Amy Nancato, the founding director of the new Parisa Tabriz School of Computing and Data Science, has commissioned a series of snow ramps on the south slope of the Orchard Downs sledding hill[1] and challenged Banjin Nerfherder, head of the Department of Electrical and Computer Engineering, to a sledding contest. Banjin and Amy will both sled down the hill, each trying to maximize their air time. The winner gets to expand their unit into Siebel Center, the ECE Building, *and* Campus Instructional Facility; the loser has to move their entire unit under the Boneyard bridge behind Everitt Lab.

Whenever Amy or Banjin reaches a ramp *while on the ground*, they can either use that ramp to jump through the air, possibly flying over one or more ramps, or sled past that ramp and stay on the ground. Obviously, if someone flies over a ramp, they cannot use that ramp to extend their jump.

1. Suppose you are given a pair of arrays as input:

   - A sorted array $Ramp[1..n]$, where $Ramp[i]$ is the distance from the top of the hill to the ith ramp.
   - An unsorted array $Length[1..n]$, where $Length[i]$ is the distance that any sledder who takes the ith ramp will travel through the air.

   Describe and analyze an algorithm to determine the maximum *total* distance that Banjin or Amy can travel through the air.

2. Uh-oh. The university lawyers heard about Amy and Banjin's little bet and immediately objected. To protect the university from both lawsuits and sky-rocketing insurance rates, they impose an upper bound on the number of jumps that either sledder can take.

   Describe and analyze an algorithm to determine the maximum total distance that Amy or Banjin can spend in the air *with at most k jumps*, given the arrays $Ramp[1..n]$ and $Length[1..n]$ and the integer k as input.

Harder problem to think about later:

3. When the lawyers realized that imposing their restriction didn't immediately shut down the contest, they added yet another restriction: No ramp may be used more than once! Disgusted by all the legal interference, Banjin and Amy give up on their bet and decide to cooperate to put on a good show for the spectators.

   Describe and analyze an algorithm to determine the maximum total distance that Amy and Banjin can spend in the air, each taking at most k jumps (so at most $2k$ jumps total), and with each ramp used at most once. As in problem 2, your input consists of the arrays $Ramp[1..n]$ and $Length[1..n]$ and the integer k.

[1]The north slope is faster, but too short for an interesting contest.