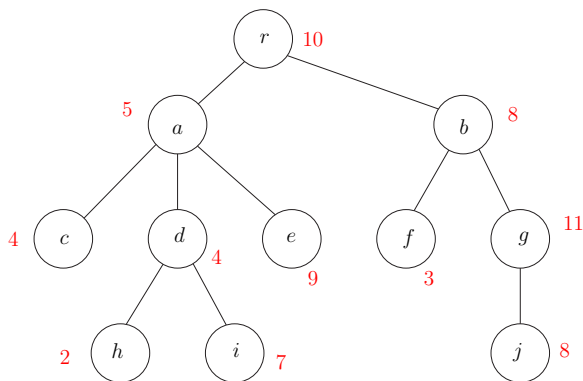


- Given a graph $G = (V, E)$, a *vertex cover* of G is a subset $S \subseteq V$ of vertices such that for each edge $e = (u, v)$ in G , u or v is in S . That is, the vertices in S cover all the edges. Given a tree $T = (V, E)$ and a non-negative weight $w(v)$ for each vertex $v \in V$, give an algorithm that computes the minimum weight vertex cover of T ; the weight of a cover S is the sum of the weights of the vertices in S . In the tree below, $\{a, d, b, j\}$ is a vertex cover while $\{r, a, f, g\}$ is not a vertex cover.



Run your iterative algorithm on the tree and compute the value of the minimum vertex cover in the tree above.

- A **basic arithmetic expression** is composed of characters from the set $\{1, +, \times\}$ and parentheses. Almost every integer can be represented by more than one basic arithmetic expression. For example, all of the following basic arithmetic expression represent the integer 14:

$$\begin{aligned}
 &1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 \\
 &((1 + 1) \times (1 + 1 + 1 + 1 + 1)) + ((1 + 1) \times (1 + 1)) \\
 &(1 + 1) \times (1 + 1 + 1 + 1 + 1 + 1 + 1) \\
 &(1 + 1) \times (((1 + 1 + 1) \times (1 + 1)) + 1)
 \end{aligned}$$

Describe and analyze an algorithm to compute, given an integer n as input, the minimum number of 1's in a basic arithmetic expression whose value is equal to n . The number of parentheses doesn't matter, just the number of 1's. For example, when $n = 14$, your algorithm should return 8, for the final expression above. The running time of your algorithm should be bounded by a small polynomial function of n .

- To think about later:** Suppose you are given a sequence of integers separated by + and - signs; for example:

$$1 + 3 - 2 - 5 + 1 - 6 + 7$$

You can change the value of this expression by adding parentheses in different places. For example:

$$\begin{aligned}
 &1 + 3 - 2 - 5 + 1 - 6 + 7 = -1 \\
 &(1 + 3 - (2 - 5)) + (1 - 6) + 7 = 9 \\
 &(1 + (3 - 2)) - (5 + 1) - (6 + 7) = -17
 \end{aligned}$$

Describe and analyze an algorithm to compute, given a list of integers separated by + and - signs, the maximum possible value the expression can take by adding parentheses. Parentheses must be used only to group additions and subtractions; in particular, do not use them to create implicit multiplication as in $1 + 3(-2)(-5) + 1 - 6 + 7 = 33$.