Let L be an arbitrary regular language over the alphabet $\Sigma = \{0, 1\}$. Prove that the following languages are also regular.

1. $\textsc{Superstrings}(L) := \{xyz \mid y \in L \text{ and } x, z \in \Sigma^*\}$. This language contains all superstrings of strings in L. For example:

   $$\textsc{Superstrings}(\{10010\}) = \{10010,\ 01010010,\ 1001011,\ 10010010010,\ \cdots\}$$

   [Hint: This is much easier than it looks.]

2. $\textsc{Substrings}(L) := \{y \mid x, y, z \in \Sigma^* \text{ and } xyz \in L\}$. This language contains all substrings of strings in L. For example:

   $$\textsc{Substrings}(\{10010\}) = \{\varepsilon,\ 0,\ 1,\ 00,\ 01,\ 10,\ 001,\ 010,\ 100,\ 0010,\ 1001,\ 10010\}$$

3. $\textsc{Cycle}(L) := \{xy \mid x, y \in \Sigma^* \text{ and } yx \in L\}$. This language contains all strings that can be obtained by splitting a string in L into a prefix and a suffix and concatenating them in the wrong order. For example:

   $$\textsc{Cycle}(\{\texttt{OOK!},\ \texttt{OOKOOK}\}) = \{\texttt{OOK!},\ \texttt{OK!O},\ \texttt{K!OO},\ \texttt{!OOK},\ \texttt{OOKOOK},\ \texttt{OKOOKO},\ \texttt{KOOKOO}\}$$

Work on these later.

5. $\textsc{Subsequences}(L)$ is the set of all *subsequences* of strings in L. A subsequence of a string w is the result of deleting zero or more symbols from w, leaving the remaining symbols in order. For example:

   $$\textsc{Subsequences}(\{10010\}) =$$
   $$\{\varepsilon,\ 0,\ 1,\ 00,\ 01,\ 10,\ 11,\ 000,\ 001,\ 010,\ 100,\ 101,\ 110,\ 0010,\ 1000,\ 1001, 10010\}$$

6. $\textsc{FlipOdds}(L) := \{\mathit{flipOdds}(w) \mid w \in L\}$, where the function *flipOdds* inverts every other bit in w, starting with the first bit. For example:

   $$\mathit{flipOdds}(0000111101010100) = 1010010111111110$$

7. $\textsc{UnflipOdd1s}(L) := \{w \in \Sigma^* \mid \mathit{flipOdd1s}(w) \in L\}$, where the function *flipOdd1s* inverts every other 1 bit of its input string, starting with the first 1. For example:

   $$\mathit{flipOdd1s}(0000111100101010) = 0000101000010000$$

8. $\textsc{FlipOdd1s}(L) := \{\mathit{flipOdd1s}(w) \mid w \in L\}$, where the function *flipOdd1s* is defined in the previous problem.

9. $\textsc{Stutter}(L) = \{\mathit{stutter}(w) \mid w \in L\}$, where the function *stutter* duplicates every symbol in the input string:

   $$\mathit{stutter}(w) := \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ aa \bullet \mathit{stutter}(x) & \text{if } w = ax \end{cases}$$

   For example, $\mathit{stutter}(1001) = 11000011$.

10. $\textsc{Unstutter}(L) = \{w \mid \mathit{stutter}(w) \in L\}$, where the function *stutter* is defined in the previous problem.