## CS/ECE 374 A ✧ Fall 2025

# Practice Final Exam 3

**May 10, 2026**

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| Name: | |
| NetID: | |

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- ***Don't panic!***
- You have 180 minutes to answer seven questions. The questions are described in more detail in a separate handout.
- If you brought anything except your writing implements, your two **hand-written** double-sided 8½" × 11" cheat sheets, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
- Please clearly print your name and your NetID in the boxes above.
- Please also print your name at the top of every page of the answer booklet, except this cover page. We want to make sure that if a staple falls out, we can reassemble your answer booklet. (It doesn't happen often, but it does happen.)
- Greedy algorithms require formal proofs of correctness to receive any credit, even if they are correct. Otherwise, proofs or other justifications beyond items listed in the standard rubrics are required for full credit if and only if we explicitly ask for them, using the word ***prove*** or ***justify*** in bold italics.

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- **Please do not write outside the black boxes on each page.** These indicate the area of the page that our scanners can actually scan. If the scanner can't see your work, we can't grade it.
- If you run out of space for an answer, please use the overflow/scratch pages at the back of the answer booklet, but **please clearly indicate where we should look**. If we can't find your work, we can't grade it.
- **Only work that is written into the stapled answer booklet will be graded**. In particular, you are welcome to detach scratch pages from the answer booklet, but any work on those detached pages will not be graded. We will provide additional scratch paper on request, but any work on that scratch paper will not be graded.
- Please return ***all*** paper with your answer booklet: your question sheet, your cheat sheet, and all scratch paper. **Please put all loose paper *inside* your answer booklet.**
- Breathe in. Breathe out. You've got this.

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| CS/ECE 374 A ✧ Fall 2025<br>Practice Final 1 Problem 1 | Name: |
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(a) Which of the following statements are *always* true?

- The problem of deciding whether $M$ accepts $x$ for a given Turing machine $M$ and a given string $x$ is decidable.

  Yes No

- The problem of deciding whether $M$ accepts $x$ within $2^{|x|}$ steps for a given Turing machine $M$ and a given string $x$ is decidable.

  Yes No

- If $P = NP$, then all decidable problems are solvable in polynomial time.

  Yes No

- If $L$ is undecidable, then $\overline{L}$ is decidable.

  Yes No

- If $L$ is not context-free, then it is undecidable.

  Yes No

(b) Suppose there is a ***polynomial-time*** reduction from some language $A$ over the alphabet $\{0, 1\}$ to some other language $B$ over the alphabet $\{0, 1\}$. Which of the following statements are ***always*** true, assuming P $\neq$ NP?

- $A$ is a subset of $B$.

  Yes No

- If $B \in P$, then $A \in P$.

  Yes No

- If $B$ is NP-hard, then $A$ is NP-hard.

  Yes No

- If $B$ is regular, then $A$ is regular.

  Yes No

- If $B$ is regular, then $A$ is decidable.

  Yes No

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(a) For each statement below, check "YES" if the statement is ***always*** true and "NO" otherwise, and give a ***brief*** (at most one short sentence) explanation of your answer. **Assume $P \neq NP$.** If there is any other ambiguity or uncertainty about an answer, write "NO".

- The solution to the recurrence $T(n) = 8T(n/2) + O(n^2)$ is $T(n) = O(n^2)$.

  Yes No ________________________________

- The solution to the recurrence $T(n) = 2T(n/8) + O(n^2)$ is $T(n) = O(n^2)$.

  Yes No ________________________________

- Every directed acyclic graph contains at least one sink.

  Yes No ________________________________

- Given *any* undirected graph $G = (V, E)$, we can compute a spanning tree of $G$ in $O(V + E)$ time using depth-first search.

  Yes No ________________________________

- Suppose $A[1..n]$ is an array of integers. Consider the following recursive function:

$$What(i,j) = \begin{cases} 0 & \text{if } i < 0 \text{ or } i > n \\ 0 & \text{if } j < 0 \text{ or } j > n \\ \max \begin{Bmatrix} What(i, j-1) \\ What(i-1, j) \\ A[i] \cdot A[j] + What(i+1, j+1) \end{Bmatrix} & \text{otherwise} \end{cases}$$

  We can memoize this function into an array $What[0..n, 0..n]$ in $O(n^2)$ time, by increasing $i$ in the outer loop and increasing $j$ in the inner loop.

  Yes No ________________________________

*Problem 2 continues on the next page.*

(b) Consider the following pair of languages:

- DirHamPath := $\{G \mid G$ is a directed graph with a Hamiltonian path$\}$
- Acyclic := $\{G \mid G$ is a directed acyclic graph$\}$

(For concreteness, assume that in both of these languages, graphs are represented by their adjacency matrices.) Which of the following statements are true, assuming $P \neq NP$?

- Acyclic $\in$ NP

  Yes No ______________________________________

- Acyclic $\cap$ DirHamPath $\in$ P

  Yes No ______________________________________

- DirHamPath is decidable.

  Yes No ______________________________________

- A polynomial-time reduction from DirHamPath to Acyclic would imply P=NP.

  Yes No ______________________________________

- A polynomial-time reduction from Acyclic to DirHamPath would imply P=NP.

  Yes No ______________________________________

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| **Practice Final 1 Problem 3** | |

(See the questions handout for the problem description.)

(a) Consider the following greedy strategy: pick a pair $(a,b) \in A \times B$ maximizing $d(a,b)$, remove $a$ from $A$ and $b$ from $B$, and recursively pick the remaining pairs. Show that this greedy strategy does not always correctly find an optimal solution. (Your counterexample may be given in the form of a picture.)

(b) Let $a$ be the leftmost point in $A$ (i.e., with the smallest $x$-coordinate). Let $b$ be the rightmost point in $B$ (i.e., with the largest $x$-coordinate). Prove that any optimal pairing $T^*$ must use the pair $(a,b)$.

*[Hint: You may use the following geometric fact: for any 4 distinct points $s,t,u,v$ on a circle in counterclockwise order, $d(s,u)+d(t,v) > d(t,u)+d(s,v)$.]*

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Suppose you are asked to tile a $2 \times n$ grid of squares with dominos ($1 \times 2$ rectangles). Each domino must cover exactly two grid squares, either horizontally or vertically, and each grid square must be covered by exactly one domino.

Each grid square is worth some number of points, which could be positive, negative, or zero. The ***value*** of a domino tiling is the sum of the points in squares covered by vertical dominos, ***minus*** the sum of the points in squares covered by horizontal dominos.

Describe an algorithm to compute the largest possible value of a domino tiling of a given $2 \times n$ grid. Your input is an array $Points[1\,..\,2, 1\,..\,n]$ of point values.

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| CS/ECE 374 A ✧ Fall 2025<br>Practice Final 1 Problem 5 | Name: |
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***Prove*** that the following problem (which we call MATCH) is NP-hard. The input is a finite set $S$ of strings, all of the same length $n$, over the alphabet $\{0, 1, 2\}$. The problem is to determine whether there is a string $w \in \{0, 1\}^n$ such that for every string $s \in S$, the strings $s$ and $w$ have the same symbol in at least one position.

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| CS/ECE 374 A ✧ Fall 2025<br>Practice Final 1 Problem 6 | Name: |
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Describe and analyze an algorithm to determine whether the language accepted by a given DFA is finite or infinite. You can assume the input alphabet of the DFA is {0,1}. *[Hint: DFAs are directed graphs.]*

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| CS/ECE 374 A ✧ Fall 2025<br>Practice Final 1 Problem 7 | Name: |
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(a) Let $L_a$ denote the set of all strings in $\{0, 1\}^*$ where every 0 is followed immediately by at least one 1. Describe a DFA or NFA that accepts $L_a$ ***and*** give a regular expression that describes $L_a$. (You do not need to prove that your answers are correct.)

(b) Let $L_b$ denote the set of all strings in $\{0, 1\}^*$ whose run lengths are increasing; that is, every run except the last is followed immediately by a *longer* run. ***Prove*** that $L_b$ is not a regular language.

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(overflow / scratch paper)

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**Some useful NP-hard problems.** You are welcome to use any of these in your own NP-hardness proofs, except of course for the specific problem you are trying to prove NP-hard.

**3Sat:** Given a boolean formula in conjunctive normal form, with exactly three distinct literals per clause, does the formula have a satisfying assignment?

**IndependentSet:** Given an undirected graph $G$ and an integer $k$, is there a subset of $k$ vertices in $G$ that have no edges among them?

**Clique:** Given an undirected graph $G$ and an integer $k$, is there a subset of $k$ vertices in $G$ where there is an edge between every pair of them?

**VertexCover:** Given an undirected graph $G$ and an integer $k$, is there a subset of $k$ vertices that touch every edge in $G$?

**DominatingSet:** Given an undirected graph $G$ and an integer $k$, is there a subset of $k$ vertices such that every vertex of $G$ is either in the set or adjacent to a member of the set?

**SetCover:** Given a collection of subsets $S_1, S_2, \ldots, S_m$ of a set $S$ and an integer $k$, is there a subcollection of $k$ of these subsets whose union is $S$?

**HittingSet:** Given a collection of subsets $S_1, S_2, \ldots, S_m$ of a set $S$ and an integer $k$, is there a subset of $S$ of size $k$ that intersects every subset $S_i$?

**3Color:** Given an undirected graph $G$, can its vertices be colored with three colors so that every edge touches vertices with two different colors?

**HamiltonianPath:** Given graph $G$ (either directed or undirected), is there a path in $G$ that visits every vertex exactly once?

**HamiltonianCycle:** Given a graph $G$ (either directed or undirected), is there a cycle in $G$ that visits every vertex exactly once?

**TravelingSalesman:** Given a graph $G$ (either directed or undirected) with weighted edges and a number $L$, is there a Hamiltonian path/cycle of weight at most $L$ in $G$?

**LongestPath:** Given a graph $G$ (either directed or undirected, possibly with weighted edges) and a number $L$, is there a simple path of length at least $L$ in $G$?

**SteinerTree:** Given an undirected graph $G$ with some of the vertices marked and an integer $k$, is there a subtree of $G$ with at most $k$ edges that contains every marked vertex?

**SubsetSum:** Given a set $X$ of positive integers and an integer $k$, does $X$ have a subset whose elements sum to $k$?

**Partition:** Given a set $X$ of positive integers, can $X$ be partitioned into two subsets with the same sum?

**3Partition:** Given a set $X$ of $3n$ positive integers, can $X$ be partitioned into $n$ three-element subsets, all with the same sum?

**Draughts:** Given an $n \times n$ international draughts configuration and an integer $k$, is there a move that can (and therefore must) capture at least $k$ pieces in a single move?