You saw the following context-free grammars in class on Thursday; in each example, the grammar itself is on the left; the explanation for each non-terminal is on the right.

- Properly nested strings of parentheses.

$$S \to \varepsilon \mid S(S) \qquad \text{properly nested parentheses}$$

  Here is a different grammar for the same language:

$$S \to \varepsilon \mid (S) \mid SS \qquad \text{properly nested parentheses}$$

- $\{0^m 1^n \mid m \neq n\}$. This is the set of all binary strings composed of some number of 0s followed by a *different* number of 1s.

$$\begin{aligned} S &\to A \mid B & \{0^m 1^n \mid m \neq n\} \\ A &\to 0A \mid 0C & \{0^m 1^n \mid m > n\} \\ B &\to B1 \mid C1 & \{0^m 1^n \mid m < n\} \\ C &\to \varepsilon \mid 0C1 & \{0^m 1^n \mid m = n\} \end{aligned}$$

Give context-free grammars for each of the following languages over the alphabet $\Sigma = \{0, 1\}$. For each grammar, describe the language for each non-terminal, either in English or using mathematical notation, as in the examples above. We probably won't finish all of these during the lab session.

1. All palindromes in Σ^*

2. All palindromes in Σ^* that contain an even number of 1s

3. All palindromes in Σ^* that end with 1

4. All palindromes in Σ^* whose length is divisible by 3

5. All palindromes in Σ^* that do not contain the substring 00

Harder problems to work on later:

6. $\{\mathtt{0}^{2n}\mathtt{1}^n \mid n \geq 0\}$

7. $\{\mathtt{0}^m\mathtt{1}^n \mid m \neq 2n\}$

   [Hint: If $m \neq 2n$, then either $m < 2n$ or $m > 2n$. Extend the previous grammar, but pay attention to parity. This language contains the string `01`*.]*

8. $\{\mathtt{0},\mathtt{1}\}^* \setminus \{\mathtt{0}^{2n}\mathtt{1}^n \mid n \geq 0\}$

   [Hint: Extend the previous grammar. What's missing?]

9. $\left\{w \in \{\mathtt{0},\mathtt{1}\}^* \;\middle|\; \#(\mathtt{0},w) = 2 \cdot \#(\mathtt{1},w)\right\}$ — Binary strings where the number of `0`s is exactly twice the number of `1`s.

*10. $\{\mathtt{0},\mathtt{1}\}^* \setminus \{ww \mid w \in \{\mathtt{0},\mathtt{1}\}^*\}$.

   [Anti-hint: The language $\{ww \mid w \in \{\mathtt{0},\mathtt{1}\}^*\}$ is ***not*** *context-free. Thus, the complement of a context-free language is not necessarily context-free!]*