1. A ***basic arithmetic expression*** is a string over the alphabet {1, +, ×, (,)} that represents a properly bracketed and unambiguoous arithmetic expression. Formally, a basic arithmetic expression is any string generated by the following context-free grammar (with starting non-terminal A):

$$\begin{aligned} A &\to S \mid P && \text{[Arithmetic expression]} \\ S &\to \texttt{1} \mid \texttt{(}P\texttt{)} \mid S\texttt{+}S && \text{[Sum]} \\ P &\to \texttt{1} \mid \texttt{(}S\texttt{)} \mid P\texttt{×}P && \text{[Product]} \end{aligned}$$

Every basic arithmetic expression has a positive integer *value*, which can be computed by interpreting + as addition and × as multiplication, exactly as you would expect. Almost every integer is the value of more than one basic arithmetic expression. For example, all of the following basic arithmetic expression have value 14:

```
1+1+1+1+1+1+1+1+1+1+1+1+1+1
((1+1)×(1+1+1+1+1))+((1+1)×(1+1))
(1+1)×(1+1+1+1+1+1+1)
(1+1)×(((1+1+1)×(1+1))+1)
```

Describe and analyze an algorithm to compute, given an integer n as input, the minimum number of 1s in a basic arithmetic expression whose value is n. The number of parentheses doesn't matter, just the number of 1s. For example, when $n = 14$, your algorithm should return the integer 8, which is the number of 1s in the final expression above. The running time of your algorithm should be polynomial in n.

Harder problem to think about later:

2. Suppose you are given a sequence of integers separated by + and − signs; for example:

$$1 + 3 - 2 - 5 + 1 - 6 + 7$$

Both addition and subtraction are left-associative, so this expression evaluates to −1. You can change the expression's value by inserting parentheses in different places. For example:

$$\begin{aligned} 1 + 3 - 2 - 5 + 1 - (6 + 7) &= -15 \\ (1 + 3 - (2 - 5)) + (1 - 6) + 7 &= 9 \\ (1 + (3 - 2)) - (5 + 1) - (6 + 7) &= -17 \end{aligned}$$

Describe and analyze an algorithm to compute the *largest* possible value the expression can take by adding parentheses. Parentheses must be used only to group additions and subtractions; in particular, do not use them to create implicit multiplication as in $1 + 3(-2)(-5) + 1 - 6 + 7 = 33$.