Describe deterministic finite-state automata that accept each of the following languages over the alphabet $\Sigma = \{0, 1\}$. You may find it easier to describe these DFAs formally than to draw pictures.

Either drawings or formal descriptions are acceptable, as long as the states Q, the start state s, the accept states A, and the transition function δ are all clear. Try to keep the number of states small.

1. All strings in which the number of 0s is even **and** the number of 1s is *not* divisible by 3.
2. All strings in which the number of 0s is even **or** the number of 1s is *not* divisible by 3.
3. All strings that are **both** the binary representation of an integer divisible by 3 **and** the ternary (base-3) representation of an integer divisible by 4.

   For example, the string 1100 is an element of this language, because it represents $2^3 + 2^2 = 12$ in binary and $3^3 + 3^2 = 36$ in ternary.

Harder problems to think about later:

4. All strings in which the subsequence 0101 appears an even number of times.

*5. All strings w such that $\binom{|w|}{2} \bmod 6 = 4$.
[Hint: Maintain both $\binom{|w|}{2} \bmod 6$ *and* $|w| \bmod 6$.*]*
[Hint: $\binom{n+1}{2} = \binom{n}{2} + n$.*]*

★6. All strings w such that $F_{\#(10,w)} \bmod 10 = 4$, where $\#(10, w)$ denotes the number of times 10 appears as a substring of w, and F_n is the nth Fibonacci number:

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{otherwise} \end{cases}$$