January 16, 2024 12:33 AM

CS 374 A: Intro to Algorithms & Models of Computation

https://courses.grainger.illinois.edu/cs374al1/sp2024

Instructors: Timothy Chan & Ruta Mehta

8 TAs, 22 CAs

HWs: 11 guided problem sets (GPSs) on Prairielearn (autograded)

+ 11 written homeworks, each with 2 problems (may work in groups of ≤ 3)

⇒ total = 33 HW problems (drop lowest 6)

No late GPS

HW: ≤ 24 hr late with 40% penalty (zero afterwards)

extenuating circumstances e.g. illness ⇒ ask instructor

Exams

Midterm 1 : Feb 19 Mon 7p-9p
Midterm 2 : Apr 8 Mon 7p-9p (conflict: TBA)
Final: TBA

Grades

HW	28%
Mid 1	21%
Mid 2	21%
Final	30%

fixed cut-offs for letter-grade
(Curved cut-offs as backup take better of two)
see web pages

Note: Sections A & B are completely independent

Resources: Jeff's book + lecture scribbles

To do well: attend all lectures (ask Qs!)
& labs
get help during OHs, ~~piazza~~ Ed Discord, ..., etc.

Overview

Introduction to CS theory

Goal 1: how to solve problems (efficiently)
← design algorithms & analysis

Goal 2: how to show that a problem can't be solved (efficiently)

mathematically prove

Outline

Part I. Models of Computation

- finite automata ⟷ reg exprs
- context-free grammar
- Turing machine

Part II. Algorithm Design Techniques

- divide & conquer
- dynamic programming *
- greedy
- graph algorithms

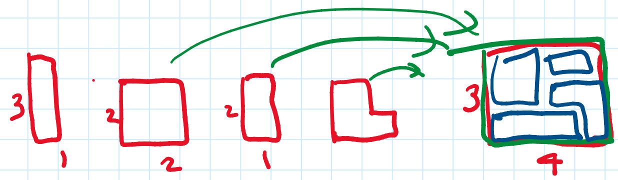
Part III. NP-Completeness * (& Undecidability)

Ex1 Given n numbers, do there exist 3 numbers summing to $\underline{100}$? "3SUM problem"

e.g. 81, $\underline{43}$, 95, 20, $\underline{32}$, 74, $\underline{25}$

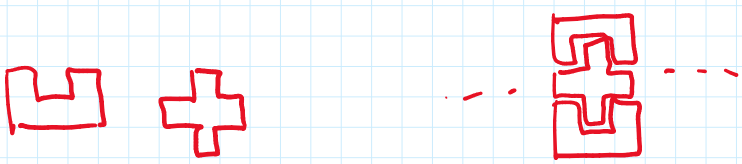
brute force: $O(n^3)$ time
smarter alg'm: $O(n^2)$ time.
fastest? OPEN $\sim O\left(\frac{n^2}{(\log n)^2}\right)$ [C'18]

Ex2 Given n polygons & rectangle, can they be packed in rectangle?



no efficient alg'm believed to be possible ("NP-complete")

Ex3 Given n polygons, can they tile the entire plane?



"undecidable"

Part I. Models of Computation

Math Preliminaries

Strings A $\underline{\text{string}}$ is a finite sequence of symbols from a finite set Σ

Strings

A **string** is a finite sequence of symbols from a finite set Σ ← alphabet

why important: input to program / machine

e.g. strings over $\Sigma = \{0,1\}$

1001, 01, 010, 1

Let ε denote the empty string

Let Σ^* denote {all strings over Σ}.

Let x, y be strings.

a) length $|x|$ e.g. $|1001| = 4$, $|\varepsilon| = 0$.

b) concatenation xy

e.g. $x = 10$, $y = 0110$, $\Rightarrow xy = 100110$

$$(xy)z = x(yz) \qquad xy \neq yx$$

$$|xy| = |x| + |y| \leftarrow$$

$$\varepsilon x = x\varepsilon = x$$

c) ith power $x^i = \underbrace{xx\cdots x}_{i \text{ times}}$

e.g. $(010)^3 = 010010010$

$$|x^i| = i|x|$$

$$x^i = x \cdot x^{i-1}$$

$$x^0 = \varepsilon$$

d) x is a substring of y if

$y = wxz$ for some strings w, z

(prefix if $w = \varepsilon$, suffix if $z = \varepsilon$)

e) other ops: x^R = reverse of x

(can be defined recursively.

$$x^R = \begin{cases} \varepsilon & \text{if } x = \varepsilon \\ y^R a & \text{if } x = ay \text{ for some } a \in \Sigma, y \in \Sigma^* \end{cases}$$

$$(xy)^R = y^R x^R$$

Languages

A language is a set of strings L

i.e. $L \subseteq \Sigma^*$.

all decision problems

e.g. $\{100, 01, 101, 0\}$

boring! finite

{ all words in English dictionary } over $\Sigma = \{'a', .., 'z'\}$.

$\{x \in \{0,1\}^* : |x| \text{ is odd}\}$

{ all syntatically valid programs in python }.

{ all prime numbers written in binary }

Let L_1, L_2 be languages.

a) union $L_1 \cup L_2$.

intersection $L_1 \cap L_2$.

complement $\overline{L_1}$ $(= L_1^c)$ $(= \Sigma^* \setminus L_1)$

difference $L_1 \setminus L_2 = L_1 \cap \overline{L_2}$

b) concatenation

$$L_1 L_2 = \{xy : x \in L_1, y \in L_2\}.$$

e.g. $L_1 = \{0, 00\}$, $L_2 = \{1, 01\}$

e.g. $L_1 = \{0, 00\}$, $L_2 = \{1, 01\}$

$L_1 L_2 = \{01, 0001, 001\}$

e.g. $L_1 = \{0, 00, 000, \dots\} = \{0^i : i \geq 1\}$

$L_2 = \{1, 11, 111, \dots\} = \{1^i : i \geq 1\}$

$L_1 L_2 = \cancel{\{0^i 1^i : i \geq 1\}}$?

$= \{0^i 1^j : i, j \geq 1\}$.