NFA → DFA

Thm If L is accepted by some NFA M, then L is accepted by some DFA M'.

Pf: idea - remember a subset of states

Given NFA $M = (Q, \Sigma, s, \delta, A)$, $\delta: Q \times (\Sigma \cup \{\varepsilon\}) \to 2^Q$

Construct DFA $M' = (Q', \Sigma, s', \delta', A')$

$\delta': Q' \times \Sigma \to Q'$

where $Q' = 2^Q$ (power set $\mathcal{P}(Q)$) ($= \{$all subsets of $Q\}$)

$s' = \varepsilon\text{-reach}(s)$.

$A' = \{ S \in Q' : S \cap A \neq \emptyset \}$

$$\delta'(S, a) = \bigcup_{q \in S} \delta^*(q, a) \qquad \forall S \in Q' \quad \forall a \in \Sigma$$

called subset construction (or power-set construction)

$\Downarrow$

Lemma

$$\delta'^*(S, x) = \bigcup_{q \in S} \delta^*(q, x) \qquad \forall S \in Q' \quad \forall x \in \Sigma^*$$

Pf: by induction (skipped). $\square$

(Then $x \in L(M') \underset{\substack{\text{def of} \\ \text{accept of} \\ \text{DFA}}}{\iff} \delta'^*(s', x) \in A'$

$\iff \bigcup_{q \in \varepsilon\text{-reach}(s)} \delta^*(q, x) \in A'$

$$\iff \delta^*(s,x) \in A'$$

$$\underset{\text{def of } A'}{\iff} \delta^*(s,x) \cap A \neq \emptyset$$

$$\underset{\text{def of accept in NFA}}{\iff} x \in L(M).$$

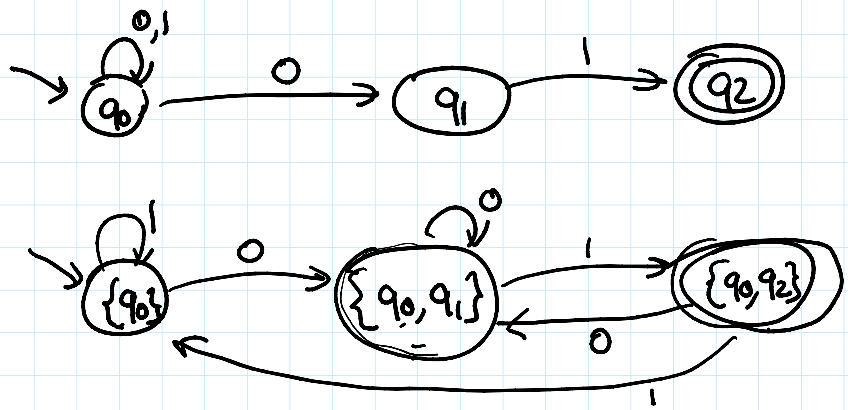
$$L(M') = L(M). \qquad \square$$

Ex1

NFA

$\Downarrow$

DFA



$$\delta'(\{q_0,q_1\},0) = \delta(q_0,0) \cup \delta(q_1,0)$$
$$= \{q_0,q_1\} \cup \emptyset = \{q_0,q_1\}.$$

$$\delta'(\{q_0,q_1\},1) = \delta(q_0,1) \cup \delta(q_1,1)$$
$$= \{q_0\} \cup \{q_2\} = \{q_0,q_2\}$$

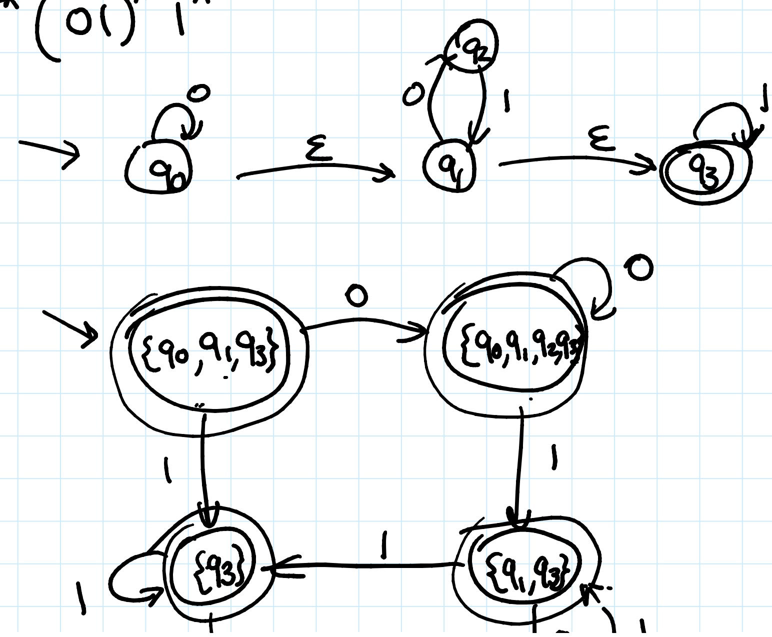
Ex2

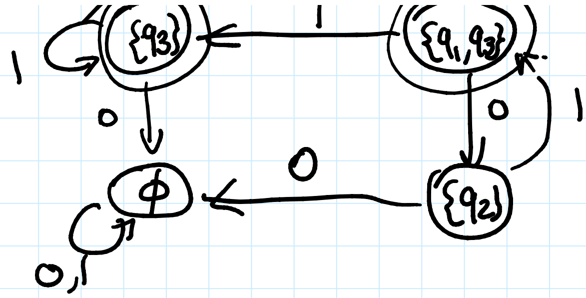
$$0^*(01)^*1^*$$

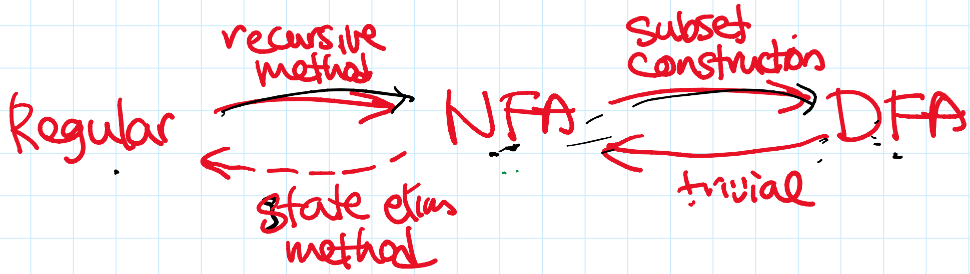
NFA

$\Downarrow$

DFA







NFA → Regular

Thm If L is accepted by NFA M then L is regular.

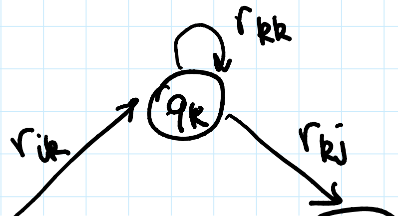
Pf. Sketch: Given $M = (Q, \Sigma, s, \delta, A)$

$Q = \{q_0, q_1, \ldots, q_{n-1}\}$

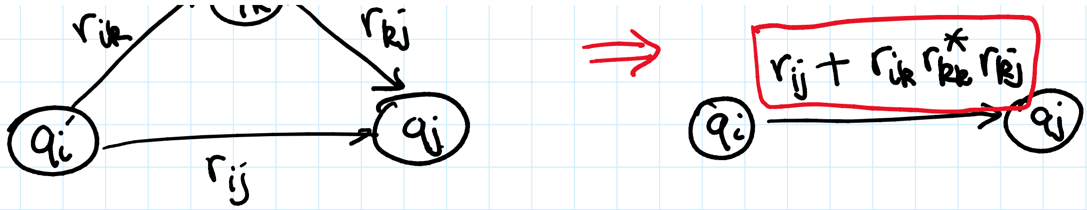
state elimination method

0. add new state s', f' with ε-transition from s' to s & from A to f

1. for $k = n-1$ to 0:
   remove q_k & apply the rule below:

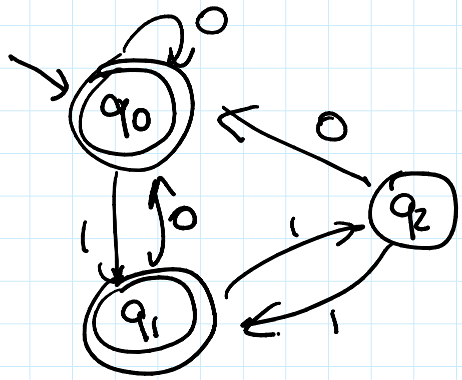


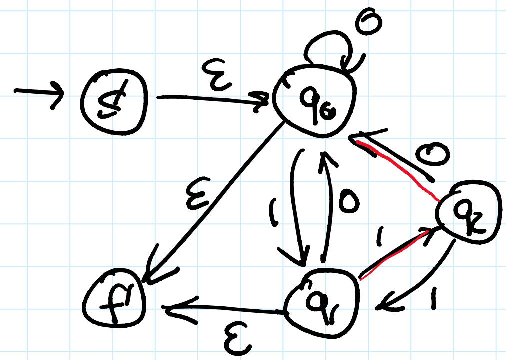
$\Rightarrow$ $\boxed{r_{ij} + r_{ik} r_{kk}^* r_{kj}}$



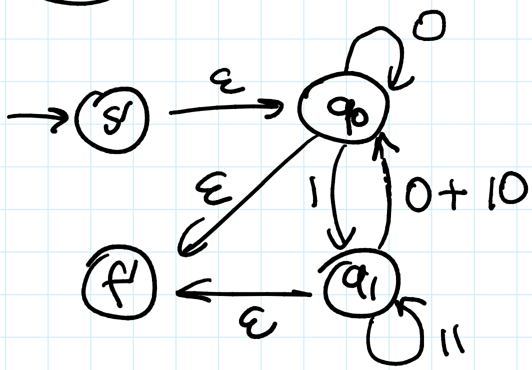
2. return label from s' to f'. □

Ex

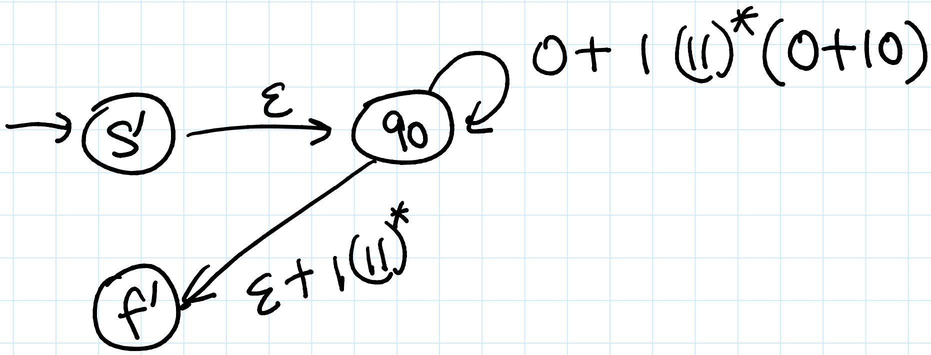




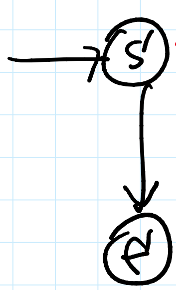
remove q_2:



remove q_1:



remove q_2:



$$\boxed{\left(0+1(11)^*(0+10)\right)^*\left(\varepsilon+1(11)^*\right)}.$$

Kleene's Thm (1956)

L is regular iff L is accepted by some DFA.

Corollaries

If L is regular, then $\bar{L}$ is also regular.

If L_1, L_2 are regular, then so is $L_1 \cap L_2$.

If L is accepted by some DFA, then so is L^*.

is L^R.

many closure props . . .