s, A)$ be an arbitrary DFA accepting L.

Build an NFA $M' = (Q', \delta', s', A')$ for the new language. Want to remember where we are in taking 2 + skipping 2.

$Q' = Q \times \{0, 1, 2, 3\}$ ← how far in pattern

$s' = (s, 0)$, $A' = A \times \{0, 1, 2, 3\}$

$a \in \Sigma$

$\delta'((q, 0), a) = \{(\delta(q, a), 1)\}$ $\forall q \in Q$

$\delta'((q, 1), a) = \{(\delta(q, a), 2)\}$ $\forall q \in Q$

$\delta'((q, 2, a) = \{(q, 3)\}$ $\forall q \in Q$

$\delta'((q, 3), a) = \{(q, 0)\}$ $\forall q \in Q$

(b) on page 6

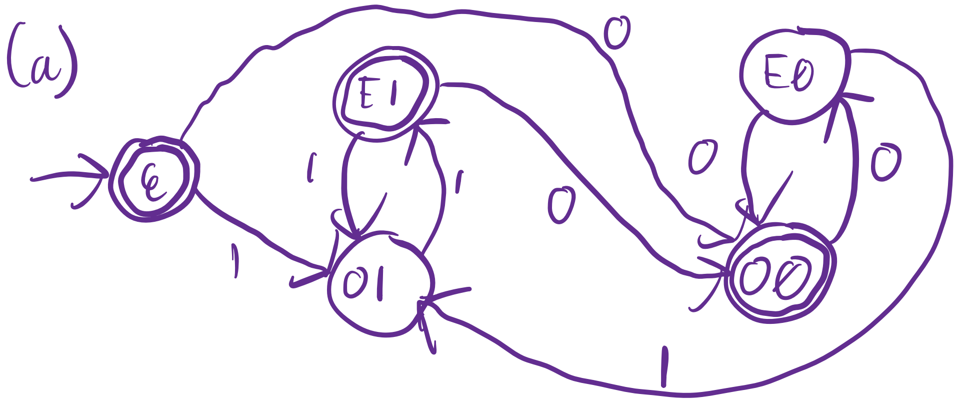
CS/ECE 374 A ✧ Spring 2026 **Midterm 1 Practice 1 Problem 5**	Name: Emily Fox

For each of the following languages L over the alphabet $\Sigma = \{0, 1\}$, describe a DFA that accepts L **and** give a regular expression that represents L. You do ***not*** need to prove that your answers are correct.

(a) All strings in which every run of 1s has even length and every run of 0s has odd length.

(b) All strings in 0^*10^* whose length is a multiple of 3.

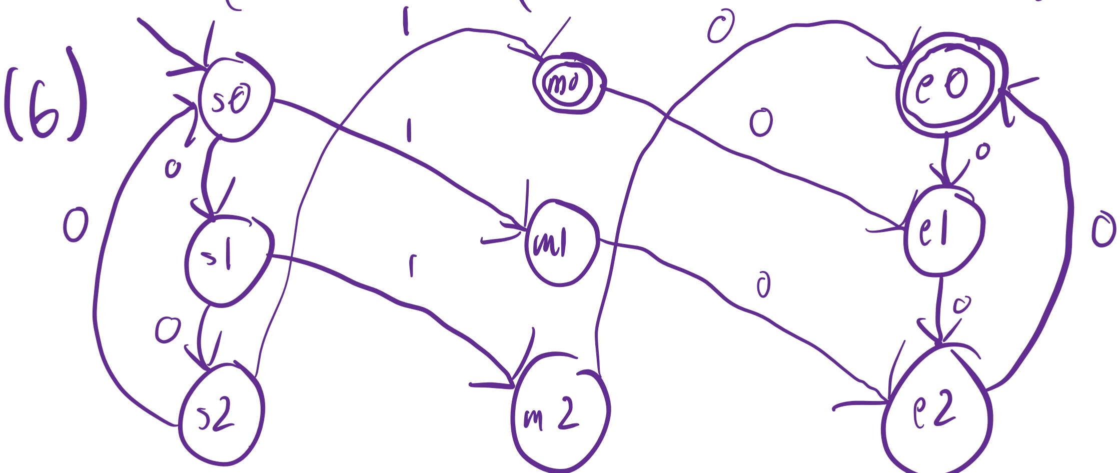
(a)



All missing transitions go to a dump state.

$$(\varepsilon + 0(00)^*)(11(11)^*0(00)^*)^*(\varepsilon + 11(11)^*)$$

(b)



All missing transitions go to a dump state.

$$(000)^*100(000)^* + 0(000)^*10(000)^* + 00(000)^*1(000)^*$$

$$\equiv (000)^*(100 + 010 + 001)(000)^*$$

don't need to optimize unless improves clarity

(scratch paper)

(6) Let $M = (Q, \delta, s, A)$ be a DFA for C. Build an NFA with ε-transitions $M' = (Q', \delta', s', A')$. Will track # characters taken or skipped.

$Q' = Q = \{0, 1, 2, 3\}$

$s' = (s, 0)$, $A' = A \times \{0, 1, 2, 3\}$

$\delta' = ((q, 0), a) = \{(\delta(q, a), 1\}$ $\forall q \in Q, a \in \Sigma$

$\delta'((q, 0), \varepsilon) = \emptyset$ $\forall q \in Q$

$\delta'((q, 1), a) = \{(\delta(q, a), 2\}$ $\forall q \in Q, a \in \Sigma$

$\delta'((q, 1), \varepsilon) = \emptyset$ $\forall q \in Q$

$\delta'((q, 2), a) = \emptyset$ $\forall q \in Q, a \in \Sigma$

$\delta'((q, 2), \varepsilon) = \{(\delta(q, 0), 3), \delta(q, 1), 3)\}$ $\forall q \in Q$

$\delta'((q, 3), a) = \emptyset$ $\forall q \in Q, a \in \Sigma$

$\delta'((q, 3), \varepsilon) = \{(\delta(q, 0), 0), \delta(q, 1), 0)\}$ $\forall q \in Q$

(scratch paper)

(scratch paper)